A Large Central Bank Balance Sheet? The Role of Interbank Market Frictions*

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Abstract

Recent quantitative easing (QE) policies implemented over the course of the Great Recession by the major central banks have had a profound impact on the working of money markets, giving rise to large excess reserves and pushing down key interbank rates against their floor –the interest rate on reserves. With macroeconomic fundamentals improving, central banks now face the dilemma as to whether to maintain this large balance sheet/floor system, or else to reduce balance sheet size towards pre-crisis trends and operate traditional corridor systems. We address this issue using a relatively simple New Keynesian model with two distinct features: heterogeneous banks that trade funds in an interbank market, and matching frictions in the latter market. We show that a large balance sheet allows for ampler 'policy space' by widening the average distance between the interest on reserves and its effective lower bound. Nonetheless, a lean-balance-sheet regime that resorts to temporary QE in response to recessions severe enough for the lower bound to bind achieves similar stabilization and welfare outcomes as a large-balance-sheet regime in which interest-rate policy is the primary adjustment margin thanks to the larger policy space. At the same time, the effectiveness of QE through the channel we model is limited. In line with the empirical evidence, the marginal effect vanishes as the balance sheet becomes very big.

Keywords: central bank balance sheet, interbank market, search and matching frictions, reserves, zero lower bound:.

JEL codes: E20, E30, G10, G20

*The views expressed in this manuscript are those of the authors and do not necessarily represent the views of the European Central Bank or Banco de España. All remaining errors are ours.

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1 Introduction

The financial crisis and the ensuing Great Recession forced central banks across the industrialized world to put in place quantitative easing (QE) policies that led to a massive increase in the size of their balance sheets. On the liabilities side, balance sheet expansion has taken the form of an unprecedented increase in aggregate excess reserves. In turn, such an increase in excess liquidity has put downward pressure on overnight interbank market rates, to the point that they have been pushed towards their floor – the interest paid on excess reserves by the respective central banks, which has thus become basically the sole determinant of such interbank rates in recent years. This 'large balance sheet regime' or 'floor system' represents a change in paradigm as regards the conduct of monetary policy vis-à-vis the one prevailing before the crisis, characterized by relatively small central bank balance sheets, near-zero aggregate excess reserves, and interbank rates in between the interest rates paid and charged by central banks on excess reserves and on their marginal lending, respectively ('lean balance sheet regime', or 'corridor system').

Figure 1 illustrates these developments for the case of the euro area. Before the crisis, the EONIA – the main index of interest rates on overnight loans in the euro area interbank market – remained very close to the middle of the corridor formed by the interest rates of the ECB's deposit and marginal lending facilities. Also, reserves in excess of regulatory requirements were negligible. Following the first large-scale liquidity injections put in place in the context of the recent crisis, the EONIA shifted towards the lower bound of the interest rate corridor, i.e. the deposit facility rate, as excess reserves scaled up to historical highs at that time. The launch of the large-scale asset purchase program (APP) in February 2015 consolidated the new large-balance-sheet, floor system regime.

As macroeconomic fundamentals slowly but steadily improve across many advanced economies, monetary policy-makers now face the dilemma as to whether to reduce the size of their balance sheets towards pre-crisis trends and return to the corridor system, or else whether to continue operating under the current floor system. This issue has drawn much attention in recent times both in academia and policy circles.[1] However, formal analyses in the context of well-suited theoretical models are relatively scarce.[2]

In this paper, we propose a relatively simple general equilibrium model designed to compare the stabilization and welfare properties of (a) the pre-crisis lean balance sheet regime and (b) the post-crisis floor system with a large balance sheet. Our framework departs from the standard New Keynesian DSGE model in two key dimensions. First, in order to motivate the existence of an interbank market, we introduce banks that collect deposits from households and have the possibility of lending to nonfinancial firms. Banks receive idiosyncratic shocks to the return that

[1] See e.g. Bernanke (2016) and Bullard (2017).

[2] See the related literature below.

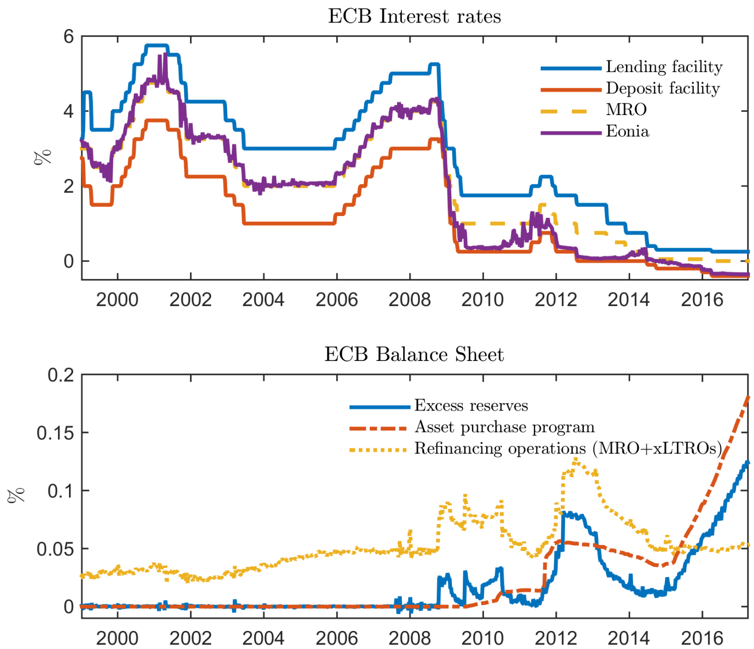

Figure 1: This figure shows the ECB interest rates and its balance sheet since the introduction of the Euro. Excess reserves are both excess reserves in current accounts as well as deposits at the deposit facility.

they can expect from the latter investment. As a result, some banks will endogenously choose to borrow in the interbank market – subject to a leverage constraint – so as to finance lending to firms, and some others will choose to lend in the same market or to hold government bonds.[3]

Second, following a recent literature, we capture the bilateral trading nature of the interbank market by assuming that the latter is characterized by search and matching frictions.[4] Every period, lending and borrowing banks search for each other and, upon matching, negotiate the interbank loan rate, with the interest rates on the central bank's deposit and lending facilities as the outside option for the lending and borrowing bank, respectively. As a result, the agreed interbank rate falls inside the interest rate corridor; its actual position within the latter is determined by the effective bargaining power of borrowers and lenders, which in turn depends on the relative tightness of the interbank market. In this setup, bank reserves are therefore the residual funds that lending banks are not able to place in the interbank market.

In the model, the size of the central bank's balance sheet plays a key role in determining

[3] In particular, our modelling of banks shares many features with Buera and Moll's (2015) modelling of entrepreneurs in a real framework, where the latter receive iid idiosyncratic shocks to the future return on their investments and can borrow from other entrepreneurs subject to an exogenous leverage constraint.

[4] See e.g. Afonso and Lagos (2015), Armenter and Lester (2017), Atkeson, Eisfeldt and Weill (2015), Bech and Monnet (2016), and Bianchi and Bigio (2014). Our modelling of search frictions in the interbank market follows Bianchi and Bigio (2014) closely.

outcomes in the interbank market. On the assets side, the central bank purchases long-term government bonds and, through its lending facility, provides funding to borrowing banks that fail to find lenders in the interbank market. Its liabilities are banks' reserves at the deposit facility. An expansion of the central bank balance sheet through bond purchases produces *ceteris paribus* a symmetric fall in banks' bond holdings and a corresponding increase of the amount of funds available for lending to other banks. The resulting slackening in the interbank market improves the bargaining position of borrowing banks and compresses the spread between the interbank rate and the deposit facility rate. Moreover, the same slackening implies that lending banks find it harder to find suitable trading partners and are thus forced to keep a larger proportion of their excess funds at the deposit facility. In equilibrium, an expansion in the monetary authority's asset holdings translates basically one for one into an increase in reserves. After calibrating our model to the euro area, we show that it replicates well the relationship between excess reserves and the spread between the interbank and deposit facility rates since 1999, including both the pre-crisis period –with basically zero excess reserves and a stable spread around 1%– and the recent period characterized by large excess reserves and a near-zero spread.

As mentioned before, our main interest is to compare the equilibrium and welfare properties of both monetary policy regimes. We first perform a comparative statics exercise in which we vary the permanent size of the central bank's balance sheet, and show that a larger balance sheet reduces steady-state welfare monotonically. The reason is that, in our model, neither central bank asset purchases per se, nor the reserves resulting from these purchases fulfill any socially useful role. This is so because, on the one hand, assets purchases per se do not affect any frictions, while, on the other hand, reserves are no more than the residual store of value of those banks that do not have sufficiently profitable investment opportunities in the real economy and cannot find suitable borrowers in the interbank market either. Moreover, because the central bank's interest rate corridor acts as a tax on the banking sector as a whole, larger bank reserves imply that the same corridor is more costly from a welfare perspective.[5] Nonetheless, the welfare losses from moving from a lean- to a large-balance-sheet steady state have a second-order magnitude.

The steady state analysis however does not inform on the usefulness of large central bank balance sheets under severe recessions in which interest-rate policy is constrained effective lower bound (ELB). To analyze this, we build a crisis scenario driven by an exogenous contraction in banks' leverage constraint, inspired by the financial origin of the Great Recession. The shock is deflationary enough to drive the central bank's deposit facility rate against its ELB, thus preventing

[5] Notice that this negative result contrasts the positive effects of a large central bank balance sheets found in the literature: In Gertler and Karadi (2013) asset purchases help circumventing leverage constraints; Curdia and Woodford (2011) assume that reserves provide payment services. While we do not wish to argue against the plausibility of these assumptions, we want to highlight the effects of balance sheet policies considered here stem from a different source.

further (standard) monetary accommodation for some time. We show that a temporary asset purchase program can reduce the severity of the recession through essentially the same mechanism explained above: by flushing the banking sector with a large amount of excess liquidity and thus strengthening the bargaining position of borrowing banks, the central bank drives interbank rates towards their floor – the deposit facility rate. Thus, ceteris paribus interbank rates fall and, since the latter are a key determinant of effective lending and borrowing rates for the real economy, both economic activity and welfare improve relative to the baseline scenario without QE. We refer to this novel mechanism as the interbank transmission channel of temporary balance-sheet policies.

We furthermore uncover two interesting properties of the effect of balance sheet policies through this channel. On the one hand, we show theoretically that the expansionary effect of balance sheet policies through the interbank transmission channel does not depend on the type of assets bought by the central bank. Whether the central bank provides liquidity by purchasing government bonds (like the ECB's public sector purchase program) or by providing loans to banks (as through the ECB's long term refinancing operations) is largely irrelevant. On the other hand, the model explains how the effectiveness of such measures depends on the total scale of the balance sheet. Starting off with a minimal balance sheet, a marginal balance sheet extension is very effective. As the balance sheet becomes large, this effectiveness goes towards zero quickly. Hence, the model explains the empirical finding in Reis (2016) that only the first QE measure in the US was effective and formalizes the economic reasoning provided there.

Finally, we show that a large-balance-sheet where interest-rate policy is the prime policy instrument to address the consequences of negative disturbances has very similar stabilization properties to a lean-balance-sheet regime where temporary balance-sheet expansions substitute for conventional policy when the latter is temporarily constrained by the ELB. As in standard DSGE models, in the steady state of our model the interest rate that determines households' consumption and saving decisions – here, the interest earned on their deposits – equals their rate of time preference, and is hence independent of monetary policy. Under a lean balance sheet, a steady-state *spread* exists between such 'natural' interest rate and the central bank's deposit facility rate, the size of which depends essentially on the width of the corridor and the relative bargaining power in the interbank market. By contrast, under a sufficiently large balance sheet, the steady-state deposit facility rate is essentially *equal* to the 'natural' rate; therefore, it is *higher* than in the lean-balance-sheet regime and *further away* from the ELB. In other words, a large balance sheet allows the conventional interest-rate policy to operate with more 'policy space' in the face of unforeseen events and is thus less constrained by the ELB. Numerically, we show that the financial crisis impacts similarly on activity and welfare whether the central bank operates a large balance sheets or keeps instead a lean balance sheet and responds to the crisis through a transitory bond purchase program.

Literature review [INCOMPLETE]. Our paper contributes to several strands of literature within the realm of DSGE models of monetary policy transmission. In analyzing the central bank's balance sheet as an instrument of monetary policy, we contribute to a by now large literature of which Gertler and Karadi (2011, 2013), Gertler and Kiyotaki (2010), and Cúrdia and Woodford (2011) are some prominent examples. We depart from these important contributions both in terms of modelling and in focus. As regards modelling, we depart from the above papers in several dimensions. Unlike in Gertler and Karadi (2011, 2013) or Cúrdia and Woodford (2011), we explicitly model the interbank market. Unlike in Gertler and Kiyotaki (2010), the interbank market in our framework emerges endogenously as a result of idiosyncratic shocks to the prospective return on banks' investments projects. This has the important implication that the fraction of borrowing and lending banks, and the relative amounts of interbank borrowing and lending orders, is endogenous to monetary policy, with the above-explained consequences for the position of the interbank rate within the central bank's corridor. In terms of focus, none of the above papers compares the pre-crisis corridor system with the current floor system in a macroeconomic model with both deposit and lending central bank facilities. Furthermore, the mechanism through which balance sheet policies have effects is fundamentally different. In the above contributions, such policies are effective either because they allow circumventing frictions in the financial system (direct lending), or due to portfolio preferences and by relaxing banks' leverage constraint (purchase of sovereign bonds). In our setup such policies are effective because, by increasing the supply of liquidity, they shift the equilibrium in the interbank market. Our welfare analysis of lean vs. large central bank balance sheets bears some resemblance with that in Cúrdia and Woodford (2011). In their model, it is optimal for the central bank to satiate the market for reserves by increasing their supply to a sufficiently high level. Key to their result is the fact that in their model reserves play a socially useful role by reducing banks' costs of providing loans.[6] By contrast, in our model reserves are the residual asset for those banks that face relatively unprofitable investment opportunities and also fail to find suitable borrowers in the interbank market. As explained before, welfare in the deterministic steady state decreases monotonically with the volume of excess reserves – though by a second order magnitude –, by increasing the implicit tax paid by the banking system as a whole associated to the central bank's corridor system.

In analyzing the transmission of interest-rate and balance-sheet policies in a dynamic model with an endogenous market for interbank loans characterized by matching frictions, our analysis is closely related to Bianchi and Bigio (2014). An important difference is how we motivate the existence of the interbank market. As mentioned before, the interbank market emerges in our framework as a result of heterogeneous investment opportunities across banks. In their model,

[6] Ireland (2014) proposes a New Keynesian model where banks' demand for reserves arises due to their role as an input in the production of banking services.

banks instead receive idiosyncratic withdrawal shocks which, coupled with mandatory reserve requirements, leads those banks with excess reserves to lend federal funds to those other banks with liquidity shortages. Also, we place our interbank market substructure into an otherwise standard New Keynesian DSGE model, which allows us to analyze the extent to which balance sheet policies complement conventional interest-rate policies. Finally, our papers largely differ in focus. Bianchi and Bigio (2014) use their framework to study quantitatively why banks have recently increased their reserve holdings but have not expanded lending despite policy efforts. By contrast, we focus on the comparison of the pre-crisis lean-balance-sheet, corridor-system regime with the current large-balance-sheet, floor-system regime (still) prevailing in the largest industrialized economies.

2 Model

Time is discrete. The economy is composed by 8 types of agents: intermediate-good firms, investment banks, retail banks, final-good producers, retailers, households, the central bank and the government. There is no aggregate uncertainty.

2.1 Households

The representative household's utility is

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[u(C_t) - v(L_t) \right],$$

where C_t is consumption and L_t is labor supply. In addition to consuming and supplying labor, households save in the form of deposits, the real value of which is denoted by D_t. They also build new capital goods K_t using the technology

$$K_t = \left[1 - S\left(\frac{I_t}{I_{t-1}} \right) \right] I_t + (1-\delta)\, \Omega_{t-1} K_{t-1},$$

where I_t are final goods used for investment purposes, and $(1-\delta)\,\Omega_{t-1}K_{t-1}$ is depreciated effective capital repurchased from firms after production in period t; in the latter term, δ is the depreciation rate and Ω_{t-1} is an effective capital index, to be defined below, which the household takes as given. The function S satisfies $S(1) = S'(1) = 0$ and $S''(1) \equiv \zeta > 0$. The budget constraint is

$$C_t + I_t + Q_t^K (1-\delta)\, \Omega_{t-1} K_{t-1} + D_t = W_t L_t + \frac{R_{t-1}^D}{P_t/P_{t-1}} D_{t-1} + Q_t^K K_t + \Pi_t^R - T_t,$$

where P_t is the aggregate price level, R_{t-1}^D is the riskless gross deposit rate, W_t is the real wage, Q_t^K is the real price of capital goods, Π_t^R are lump-sum real dividend payments from the household's ownership of retailers and T_t are lump-sum taxes. The first order conditions are

$$
\begin{aligned}
1 &= \Lambda_{t,t+1}\frac{R_t^D}{1+\pi_{t+1}}, &(1)\\
W_t &= \frac{v'(L_t)}{u'(C_t)},
\end{aligned}
$$

$$1 = Q_t^K\left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right] + \Lambda_{t,t+1}Q_{t+1}^K S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2, \tag{2}$$

where $\Lambda_{t,t+1} = \beta\frac{u'(C_{t+1})}{u'(C_t)}$ is the stochastic discount factor and $\pi_t \equiv P_t/P_{t-1} - 1$ is the inflation rate.

We assume that household may save in a non-modelled technology ('mattress') at a rate $-\kappa$, where κ is a positive constant. Therefore there is an effective lower bound (ELB) on the nominal interest rate on deposits:

$$R_t^D \geq 1 - \kappa.$$

2.2 Intermediate good firms

As in Kiyotaki and Moore (2008), we assume that firms are segmented across a continuum of 'islands', indexed by $j \in [0,1]$. The representative firm in island j produces units of the intermediate good, Y_t^j, according to a Cobb-Douglas technology,

$$Y_t^j = Z_t(\omega_{t-1}^j K_{t-1}^j)^\alpha (L_t^j)^{1-\alpha}, \tag{3}$$

where Z_t is an exogenous aggregate total factor productivity (TFP) process, L_t^j is labor, K_{t-1}^j is the pre-determined stock of installed capital, and ω_{t-1}^j is an island-specific shock to effective capital. These island-specific shocks are iid over time and across islands and their cumulative distribution function is denoted by $F(\omega)$. The mean of the distribution is assumed to be time-invariant and is normalized to one: $\int \omega dF(\omega) = 1$. The shock affecting capital efficiency at time t, ω_{t-1}^j, is known one period in advance, at the end of period $t-1$. At this point each firm needs to install capital on its island, which it buys from the household at price Q_{t-1}^K.[7] In order to finance this purchase, the firm turns to the investment bank on its island, who provides the firm with $Q_{t-1}^K K_{t-1}^j$ funds in return for A_{t-1}^j claims on the period-t cash flow, each of which has the price Q_{t-1}^K. The firm's

[7] The assumption that firms purchase (or repurchase) their entire capital stock each period is standard in the macro-finance literature (see e.g. Bernanke, Gertler and Gilchrist, 1999; Gertler and Karadi, 2011; Christiano, Motto and Rostagno, 2014). As explained by Bernanke, Gertler and Gilchrist (1999), this modeling device ensures, realistically, that leverage restrictions or other financial constraints apply to the constrained borrowers (in this case, the banks) as a whole, not just to the marginal investment.

balance sheet constraint at the end of period $t-1$ is thus $Q^K_{t-1}K^j_{t-1} = Q^K_{t-1}A^j_{t-1}$. In period t the firm then hires labor and produces.

Each firm j chooses labor in order to maximize operating profits, $P^Y_t Y^j_t - P_t W_t L^j_t$, subject to (3), where P^Y_t is the price of the intermediate good. The first order condition with respect to labor implies that the effective capital-labor ratio is equalized across islands,

$$\frac{\omega^j_{t-1}K^j_{t-1}}{L^j_t} = \left(\frac{W_t}{M_t\left(1-\alpha\right)Z_t}\right)^{1/\alpha}, \tag{4}$$

for all j, where $M_t \equiv P^Y_t/P_t$ is the inverse of the average gross markup of final goods prices over the intermediate good price, as explained below. The firm's nominal profits then equal

$$P^Y_t Y^j_t - P_t W_t L^j_t = P_t R^k_t \omega^j_{t-1}K^j_{t-1},$$

where

$$R^k_t \equiv \alpha M_t Z_t \left[\frac{\left(1-\alpha\right)M_t Z_t}{W_t}\right]^{(1-\alpha)/\alpha}$$

is the common return on effective capital. After production, the firm sells the depreciated effective capital $\left(1-\delta\right)\omega^j_{t-1}K^j_{t-1}$ to households at price Q^K_t. The total real cash flow from the firm's investment project equals the sum of operating profits and proceeds from the sale of depreciated capital,

$$R^k_t\omega^j_{t-1}K^j_{t-1} + \left(1-\delta\right)Q^K_t\omega^j_{t-1}K^j_{t-1} = \left[R^k_t + \left(1-\delta\right)Q^K_t\right]\omega^j_{t-1}K^j_{t-1}. \tag{5}$$

Since the capital purchase is financed entirely by state-contingent debt, the cash flow in (5) is paid off entirely to the lending banks.

2.3 Investment banks

In each island there exists a representative investment bank operated by a banker. Only the bank on island j has the technology to obtain perfect information about firms on that island, monitor them, and enforce their contractual obligations.[8] This effectively precludes firms from obtaining funding from other sources, including households or retail banks. As indicated before, banks finance firms investment in the form of perfectly state-contingent debt A^j_t. After production in period $t+1$, island j's firm pays the bank the entire cash flow from the investment project,

$$\left[R^k_{t+1} + \left(1-\delta\right)Q^K_{t+1}\right]\omega^j_t A^j_t = \frac{R^k_{t+1} + \left(1-\delta\right)Q^K_{t+1}}{Q^K_t}\omega^j_t Q^K_t A^j_t.$$

[8] The costs of these activities for the bank are assumed to be negligible.

The gross return on the bank's real assets $Q_t^K A_t^j$ is thus the product of an aggregate component,

$$R_{t+1}^A \equiv \frac{R_{t+1}^k + (1-\delta)\, Q_{t+1}^K}{Q_t^K},$$

and an island-specific component, ω_t^j. Besides investing into the local firm, the bank may borrow or lend funds in the interbank market by means of one-period contracts. B_t^j denotes the amount borrowed at t in real terms.[9] For each unit lent at the interbank market at the end of period t the bank receives a noncontingent nominal return R_t^L at period $t+1$, whereas each unit borrowed at t costs the bank the noncontingent gross nominal rate R_t^B at $t+1$. The borrowing and lending rates may differ and the bank takes these returns as given. We explain how they are determined below. In addition, the investment bank can purchase a positive amount $B_t^{j,G} \geq 0$ of nominal long-term treasury bonds. These bonds pay a coupon equal to a fraction ζ of the nominal value each period and decay at the same rate. They are traded at the nominal price Q_t^G. Hence, the nominal return in $t+1$ of a bond purchased at t is given by

$$R_{t+1}^G \equiv \frac{\zeta + (1-\zeta) Q_{t+1}^G}{Q_t^G}.$$

For simplicity we assume that the the other private agents can not trade these bonds.[10] We conjecture that in equilibrium $R_t^B \geq R_t^L = R_{t+1}^G$ and we verify this conjecture below.

Bank j's pre-dividend equity is given by the sum of the return on its investment in firms and government bonds minus the value of outstanding interbank debt. Denoting the banks pre dividend equity in real terms by E_t^j and defining the real market value of debt $b_t^{j,G} \equiv Q_t^G B_t^{j,G}/P_t$,[11] we can write this as:

$$E_t^j = R_t^A \omega_{t-1}^j Q_{t-1}^K A_{t-1}^j - \frac{B_{t-1}^j}{1+\pi_t}\left(\mathbf{1}_{B_{t-1}^j>0} R_{t-1}^B + \mathbf{1}_{B_{t-1}^j<0} R_{t-1}^L\right) + \frac{R_t^G}{(1+\pi_t)} b_{t-1}^{j,G}. \tag{6}$$

These funds can either be consumed as dividends (Π_t^j) or retained as post-dividend equity (N_t^j):

$$E_t^j = N_t^j + \Pi_t^j. \tag{7}$$

[9] Anticipating the equilibrium condition that the borrowing rate can not be smaller than the lending rate, and that it is hence not profitable to borrow and lend at the same time, we denote borrowing by B_{t-1}^j and lending by $-B_{t-1}^j$.

[10] This assumption is innocous under certainty. If we were to allow both retail banks and households to also trade government bonds, the individual portfolios would remain undetermined and the equilibrium would otherwise not be affected. However, this assumtion does matter when we consider unanticipated shocks later, because such shocks can lead to revaluation gains/losses. These effects disappear as the maturity of debt goes towards 1 period ($\zeta = 1$). Notice also, that this assumption places an upper bound on the exogenous amount of total sovereign debt, as we shall see later. This bound does not bind in our calibration.

[11] This definition allows us to write the model compactly and in terms of stationary variables.

The balance sheet constraint is thus

$$Q_t^K A_t^j + b_t^{j,G} = N_t^j + B_t^j. \tag{8}$$

Furthermore, investment banks face a capital constraint or equivalently a maximum leverage ratio

$$Q_t^K A_t^j \leq \phi N_t^j, \quad \phi > 1, \tag{9}$$

such that the maximum real debt is

$$B_t^j = Q_t^K A_t^j - N_t^j + b_t^{j,G} \leq (\phi - 1)\, N_t^j + b_t^{j,G}.$$

Notice that we are assuming that public debt is not capital constrained, and thus it does not affect the maximum leverage ratio in (9). However, as we will see below, the leverage constraint will in equilibrium be slack for those banks, who will choose to invest in sovereign debt. Therefore this assumption is innocuous.

The bankers problem hence consists of choosing paths for his consumption Π_t^j and his balance sheet N_t^j, A_t^j , B_t^j, and $b_t^{j,G}$ subject to the evolution of pre-dividend equity and the balance sheet and leverage constraints such as to maximize his lifetime utility function:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \widehat{\beta}^t \log(\Pi_t^j), \tag{10}$$

where $\widehat{\beta}$ is the banker's subjective discount factor.

The solution of the dynamic programming problem of the investment bankers is given by the following lemma, proved in the Appendix.

Lemma 1 (Banker's problem) *The solution to the banker's problem is given by a dividend policy*

$$\Pi_t^j = \left(1 - \widehat{\beta}\right) E_t^j, \tag{11}$$

a retained earnings policy

$$N_t^j = \widehat{\beta} E_t^j, \tag{12}$$

an asset demand

$$A_t^j = \mathbf{A}_t\left(N_t^j, \omega_t^j\right) = \begin{cases} \phi N_t^j / Q_t^K, & \textit{if } \omega_t^j \geq \omega_t^B \equiv \frac{R_t^B/(1+\pi_{t+1})}{R_{t+1}^A}, \\ N_t^j / Q_t^K, & \textit{if } \omega_t^L \leq \omega_t^j < \omega_t^B, \\ 0, & \textit{if } \omega_t^j < \omega_t^L \equiv \frac{R_t^L/(1+\pi_{t+1})}{R_{t+1}^A}, \end{cases} \tag{13}$$

a public debt demand

$$b_t^{j,G} = \mathbf{b}_t^G \left(N_t^j, \omega_t^j\right) = \begin{cases} 0, & \textit{if } \omega_t^j \geq \omega_t^B, \\ 0, & \textit{if } \omega_t^L \leq \omega_t^j < \omega_t^B, \\ b_t^{j,G} \in \left[0, N_t^j\right], & \textit{if } \omega_t^j < \omega_t^L, \end{cases} \tag{14}$$

and a demand for interbank loans

$$B_t^j = \mathbf{B}_t \left(N_t^j, \omega_t^j\right) = \begin{cases} (\phi - 1)\, N_t^j, & \textit{if } \omega_t^j \geq \omega_t^B, \\ 0, & \textit{if } \omega_t^L \leq \omega_t^j < \omega_t^B, \\ \left(b_t^{j,G} - N_t^j\right) \in \left[-N_t^j, 0\right], & \textit{if } \omega_t^j < \omega_t^L. \end{cases} \tag{15}$$

Thus, investment bankers pay a fixed share $1 - \widehat{\beta}$ of their net earnings as dividends and keep the rest on their balance sheet as equity. Furthermore, according to the value of the island-specific capital quality shock, they endogenously split into banks that borrow from the interbank market up to the limit to invest in the real economy, banks that only invest their own funds and banks that do not invest in real assets and only lend in the interbank market and to the government.[12] Notice finally that leveraged banks (those with $\omega_t^j \geq \omega_t^B$) do not purchase government bonds, which verifies our earlier claim that including such bonds in the leverage constraint or not is irrelevant.

2.4 Retail banks

Retails banks collect deposits from the households and to lend these funds out through the interbank market. Their real profits Π_t^{RB} are given by

$$\Pi_t^{RB} = \frac{1}{1+\pi_t} \left(R_{t-1}^L B_{t-1}^L - R_{t-1}^D D_{t-1}\right),$$

where B_t^L are the funds lent at the interbank market. The balance sheet constraint of retail banks is

$$D_t = B_t^L.$$

Assuming free-entry into the retail banking sector, profits are zero $\Pi_t^{RB} = 0$ and, as long as it is above the ELB, the deposit rate equals the interbank rates: $R_t^D = R_t^L$. If $R_t^L < 1 - \kappa$ then the retail banks stop accepting deposits: $D_t = 0$.

[12] Notice that the amount of public debt is undetermined at the individual level, but it will be determined at the aggregate levelo below.

2.5 The interbank market

The interbank market is a directed over-the-counter (OTC) market similar lo the ones in Bianchi and Bigio (2014) or Afonso and Lagos (2012). Banks (both investment and retail) who wish to lend can place a lending order whereas (investment) banks who wish to borrow can place a borrowing order. Orders are placed on a per-unit basis as in Atkeson et al. (2012). Orders are randomly matched.

Let $H_t(N, \omega)$ be the endogenous cumulative distribution function of net worth and island-specific shocks. Given (7), ω_t^j and N_t^j are distributed independently: $H_t(N, \omega) = G_t(N) F(\omega)$. The probability that a lending or borrowing order finds a match depends on the relative mass on each side of the market. We know from (15) that banks with $\omega_t^j \geq \omega_t^B$ borrow in the amount $(\phi - 1) N_t^j$ and those with $\omega_t^j < \omega_t^L$ lend in the amount N_t^j. Therefore, the mass of *borrowing* and *lending* orders are given respectively by

$$
\begin{aligned}
\Phi_t^B &\equiv \int_{[0,\infty)} \int_{\left(\omega \geq \omega_t^B\right)} (\phi - 1) N dH_t(N, \omega) = (\phi - 1) N_t \left[1 - F\left(\omega_t^B\right)\right], \\
\Phi_t^L &\equiv \int_{[0,\infty)} \int_{\left(\omega \leq \omega_t^L\right)} \left(N - \mathbf{b}_t^G(N, \omega)\right) dH_t(N, \omega) + B_t^L = F\left(\omega_t^L\right) N_t - b_t^G + B_t^L. \qquad (16)
\end{aligned}
$$

where $N_t \equiv \int_0^1 N_t^j dj$ is aggregate bank equity and $b_t^G \equiv \int_0^1 b_t^{j,G} dj$ are aggregate purchases of public debt by investment banks.

Matches in the interbank market are given by a matching function,

$$
\Upsilon\left(\Phi_t^L, \Phi_t^B\right).
$$

We assume that $\Upsilon_1, \Upsilon_2 \geq 0$. Assuming constant returns to scale, the probability that a borrowing order finds a lending order is given by

$$
\frac{\Upsilon\left(\Phi_t^L, \Phi_t^B\right)}{\Phi_t^B} = \Upsilon\left(\frac{\Phi_t^L}{\Phi_t^B}, 1\right) \equiv \Gamma_t^B, \qquad (17)
$$

and the probability that a lending order finds a borrowing order is

$$
\frac{\Upsilon\left(\Phi_t^L, \Phi_t^B\right)}{\Phi_t^L} = \Upsilon\left(1, \frac{\Phi_t^B}{\Phi_t^L}\right) \equiv \Gamma_t^L. \qquad (18)
$$

Banks use a multi-round Nash bargaining to split the surplus of the dollar transfer. Details are provided in the Appendix. In the bargaining problem that emerges, the outside option for the lending bank in the final round is to deposit the funds at the central bank, receiving the *deposit facility rate* R_t^{DF}, also known as the interest on reserves. For the borrowing bank, the final round outside option is to borrow at the marginal *lending facility rate* R_t^{LF}.[13] Banks bargain only about the (gross) *interbank rate*, which we denote by R_t^{IB}. If an order does not find a match, the bank borrows from (lends to) the central bank at rate R_t^{LF} (R_t^{DF}). The *effective borrowing rate* for a borrowing bank is thus

$$R_t^B = \Gamma_t^B R_t^{IB} + \left(1 - \Gamma_t^B\right) R_t^{LF}, \tag{19}$$

and the *effective lending rate* for a lending bank is

$$R_t^L = \Gamma_t^L R_t^{IB} + \left(1 - \Gamma_t^L\right) R_t^{DF}. \tag{20}$$

Let $\xi \in (0,1)$ denote the bargaining power of the borrowers during each negotiation of the multi-round bargaining process. We assume that, if a matched pair of banks fail to agree on the interest rate at a given round, then with probability $\vartheta \in (0,1)$ both banks search for a new trading partner, otherwise the lending bank deposits the dollar at the deposit facility and the borrowing bank borrows it from the lending facility. The bargaining problem has the following solution (the proof can be found in the Appendix):

Proposition 1 (Bargaining problem) *The interbank rate is given by*

$$R_t^{IB} = \varphi_t R_t^{DF} + (1 - \varphi_t) R_t^{LF}, \tag{21}$$

where φ_t *is*

$$\varphi_t = \frac{\xi \left(1 - \vartheta \Gamma_t^L\right)}{1 - \xi \vartheta \Gamma_t^L - (1 - \xi) \vartheta \Gamma_t^B}. \tag{22}$$

The solution for the interbank rate, equation (21), allows us to write the effective borrowing and lending rates as functions of the two policy rates (R_t^{DF}, R_t^{LF}) and the borrowers *effective* bargaining power φ_t,

$$R_t^B = \varphi_t \Gamma_t^B R_t^{DF} + \left[1 - \varphi_t \Gamma_t^B\right] R_t^{LF}, \tag{23}$$

$$R_t^L = (1 - \varphi_t) \Gamma_t^L R_t^{LF} + \left(1 - (1 - \varphi_t) \Gamma_t^L\right) R_t^{DF}. \tag{24}$$

Therefore, changes in the policy rates will affect borrowing and lending rates in the banking sector both directly as well as indirectly through the endogenous interbank matching probabilities Γ_t^B and Γ_t^L. Finally we assume that banks have access to the same storage technology as the household. This constrains the rates R_t^{DF}, R_{t+1}^G and R_t^L to be larger than $(1 - \kappa)$.

[13] We use the language of the institutional setting of the Eurosystem. In the case of the Federal Reserve system the lending facility rate would be the *discount rate* and the deposit facility rate would be the *interest rate on excess reserves.*

2.6 Final good producers

A competitive representative final good producer aggregates a continuum of differentiated retail goods indexed by $i \in [0,1]$ using a Dixit-Stiglitz technology,

$$Y_t = \left(\int_0^1 Y_{i,t}^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{25}$$

where $\epsilon > 1$ is the elasticity of substitution across retail goods. Cost minimization implies

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t} \right)^{-\epsilon} Y_t \equiv Y_t^d \left(P_{i,t} \right), \text{ where } P_t = \left(\int_0^1 P_{i,t}^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}. \tag{26}$$

Total spending in intermediate inputs then equals $\int_0^1 P_{i,t} Y_{i,t} di = P_t Y_t$. Free entry implies zero profits, such that the equilibrium price of the final good is exactly P_t.

2.7 Retail goods producers

We assume that the monopolistic competition occurs at the retail level. Retailers purchase units of the intermediate good firms, transform them one-for-one into retail good varieties, and sell these to final good producers. Each retailer i sets a price $P_{i,t}$ as in the sticky price model of Calvo (1983) taking as given the demand curve $Y_t^d \left(P_{i,t} \right)$ and the price of the intermediate good, P_t^y . Specifically, during each period a fraction of firms $(1-\theta)$ are allowed to change prices, whereas the other fraction, θ, do not change. Retailers that are able to change prices in period t choose a new optimal price in order to maximize its expected discounted stream of profits,

$$\max_{P_{i,t}} \sum_{k=0}^{\infty} \theta^k \mathbb{E}_t \left[\Lambda_{t,t+k} \left(\frac{P_{i,t}}{P_{t+k}} - M_{t+k} \right) \left(\frac{P_{i,t}}{P_{t+k}} \right)^{-\epsilon} Y_{t+k} \right]. \tag{27}$$

The solution is the usual price condition,

$$\sum_{k=0}^{\infty} \theta^k \mathbb{E}_t \Lambda_{t,t+k} \left(\frac{P_t^*}{P_{t+k}} - \frac{\epsilon}{\epsilon - 1} M_{t+k} \right) P_{t+k}^{\epsilon} Y_{t+k} = 0.$$

In the Calvo model, the price level evolves according to $P_t^{1-\epsilon} = \theta P_{t-1}^{1-\epsilon} + (1-\theta) \left(P_t^* \right)^{1-\epsilon}$. In terms of stationary variables, the latter two equations can be written as

$$p_t^* = \frac{\Xi_t^1}{\Xi_t^2}.$$

$$\Xi_t^1 \equiv \sum_{k=0}^{\infty} \theta^k \mathbb{E}_t \Lambda_{t,t+k} \left(\frac{P_{t+k}}{P_t}\right)^{\epsilon} \frac{\epsilon}{\epsilon-1} M_{t+k} Y_{t+k} = \frac{\epsilon}{\epsilon-1} M_t Y_t + \theta \mathbb{E}_t \Lambda_{t,t+1} \left(1+\pi_{t+1}\right)^{\epsilon} \Xi_{t+1}^1$$

$$\Xi_t^2 \equiv \sum_{k=0}^{\infty} \theta^k \mathbb{E}_t \Lambda_{t,t+k} \left(\frac{P_{t+k}}{P_t}\right)^{\epsilon-1} Y_{t+k} = Y_t + \theta \mathbb{E}_t \Lambda_{t,t+1} \left(1+\pi_{t+1}\right)^{\epsilon-1} \Xi_{t+1}^2$$

$$1 = \theta \left(1+\pi_t\right)^{\epsilon-1} + (1-\theta) \left(p_t^*\right)^{1-\epsilon}, \tag{28}$$

where $p_t^* \equiv P_t^*/P_t$.

2.8 Central Bank

The central bank sets the stance of monetary policy by controlling two nominal policy rates, the deposit facility rate R_t^{DF} and the marginal lending facility rate R_t^{LF}, and the real market value of its government bond holdings, $b_t^{G,CB}$. We assume that the policy rates are set such that (i) a constant corridor of width $\chi > 0$ is maintained and that (ii) the interbank rate, which is the central bank's operational target, achieves a certain target level. This target level is described by a conventional Taylor rule,

$$R_t^{IB^*} = \rho(R_{t-1}^{IB}) + (1-\rho)\left[\bar{R} + \upsilon\left(\pi_t - \bar{\pi}\right)\right], \tag{29}$$

where $\bar{R} \geq 1/\beta$ is the long-run nominal interbank rate, $\bar{\pi}$ is the inflation target, $\rho \in (0,1)$ is the persistence parameter, and $\upsilon > 1$ is the inflation coefficient. However, the central bank is constrained by the effective lower bound implied by the storage technology. Anticipating that in equilibrium $R_t^D = R_t^L \geq R_t^{DF}$, this implies that the central bank needs to ensure that $R_t^{DF} \geq 1-\kappa$. In case this constraint binds, the central bank is forced to tolerate temporary deviations from the target level $R_t^{IB^*}$. Combining the relationship between the central bank's policy rates and its operational target in equation (21), the condition that the central bank ensures $R_t^{IB^*} = R_t^{IB}$ whenever possible and the definition of the target level (29), we obtain that the central bank sets its policy rates according to the following rule:

$$R_t^{DF} = \max\left\{\rho(R_{t-1}^{DF} + \left(1-\varphi_{t-1}\right)\chi) + (1-\rho)\left[\bar{R} + \upsilon\left(\pi_t - \bar{\pi}\right)\right] - \left(1-\varphi_t\right)\chi, 1-\kappa\right\}, \tag{30}$$

$$R_t^{LF} = R_t^{DF} + \chi. \tag{31}$$

Furthermore we assume that the central bank's government bond holdings evolve according to the following rule,

$$b_t^{G,CB} = (1-\zeta)\, b_{t-1}^{G,CB} + \zeta \bar{b}^{G,CB} + np_t + \zeta(b_{t-1}^{G,CB} - \bar{b}^{G,CB})ri_t, \tag{32}$$

where np_t and ri_t are extraordinary real net purchases and extraordinary real reinvestment, which are generally zero. This rule says that, in the absence of extraordinary measures, the central bank keeps the real value of its bond portfolio fixed at $\bar{b}^{G,CB}$. The real net purchases give the central bank a tool to increase the balance sheet size in extraordinary times, while reinvestment allows the central bank to keep the balanced sheet fixed for a while after net purchases have been phased out.

The central bank's assets are government bonds $b_t^{G,CB}$ and loans to banks extended by its lending facility, i.e. the mass of borrowing orders that did not find matches in the interbank market,

$$\Phi_t^B \left(1 - \Gamma_t^B\right).$$

The central bank's liabilities are banks' deposits at its deposit facility, i.e. the mass of lending orders from both investment and retail banks that did not find matches in the interbank market,

$$\Phi_t^L \left(1 - \Gamma_t^L\right).$$

We assume that the central bank has no equity and that it pays all its profits to –or eventually collects all its losses from– the government. The *balance sheet* of the central bank is therefore

$$b_t^{G,CB} + \Phi_t^B \left(1 - \Gamma_t^B\right) = \Phi_t^L \left(1 - \Gamma_t^L\right). \tag{33}$$

Its real profits or losses are

$$\Pi_t^{CB} = \frac{1}{1+\pi_t} \left[R_t^G b_{t-1}^{G,CB} + R_{t-1}^{LF} \Phi_{t-1}^B \left(1 - \Gamma_{t-1}^B\right) - R_{t-1}^{DF} \Phi_{t-1}^L \left(1 - \Gamma_{t-1}^L\right)\right].$$

Finally, we define the *monetary base* as the nominal amount of reserves at the central bank,

$$M_t \equiv P_t \Phi_t^L \left(1 - \Gamma_t^L\right).$$

2.9 Government

The budget constraint of the government expressed in real terms is given by

$$\bar{b}_{t-1} \frac{R_t^G}{1+\pi_t} = \bar{b}_t + T_t + \Pi_t^{CB},$$

reflecting how the debt is serviced with lump-sum taxes and profits from the central bank.[14] Without loss of generality, we assume the government keeps the real market value of outstanding government debt constant at some level $\bar{b}$,

$$\bar{b}_t = \bar{b}.$$

2.10 Aggregation and market clearing

Before we close the model it is useful to use some simple no-arbitrage conditions to simplify the notation. Zero profits at the retail bank, together with the ELB, implies that the deposit rate is given by

$$R_t^D = \max\left\{R_t^L, 1-\kappa\right\}.$$

Also, since government bonds and interbank lending are perfect substitutes for investment banks, no arbitrage requires that the *ex-ante* yield on public debt equals the effective lending rate,[15]

$$R_t^L = R_{t+1}^G.$$

(Ex-post the above condition may fail to hold if the economy is hit by an unanticipated shock, and only on the impact period). The difference between the effective borrowing and lending rates is

$$\begin{aligned} R_t^B - R_t^L &= \left(R_t^{LF} - R_t^{DF}\right)\left[\left(1-\Gamma_t^B\right) + \varphi_t\left(\Gamma_t^B - \Gamma_t^L\right)\right] \\ &= \chi\left[1 - \left(\left(1-\varphi_t\right)\Gamma_t^B + \varphi_t\Gamma_t^L\right)\right] \geq 0.^{16} \end{aligned}$$

Aggregate net worth of investment banks after dividend payments in period t is $N_t = \int_0^1 N_t^j dj = \int N dH_t\left(N,\omega\right)$. Market clearing for capital requires that total demand by intermediate firms equals total supply by households, $K_{t-1} = \int_0^1 K_{t-1}^j dj = \int_0^1 A_{t-1}^j dj$. Given the optimal demand for assets A_{t-1}^j by investment banks (Lemma 1), and given that ω_t^j and N_t^j are independently distributed, we have

$$\begin{aligned} K_{t-1} &= \int \mathbf{A}_{t-1}\left(N,\omega\right) dH_{t-1}\left(\omega,N\right) \\ &= \frac{N_{t-1}}{Q_{t-1}^K}\left\{\phi\left[1-F\left(\omega_{t-1}^B\right)\right] + \left[F\left(\omega_{t-1}^B\right) - F\left(\omega_{t-1}^L\right)\right]\right\}. \end{aligned} \tag{34}$$

[14] This govenment's real budget constraint can be derived, using the definitions of b_t^G and R_t^G, from the nominal budget constraint: $\zeta\overline{B}_{t-1} = \overline{B}_t^{new} Q_t^G + P_t\left(T_t + \Pi_t^{CB}\right)$, where $\overline{B}_t^{new} = \overline{B}_t - (1-\zeta)\overline{B}_{t-1}$ is gross new bond issuance.

[15] Notice that if $R_t^L > R_{t+1}^G$ then lending investment banks would invest all their funds in the interbank market an not in public debt. Then bond markets would not clear. Analogously, if $R_t^L < R_{t+1}^G$ then lending investment banks would invest all their funds in public debt and nothing in the interbank market. We assume that the value of public debt is to small to allow the bank to satisfy this demand. This restriction holds for our calibration.

Total issuance of state-contingent claims by firms must equal total demand by banks, $K_t = A_t$. Using (4) to solve for firm j's labor demand L_t^j, we have that aggregate labor demand equals

$$\int_0^1 L_t^j dj = \left(\frac{(1-\alpha) Z_t M_t}{W_t}\right)^{1/\alpha} \int_0^1 \omega_{t-1}^j K_{t-1}^j dj.$$

It can be shown that

$$\begin{aligned}
\int_0^1 \omega_{t-1}^j K_{t-1}^j dj &= \int \omega \mathbf{A}_{t-1}(N,\omega)\, dH_{t-1}(N,\omega) \\
&= \frac{\phi N_{t-1}}{Q_{t-1}^K} \int_{\omega_{t-1}^B} \omega dF(\omega) + \frac{N_{t-1}}{Q_{t-1}^K} \int_{\omega_{t-1}^L}^{\omega_{t-1}^B} \omega dF(\omega) \\
&= \frac{\phi N_{t-1}}{Q_{t-1}^K} \left[1 - F\left(\omega_{t-1}^B\right)\right] \mathbb{E}\left(\omega \mid \omega \geq \omega_{t-1}^B\right) \\
&\quad + \frac{N_{t-1}}{Q_{t-1}^K} \left[F\left(\omega_{t-1}^B\right) - F\left(\omega_{t-1}^L\right)\right] \mathbb{E}\left(\omega \mid \omega_{t-1}^L \leq \omega < \omega_{t-1}^B\right),
\end{aligned}$$

where we have used the fact that ω_{t-1}^j and N_{t-1}^j are distributed independently. Using (34), we can express the above equation more compactly as

$$\int_0^1 \omega_{t-1}^j K_{t-1}^j dj = \Omega_{t-1} K_{t-1} \tag{35}$$

where

$$\Omega_{t-1} \equiv \frac{\phi\left[1 - F\left(\omega_{t-1}^B\right)\right] \mathbb{E}\left(\omega \mid \omega \geq \omega_{t-1}^B\right)}{\phi\left[1 - F\left(\omega_{t-1}^B\right)\right] + \left[F\left(\omega_{t-1}^B\right) - F\left(\omega_{t-1}^L\right)\right]} + \frac{\left[F\left(\omega_{t-1}^B\right) - F\left(\omega_{t-1}^L\right)\right] \mathbb{E}\left(\omega \mid \omega_{t-1}^L \leq \omega < \omega_{t-1}^B\right)}{\phi\left[1 - F\left(\omega_{t-1}^B\right)\right] + \left[F\left(\omega_{t-1}^B\right) - F\left(\omega_{t-1}^L\right)\right]} \tag{36}$$

is an index of capital efficiency.[17] Labor market clearing then requires, $L_t = \int_0^1 L_t^j dj$, or equivalently,

$$L_t = \left(\frac{(1-\alpha) Z_t M_t}{W_t}\right)^{1/\alpha} \Omega_{t-1} K_{t-1}. \tag{37}$$

Equations (34), (4) and (37) then imply that

$$\frac{\omega^j K_{t-1}^j}{L_t^j} = \frac{\Omega_{t-1} K_{t-1}}{L_t}. \tag{38}$$

[17] In the limiting case in which $\omega_{t-1}^B = \omega_{t-1}^L \equiv \bar{\omega}_{t-1}$, Ω_t collapses to $\mathbb{E}\left(\omega \mid \omega \geq \bar{\omega}_{t-1}\right)$.

Using $Y_t^j = Z_t(L_t^j/\omega^j K_{t-1}^j)^{1-\alpha}\omega^j K_{t-1}^j$ and equations (35, 38), aggregate supply of the intermediate good equals

$$\int_0^1 Y_t^j dj = Z_t \left(\frac{L_t}{\Omega_{t-1}K_{t-1}}\right)^{1-\alpha} \int_0^1 \omega_{t-1}^j K_{t-1}^j dj = Z_t L_t^{1-\alpha} \left(\Omega_{t-1}K_{t-1}\right)^{\alpha},$$

and aggregate demand of the intermediate good is[18]

$$\int_0^1 Y_t^j dj = Y_t \int_0^1 \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} dj = Y_t \Delta_t,$$

where $\Delta_t \equiv \int_0^1 \left(P_{i,t}/P_t\right)^{-\epsilon} dj$ is an index of relative price dispersion with law of motion

$$\Delta_t = (1-\theta)\left(p_t^*\right)^{-\epsilon} + \theta\left(1+\pi_t\right)^{\epsilon} \Delta_{t-1}.$$

Market clearing for the intermediate good thus requires

$$Y_t = \frac{Z_t}{\Delta_t} L_t^{1-\alpha} \left(\Omega_{t-1}K_{t-1}\right)^{\alpha}.$$

The aggregate real profit from retailers is

$$\begin{aligned} \Pi_t^R &= \int_0^1 \left(\frac{P_{i,t}}{P_t} - M_t\right) Y_t^i \left(P_{i,t}\right) di = \int_0^1 \left(\frac{P_{i,t}}{P_t} - M_t\right)\left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} Y_t di \\ &= Y_t \int_0^1 \left(\frac{P_{i,t}}{P_t}\right)^{1-\epsilon} di - M_t Y_t \int_0^1 \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} di = Y_t \left(1 - M_t \Delta_t\right). \end{aligned}$$

Furthermore, total supply of the final good must equal consumption and investment demand by households and investment bankers' dividends/consumption

$$Y_t = C_t + I_t + \frac{1-\beta}{\beta} N_t$$

and bond markets must clear

$$\bar{b}_t = b_t^{G,CB} + b_t^G.$$

[18] See, for example, Yun (2005).

Finally, we can aggregate (12) across banks (with E_t^j given by 6) to obtain

$$\begin{aligned}
\frac{N_t}{\widehat{\beta}} &= R_t^A Q_{t-1}^K \int_0^1 \omega_{t-1}^j A_{t-1}^j dj - \left(\frac{R_{t-1}^B}{1+\pi_t} \int_0^1 \mathbf{1}_{B_{t-1}^j>0} B_{t-1}^j dj + \frac{R_{t-1}^L}{1+\pi_t} \int_0^1 \mathbf{1}_{B_{t-1}^j<0} B_{t-1}^j dj \right) + \frac{R_t^G}{(1+\pi_t)} \int_0^1 b_{t-1}^{j,G} dj \\
&= R_t^A Q_{t-1}^K \Omega_{t-1} K_{t-1} - \left(\frac{(\phi-1) R_{t-1}^B}{1+\pi_t} \left[1 - F\left(\omega_{t-1}^B\right)\right] N_{t-1} + \frac{R_{t-1}^L}{1+\pi_t} \left[b_{t-1}^G - F\left(\omega_{t-1}^L\right) N_{t-1}\right] \right) + \frac{R_t^G}{(1+\pi_t)} b_{t-1}^G,
\end{aligned}$$

where we have used $A_{t-1}^j = K_{t-1}^j$ for all j, equation (35), and the fact that

$$\int_0^1 \mathbf{1}_{B_{t-1}^j>0} B_{t-1}^j dj = \int_0^1 \mathbf{1}_{B_{t-1}^j>0} \left((\phi-1) N_{t-1}^j\right) dj = \left[1 - F\left(\omega_{t-1}^B\right)\right] (\phi-1) N_{t-1},$$

and

$$\int_0^1 B_{t-1}^j \mathbf{1}_{B_{t-1}^j<0} dj = \int_0^1 \left(b_{t-1}^{j,G} - N_{t-1}^j\right) \mathbf{1}_{B_{t-1}^j<0} dj = b_{t-1}^G - F\left(\omega_{t-1}^L\right) N_{t-1}.$$

The household's budget constraint is redundant by Walras' Law.

3 On the effects of the central bank's balance sheet size

In this section we compare the properties of a "lean" central bank balance sheet regime with those of a "large" balance sheet regime, in which the central bank has permanently expanded its balance sheet size through an asset purchasing program.[19] As will be clear below the properties of the interbank market, as captured by the matching function $\Upsilon\left(\Phi_t^L, \Phi_t^B\right)$, are key to understand the relative merits of both scenarios.

Definition 1 (Match-efficient interbank market) *The interbank market is match-efficient if* $\Upsilon(1,1) = 1$. *Otherwise* ($\Upsilon(1,1) < 1$) *the interbank market is match-inefficient. A match-inefficient market is asymptotically match-efficient if* $\lim_{x\to\infty} \Upsilon(x,1) = \lim_{x\to\infty} \Upsilon(1,x) = 1$.

This definition states that the interbank market is match-efficient if all orders find a match, when there is the same volume of lending and borrowing orders; it is match-inefficient if some orders do not find a match, even when the volume of borrowing and lending orders is equal. Asymptotic match-efficiency implies that, if the volume of orders on one of the two sides is very large, then the number of matches equals the volume of orders on the short side.

Examples of commonly used match-efficient matching technologies include the Leontieff matching function, $\Upsilon\left(\Phi^L, \Phi^B\right) = \min\left(\Phi^L, \Phi^B\right)$; as well as the Cobb-Douglas matching function $\Upsilon\left(\Phi^L, \Phi^B\right) = \min\{\lambda_2 \left(\Phi^L\right)^{\lambda_1} \left(\Phi^B\right)^{1-\lambda_1}, \Phi^B, \Phi^L\}$ for the case with $\lambda_2 \geq 1$. For $\lambda_2 < 1$, this function is neither match-efficient, nor asymptotically match-efficient. The functional form proposed by Den Haan et al. (2000),

[19] Notice that we take extreme interpretations of the words "lean" and "large", by which we mean 0 or infinity.

$\Upsilon\left(\Phi^{L},\Phi^{B}\right)=\Phi^{L}\Phi^{B}[\left(\Phi^{L}\right)^{\lambda}+\left(\Phi^{B}\right)^{\lambda}]^{-1/\lambda}$, is match-efficient only in the limit $\lambda\rightarrow\infty$, where it becomes equivalent to the Leontieff function.[20] However it is always asymptotically match-efficient.

3.1 The case of a match-efficient interbank market

It is instructive to start with the case of a match-efficient interbank market, in which the number of matches always equals the 'short side' of the market. We analyze both the case of a 'lean' balance sheet, and that of a 'large' balance sheet.

3.1.1 Lean balance sheet

By 'lean' balance sheet we refer to a scenario in which central bank asset holdings are arbitrarily small. In particular, we consider for illustration the limiting case in which the central bank does not hold any government bond: $b_t^{G,CB}=0$. Its balance sheet (33) then results in

$$\Phi_t^B\left(1-\Gamma_t^B\right)=\Phi_t^L\left(1-\Gamma_t^L\right),$$

which combined with the identity $\Gamma_t^B\Phi_t^B=\Gamma_t^L\Phi_t^L$ yields

$$\Phi_t^B=\Phi_t^L.$$

That is, the volume of borrowing orders in the interbank market equals the volume of lending orders. Under the assumption of match-efficiency, the above equation implies that matching probabilities are one,

$$\begin{aligned}\Gamma_t^L &= \Upsilon\left(1,\Phi_t^B/\Phi_t^L\right)=\Upsilon(1,1)=1,\\ \Gamma_t^B &= \Upsilon\left(\Phi_t^L/\Phi_t^B,1\right)=\Upsilon(1,1)=1.\end{aligned}$$

Substituting these values into the interest rate equations (19-21) we obtain the following proposition.

Proposition 2 (Lean balance sheet) *If the interbank market is match-efficient and* $b_t^{G,CB}=0$*, then*

$$R_t^L=R_t^B=R_t^{IB}=\xi R_t^{DF}+(1-\xi)R_t^{LF},$$

and the amount of central bank reserves is zero,

$$M_t/P_t=\Phi_t^L\left(1-\Gamma_t^L\right)=0.$$

This proposition shows how, in the case of a match-efficient interbank market, all the relevant interest rates coincide with the interbank rate, which in turn equals a weighted average of the deposit and lending facility interest rate with the weight given by the Nash bargaining parameter ξ. Under the assumption

[20] In our baseline calibration below we will consider the $\lambda\rightarrow\infty$ limit of the matching function in Den Haan et al. (2000).

of symmetric bargaining power ($\xi = 1/2$), the interbank rate is in the middle of the corridor formed by the deposit and lending facility rates. If $\xi \neq 1/2$ the interbank rate will be closer to one of the two policy rates. In any case, a given interbank market is consistent with multiple corridor widths, as shown by the following corollary

Proposition 3 (Irrelevance of the corridor width) *If the interbank market is match-efficient and* $b_t^{G,CB} = 0$*, and provided the ELB does not bind (*$R_t^{DF} > 1 - \kappa$*), then given an arbitrary corridor width* χ *the policy interest rates can be chosen as*

$$\begin{aligned} R_t^{DF} &= R_t^{IB} - (1 - \xi)\,\chi, \\ R_t^{LF} &= R_t^{IB} + \xi\chi, \end{aligned} \tag{39}$$

and thus the equilibrium allocation is independent of the corridor width.

The equilibrium allocation is independent of the corridor width as it only depends on the borrowing and lending interest rates $R_t^L = R_t^B = R_t^{IB}$. Notice that the proposition only holds if the ELB is not binding, i.e. $R_t^{DF} > 1 - \kappa$. In this sense, it is important to gauge the extent to which such ELB is likely to constraint interest rate policy in this match-efficient, lean-balance-sheet scenario. The following result obtains the steady-state distance between the floor of the interest rate corridor and the ELB.

Proposition 4 (Policy space - corridor system) *If the interbank market is match-efficient and* $b_t^{G,CB} = 0$*, the steady-state distance between the deposit facility rate and the ELB is*

$$R^{DF} - (1 - \kappa) = 1/\beta - (1 - \xi)\,\chi - (1 - \kappa).$$

To obtain the above result, notice first that, in the zero inflation steady state ($\bar{\pi} = 1$), the value of R^L is pinned down by the household's Euler equation, $R^D = 1/\beta$, and by the retail banks' zero profit condition: $R^L = R^D$. From Proposition 2, the steady-state interbank rate is then $R^{IB} = 1/\beta$,[21] which together with equation (39) implies a steady-state deposit facility rate $R^{DF} = 1/\beta - (1 - \xi)\chi$.

The last two propositions indicate that, even if the size of the corridor plays no role when the economy is not constrained by the ELB, it plays a major role in determining *how often the economy will hit the lower bound* when the dynamics are affected by aggregate shocks.[22]

Finally, in the match-efficient, lean-balance-sheet case, capital efficiency (equation 36) simplifies to

$$\Omega_{t-1} = \mathbb{E}\left(\omega \mid \omega \geq \omega_{t-1}^B\right) = \mathbb{E}\left(\omega \mid \omega \geq \omega_{t-1}^L\right),$$

such that investment banks split either into fully leveraged banks investing in the real economy or banks that use their equity to purchase public debt and lend in the interbank market. The segment of banks

[21] This implies in turn that the intercept in the Taylor rule (equation 29) consistent with a zero inflation steady state is ($\pi = \bar{\pi} = 1$) is $\bar{R} = 1/\beta$.

[22] Notice that, up to a first-order approximation, the deterministic steady state coincides with the mean of the stationary distribution of the dynamic states.

that invest in the real economy without borrowing from the interbank market (those with $\omega_t^L \leq \omega_t < \omega_t^B$) is of zero measure.

3.1.2 Large balance sheet

Consider now the case in which the size of the balance sheet is enlarged through an asset purchase program: $b_t^{G,CB}$. In the case of a large balance sheet, the balance sheet equation (33) combined with the identity $\Gamma_t^B \Phi_t^B = \Gamma_t^L \Phi_t^L$ yields

$$\Phi_t^L - \Phi_t^B = b_t^{G,CB}.$$

That is, the difference between lending and borrowing orders in the interbank market equals the value of bonds held by the central bank. In this case, $\Phi_t^B/\Phi_t^L < 1$, i.e. the interbank market is more *slack* that in the case of a lean balance sheet. Under match-efficiency, the latter in turn implies $\Gamma_t^B = \Upsilon\left(\Phi_t^L/\Phi_t^B, 1\right) = 1$ and $\Gamma_t^L = \Upsilon\left(1, \Phi_t^B/\Phi_t^L\right) < 1$, that is, all borrowing orders are matched to lending orders, but some lending orders fail to find a match. Given the interest rate equations (19, 20, 21), this introduces a wedge between the interbank rate and the effective lending rate,

$$R_t^{DF} \leq R_t^L < R_t^{IB} = R_t^B < R_t^{LF}.$$

From now onwards, we focus on the limiting case where the balance sheet size tends towards infinity $b_t^{G,CB} \to \infty$.[23] This limit is empirically relevant since, under the calibration considered below, the economy converges to it very quickly (i.e. for realistic levels of the balance sheet size). The following proposition characterizes the interest rates in this case.

Proposition 5 (Large balance sheet) *If the interbank market is match-efficient and $b_t^{G,CB} \to \infty$ such that $\Gamma_t^L \searrow 0$ and $\Gamma_t^B = 1$, then interest rates converge to the following values,*

$$\begin{aligned} R_t^L &\searrow R_t^{DF}, \\ R_t^B &\searrow R_t^{IB}, \\ R_t^{IB} &\searrow \varphi^\infty R_t^{DF} + (1-\varphi^\infty) R_t^{LF}, \end{aligned}$$

where

$$\varphi^\infty \equiv \frac{\xi}{1-(1-\xi)\vartheta}.$$

The real amount of central bank reserves converges to

$$M_t/P_t = \Phi_t^L\left(1-\Gamma_t^L\right) \nearrow \Phi_t^L = b^{G,CB} + \Phi_t^B.$$

[23] Naturally, the maximum size of the central bank balance sheet is limited by the total size of the bond market: $b_t^{G,CB} \leq \bar{b}_t$. Hence, formally we need to consider the limit where both the banks holdings of bond and the total stock of bonds go towards infinity: $b_t^{G,CB} \to \infty$ and $\bar{b}_t \to \infty$ and $b_t^{G,CB} \leq \bar{b}_t$. To abreviate notation we will just write $b_t^{G,CB} \to \infty$.

Notice that in this case (i) the effective lending rate R_t^L converges to the discount facility rate R_t^{DF} and (ii) there is a wedge between these two rates, on the one hand, and the interbank and effective borrowing rates (R_t^{IB}, R_t^B) on the other, where the latter two rates coincide too in the limit. Notice that the spread between the effective borrowing and lending rates $R_t^B - R_t^L$ distorts the intertemporal decision of the private agents. It acts similarly to a tax on capital returns and as such it generates profits for the central bank. It hence potentially affects welfare *vis-à-vis* the corridor system with a lean balance sheet. The size of such spread is determined by φ^∞, that is, borrowers' bargaining power in the limiting case of an arbitrarily large central bank balance sheet. In particular, the spread $R_t^B - R_t^L$ disappears as ϑ (the probability of enjoying further bargaining rounds in the interbank market upon failing to reach an agreement) goes to 1.[24] In particular:

Proposition 6 (Floor system) *If the interbank market is match-efficient, $b^{G,CB} > 0$, and $\vartheta = 1$ then the central bank operates a 'floor system' with*

$$R_t^B = R_t^{IB} = R_t^L = R_t^{DF},$$

and the marginal lending facility rate R_t^{LF} and hence the corridor width χ does not affect the equilibrium.

The preceding proposition states that if $\vartheta = 1$ then, given a positive balance sheet size, monetary policy is conducted according to a floor system, in which the only relevant policy rate is the discount facility rate.

Proposition 7 (Decreasing marginal effect) *Under the conditions of proposition 6, any quantitative easing intervention consisting of additional government bond purchases, $b^{G,CB} + \Delta b_t^{G,CB}$, will have no effect on interest rates. In the more general case that $\vartheta < 1$ the marginal effect of any additional government bond purchases on interest rates is decreasing, if the implicit function $\Phi_t^B(b_t^{G,CB})$ defined through the full equilibrium system is close enough enough to constant, i.e. $\frac{\partial^2 R_t^{IB}}{(\partial b_t^{G,CB})^2} > 0$ if $|\Phi_t^{B\prime\prime}|$ and $|\Phi_t^{B\prime}|$ are small enough.*

The first part of this result can be derived directly from Proposition 6. As $R_t^B = R_t^{IB} = R_t^L = R_t^{DF}$ any additional increase, be that temporary or permanent, will not affect interest rates. The result for the more general case resonates proposition 5. It can be derived by implicit differentiation from the equilibrium conditions of the interbank market(17), (18), (17), (22), the CB balance sheet (33) taking as given the mapping from $b_t^{G,CB}$ to Φ_t^B. This mapping is determined by the full set of the equilibrium conditions. Considering it an unknown function allows us to consider the 5 above equations in isolation, i.e. to apply partial equilibrium logic. The restrictions on $\Phi_t^B(R^B)$ ensure that the general equilibrium effects do not overturn the partial equilibrium effects. They can be expected to be fulfilled for almost any calibration of the model. Below we exemplify numerically that this is the case for our baseline calibration, and relate this finding to the empirical finding in Reis (2016).

[24] As we will see in the calibration section, a value of ϑ very close to 1 turns out to be indeed the empirically relevant case for the euro area.

Finally, Proposition 6 and the fact that $R^L = 1/\beta$ in the zero-inflation steady state imply the following key result.

Proposition 8 (Policy space - floor system) *Under a floor system, the steady-state distance between the deposit facility rate and the ELB is*

$$R^{DF} - (1-\kappa) = 1/\beta - (1-\kappa).$$

Thus, under a floor system, the steady-state distance between the deposit facility rate and the ELB is *wider* than in the case of a lean balance sheet, where such distance decreases with the corridor width (see Proposition 4). In other words, a floor system allows the central bank to operate on average with more 'policy space' in its conventional interest-rate policy. This result will play a key role in our numerical results, when we compare crisis scenarios under both the lean-balance-sheet/corridor system and the large-balance-sheet/floor system.

As a summary of our theoretical results thus far, the comparison between the case of a lean balance sheet with that of an expanded balance sheet casts some light on the relative features of each system when the interbank market is match-efficient. First, the two systems have different implications regarding interest rate distortions. In the case of a lean balance sheet, all the relevant interest rates coincide with the interbank market, so that no distortion due to spreads between borrowing and lending rates arises. On the contrary, in the case of a large balance sheet there is a spread between the interbank rate, which equals the effective borrowing rate, and the discount facility rate, which equals the effective lending rate. This spread distorts the allocation of credit and may hence be a source of inefficiency; the size of such potential welfare losses will be explored in the quantitative section below. Second, the two systems differ with respect to the policy space for interest-rate policy *vis-à-vis* the ELB. A large balance sheet implies operating a floor system, which implies that all interest rates coincide. A small balance sheet implies operating a corridor system, where the deposit facility rate is *lower* then the interest rate that is relevant for households' consumption and saving decisions. Hence, a corridor system provides less policy space then a floor system.

3.2 The case of a match-inefficient interbank market

Consider now the case with a match-inefficient interbank market. Importantly, we assume that the interbank market remains *asymptotically* match-efficient, i.e. orders on the short side of the market always find a match if orders on the other side are in large numbers.

3.2.1 Lean balance sheet

In the case of a *lean* balance sheet, now we have

$$\begin{aligned}\Gamma_t^L &= \Upsilon\left(1, \Phi_t^B/\Phi_t^L\right) = \Upsilon(1,1) < 1, \\ \Gamma_t^B &= \Upsilon\left(\Phi_t^L/\Phi_t^B, 1\right) = \Upsilon(1,1) < 1.\end{aligned}$$

That is, both lending and borrowing orders fail to find matches in the interbank market. We obtain the following proposition.

Proposition 9 (Lean balance sheet - match-inefficient IB market) *If the interbank market is inefficient and $b_t^{G,CB} = 0$ then*

$$R_t^L < R_t^{IB} < R_t^B,$$

where these interest rates are given by the equations (21-24). The amount of reserves at the central bank is positive,

$$M_t/P_t = \Phi_t^L\left(1 - \Gamma_t^L\right) > 0.$$

The match-inefficiency in the interbank market thus generates a misallocation of liquidity due to the spread between the borrowing and lending rates. In this case the size of the corridor matters and hence the results of Proposition 3 do not hold. Moreover, the size of the policy space may be increased or reduced with respect to the efficient case (Proposition 4), as the value of φ_t can be smaller or larger than ξ depending on whether Γ_t^L is now larger or smaller than Γ_t^B.

3.2.2 Large balance sheet

In the case of a *large* balance sheet, and particularly in the limit as $b^{G,CB} \to \infty$, asymptotic match-efficiency continues to implies that $R_t^{IB} = R_t^B$ and $R_t^L = R_t^{DF}$, as in Proposition 5. Finally, in the limiting case $\vartheta \to 1$, the central bank continues to operates a *floor system*, as in Proposition 6. The latter in turn implies that the steady-state distance of R_t^{DF} to the ELB remains as in Proposition 8. Therefore, the results in Propositions 5, 6 and 8 continue to hold despite the match-inefficiency of the interbank market.

The conclusion is that whereas the match-inefficiency of the interbank market may play a role in the case of a central bank with a lean balance sheet operating a corridor system, it is innocuous under a floor system as long as the interbank market is still *asymptotically* match-efficient.

3.3 Central bank's asset composition

In addition to the purchase of government bonds, central banks may also engage in lending programs to commercial banks. Here we consider the case in which the central bank lends directly to investment banks at the effective borrowing rate R_t^B. Notice that this rate is lower than the marginal lending facility rate R_t^{LF}, so it is actually subsidized lending. We think of these loans as LTROs. It is trivial to check that the demand for interbank loans in Lemma 1 is now

$$B_t^j = \mathbf{B}_t\left(N_t^j, \omega_t^j\right) = \begin{cases} (\phi-1)\,N_t^j - B_t^{j,CB} \in \left[0, (\phi-1)\,N_t^j\right], & \text{if } \omega_t^j \geq \omega_t^B, \\ 0, & \text{if } \omega_t^L \leq \omega_t^j < \omega_t^B, \\ \left(b_t^{j,G} - N_t^j\right) \in \left[-N_t^j, 0\right], & \text{if } \omega_t^j < \omega_t^L, \end{cases}$$

where $B_t^{j,CB}$ is the amount of funds borrowed directly from the central bank through this program. Defining $B_t^{CB} \equiv \int_0^1 B_t^{j,CB} dj$, the total demand of funds in the interbank market is now

$$\Phi_t^B = (\phi-1)N_t\left[1 - F\left(\omega_t^B\right)\right] - B_t^{CB}. \tag{40}$$

The central bank's balance sheet equation should now include the new asset class:

$$B_t^{CB} + b_t^{G,CB} + \Phi_t^B\left(1 - \Gamma_t^B\right) = \Phi_t^L\left(1 - \Gamma_t^L\right).$$

All other equilibrium conditions remain unchanged.

The main difference between bond purchases and central bank lending is that, *ceteris paribus*, the former increases the supply of funds to the interbank market, Φ_t^L, whereas the latter reduces the demand of funds, Φ_t^B. A second difference is that the maximum size of the lending program is $(\phi-1)N_t\left[1 - F\left(\omega_t^B\right)\right]$, such that net borrowing is zero in (40), whereas the maximum total size of the bond purchase program is the size of the bond market $\bar{b}_t$.

However, it can be shown that any equilibrium allocation that can be sustained with bond purchases can also alternatively be sustained with loans to banks.

Proposition 10 (equivalence of loan and bond programs) *Be X_t the set of time t equilibrium variables (prices, quantities and exogenous processes). Consider a particular set of sequences of equilibrium variables ranging from $t = 0$ until $t = \infty$ denoted $\left\{\{x_t^*\}_{x_t \epsilon X_t}\right\}_{t=0}^{\infty}$ and some particular initial conditions, which satisfy the following conditions:*

1. *the sequences - together with the initial conditions - constitute an equilibrium*
2. *the central bank holds some government bonds ($b_t^{G,CB*} > 0$ for some t)*
3. *the central bank never extends any loans to banks through direct lending programs ($B_t^{CB*} = 0$, $\forall t \geq 0$).*

Then there exists a second set of sequence of equilibrium variables ranging from $t = 0$ until $t = \infty$ denoted by $\left\{\{x_t^+\}_{x_t \epsilon X_t}\right\}_{t=0}^{\infty}$, which - together with the same initial conditions - constitute an equilibrium and which satisfies:

1. *The central bank purchases no bonds $b_t^{G,CB+} = 0$ $\forall t \geq 0$*

2. *At each t at which the central bank holds bonds in the previous sequence, it extends a positive amount of bank loans. In particular* $B_t^{CB+} = b_t^{G,CB*}\Gamma_t^{L*} \leq b_t^{G,CB*} \forall t \geq 0$

3. *The sequences of all equilibrium variables besides* B_t^{CB+}, $b_t^{G,CB+}$, Φ_t^{B+}, b_t^{G+} *and* Φ_t^{L+} *are the same as in the previous sequence*

The proof can be found in Appendix A.3. This proposition says that it is essentially irrelevant which of the two instruments the central bank uses to affect the liquidity in the interbank market. The intuition is that by purchasing bonds the central bank increases the supply of funds on the interbank market, while by providing loans it reduces the demand. Both interventions can equally increase the ratio of lending over borrowing orders, which in turn affects the matching probabilities and the effective bargaining weight. Notice that this statement holds without any requirements on the matching function. Hence all the propositions in the previous two subsections have counterparts where the central bank lends to banks instead of purchasing bonds.

In the numerical section below we investigate how the central bank's asset composition matters for the effect of temporary QE under a lean balance sheet, when the size of the balance sheet of the central bank is fixed across the two policies; and by how much the balance sheet size differs, if the effect is kept constant.

4 Quantitative analysis

4.1 Calibration

For our subsequent numerical analysis, we calibrate the model. We use a standard CRRA utility function with additively separable labor,

$$u(C_t) - v(L_t) = C_t^{\gamma}/(1-\gamma) + L_t^{1+\psi}/(1+\psi).$$

We also use a standard quadratic specification for investment adjustment costs,

$$S(x) = \frac{\iota}{2}(x-1)^2,$$

where ι is a scale parameter. Idiosyncratic shocks are assumed to be distributed according to a lognormal distribution with parameters μ and σ. The matching function is as in Den Haan et al. (2000)[25]

$$\Upsilon\left(\Phi_t^L, \Phi_t^B\right) = \frac{\Phi_t^L \Phi_t^B}{\left(\left(\Phi_t^L\right)^{\lambda} + \left(\Phi_t^B\right)^{\lambda}\right)^{1/\lambda}}.$$

[25]Notice that this function converges to the Leontief matching function as λ goes to infinity. For numerical reasons we approximate this function by the Leontieff function for values of $\lambda > 1000$.

The parameters of the production function (α, δ), the utility function (γ, ψ) and the New Keynesian elements $(\theta, \epsilon, \iota, \upsilon, \rho)$ take standard values for quarterly models with zero steady-state inflation and no growth; they are reported in Table 1 below.

Most of the remaining parameters are non-standard and relate to the banking sector. We set κ to -0.4%, which is the lowest value which the ECB's deposit rate has taken to date. For the leverage constraint we assume a maximum assets-to-equity ratio ϕ of 5. For the remaining parameters, we target moments from Eurozone data for the period covering the 2 decades before the crisis, i.e. 1988-2007. The households discount factor β is set in order to obtain a steady-state government bond yield of 2% per annum. Banks' discount factor $\widehat{\beta}$ is chosen to match the historical average real return on Bank equity of 10% (9.6).

The matching function parameter λ is set such as to target the steady state ratio between the volume of central bank excess reserves and the volume of interbank claims in the EMU of 0.023%.

The parameter defining the maturity of government debt ζ is set to 0.05, which yields an average maturity of 5 years. The stock of government debt $\overline{B}$ is set to 60% (59.05) of steady-state GDP, while the share of this debt held by the central bank $\overline{B}^{G,CB}$ is set to 0, reflecting the absence of QE measures or excess reserves prior to the crisis.

The parameter defining the corridor width χ is set to 0.5% per quarter, which implies a corridor width of 2% per annum, as used by the ECB before the crisis. The bargaining power of lenders ξ is set to 0.5, implying an interbank rate in the middle of the corridor.[26]

The mean of the iid shocks to island specific capital efficiency μ is normalized such that the steady state capital efficiency $\overline{\Omega}$ is 1. The standard deviation of these shocks is chosen in order to set the steady state ratio of redistributive (government debt and interbank loans) over productive assets (anything else) on the investment bank's balance sheet $\frac{F(\omega_L)N}{K}$ to its empirical counterpart for the aggregate balance sheet of monetary and financial institutions in the Eurozone of 0.6.

Finally, the parameter ϑ –the probability that another round of negotiations is reached after a failed match– is set such that the model reproduces the empirical relationship between excess reserves over GDP and the interbank-deposit facility rate spread during the entire euro period (1999-2017). In particular, we choose ϑ to minimize the weighted mean absolute error between the data and the model prediction, which yields $\vartheta = 0.998$.[27] As discussed in Section 3, a value of ϑ close to unity implies that a central bank with a sufficiently large balance sheet essentially operates a *floor system* ($R_t^{IB} = R_t^{DF}$). As shown

[26] We calibrate the model in line with the european institutional set up. In the US the corridor system was only implemented in 2008. At the same time the Fed flooded the market with liquidity, which drove the fed funds rate close to the interest-on-excess-reserves floor.

[27] We use weekly observations from 1999-2017. We weight the observations by the probability density function of the data, which we approximate by a histogram of 30 equal spaced bins. This allows us to get a good overall fit of the relationship despite having many more obseravtions close to zero than for higher values of excess reserves.

Furthermore notice that while we only consider the realtionship in steady state, the numerical experiments carried out later show that the this realtionship has a very similar shape in the dynamics, which is to say that dynamic general equilibrium effects are of less relevance.

Finally notice that in the data the corridor was changed over time while we abstract from this policy in the model. Normalization by division through the corridor width yields similar results.

Parameter		Value	Target
α	Capital share	0.33	Literature
δ	Depreciation	0.025	Literature
γ	Risk aversion	2	Literature
ψ	Inverse Frisch elasticity	1.5	Literature
θ	Calvo parameter	0.7	Literature
ϵ	Markup	5	Literature
ι	Investment adjustment	2.4	Literature
υ	Taylor rule inflation	1.5	Literature
ρ	Taylor rule persistence	0.8	Literature
κ	Effective lower bound	-0.004	minimal ECB deposit rate
ϕ	Leverage constraint	5	Literature
β	Discount factor HH	0.995	Sovereign yield
ζ	Bond maturity	0.05	Literature
$\overline{B}/\overline{Y}$	Government debt	2.4	Debt to GDP
$\overline{B}^{G,CB}/\overline{B}$	Government debt held by CB	0	no QE
ξ	Bargaining power lenders	0.5	Symmetric corridor in the EMU
ϑ	Probability of reaching next round	0.998	Relationship between spread and excess reserves
χ	Corridor width	0.5%	Pre-crisis corridor width in the EMU
$\widehat{\beta}$	Discount factor bank	0.975	RoE EMU banks
μ	Mean of idiosyncratic shocks	-0.035	Normalize $\overline{\Omega} = 1$
σ	Std of idiosyncratic shocks	0.27	Ratio of redistr. to productive assets
λ	Matching function	3000	Ratio of interbank to CB lending

Table 1: Calibrated parameter values

in Figure 2, our calibrated model replicates fairly well the relationship between excess reserves and the interbank-DFR spread, both for the pre-crisis corridor system in which such spread fluctuated around 1% (i.e. one half of the corridor width) and for the current floor system in which such spread is essentially zero.

4.2 Numerical experiments

In this section we use the calibrated model do illustrate the theoretical results in the previous section, as well as to compare the stabilization properties of both regimes (corridor vs. floor system) in the face of financially-driven recessions. We start by comparing the effect of the balance sheet size on the steady state equilibrium, before considering the effect of unanticipated shocks.

4.2.1 Comparative statics: the role of the central bank balance-sheet size

As shown above, the balance sheet size of the central bank $b^{G,CB}$ has implications for the steady state of the economy. Figure 3 illustrates such implications for the calibrated model. For each value of $b^{G,CB}$ –starting from zero–, the endogenous ratio between excess reserves and GDP is computed; this is the variable displayed in the x-axes. The bottom panels illustrate the response of interest rates to the balance sheet size. As the level of excess reserves increases, both $R^L - R^{DF}$ and $R^B - R^{DF} = R^{IB} - R^{DF}$ converge

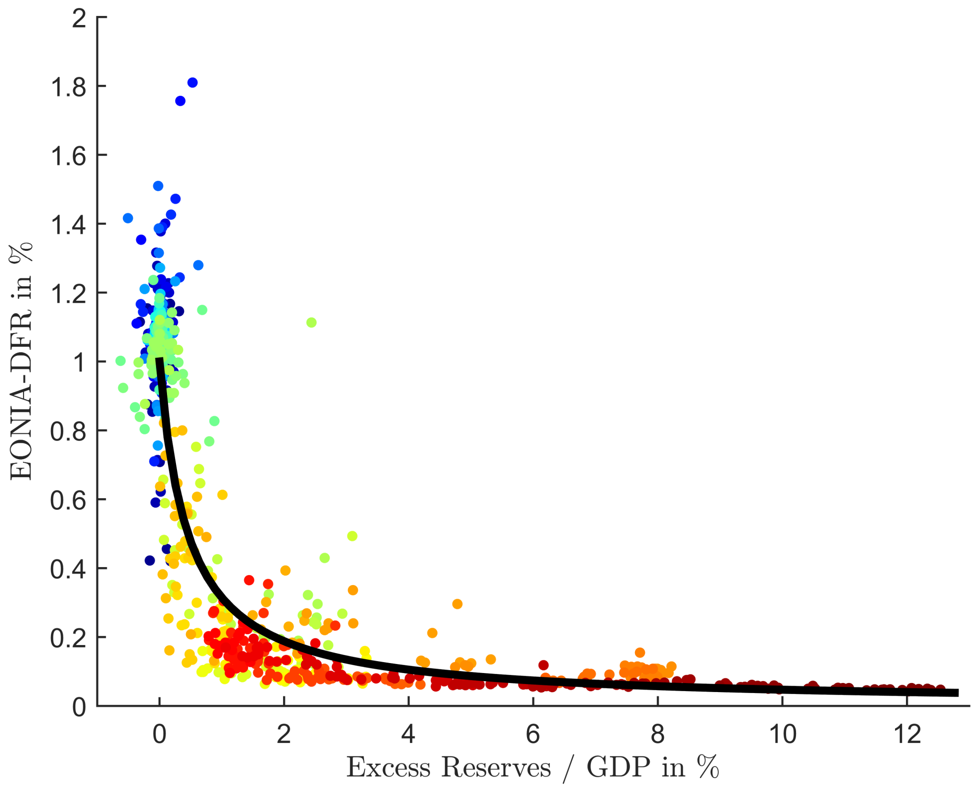

Figure 2: This figure shows how the spread between the interbank rate (EONIA) and the deposit facility rate is realted to the amount of excess reserves in the steady state of the model (black line) and in weekly EMU data (coloured dots, colours indicate time ranging from 1999 in blue to 2017 in red).

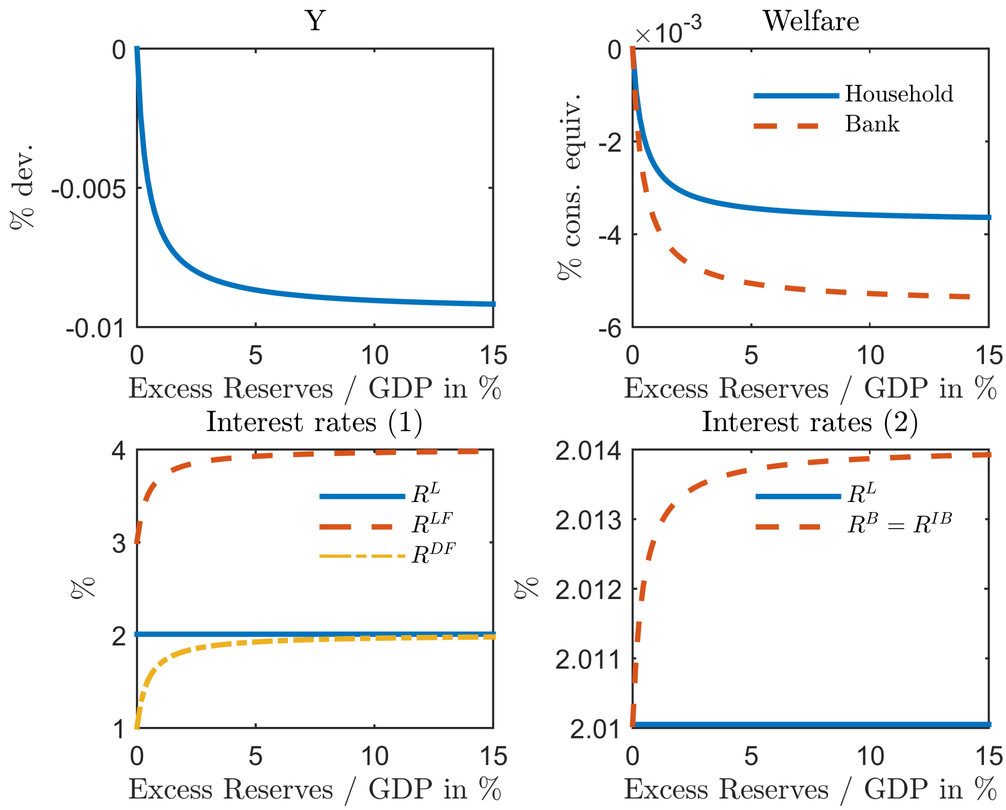

Figure 3: This figure illustrates the effect of the size of the central banks balance sheet size in steady state. The upper panels are expressed in deviations from the steady state associated with a balance sheet size of 0. The upper right panel furthermore expressed this difference in marginal consumption units.

to a value very close to zero,[28] because borrowers' effective bargaining power φ converges towards a value very close to 1 as the central bank floods the market with excess liquidity. However, $R^L - R^{DF}$ converges faster than $R^B - R^{DF}$, because the latter depends only on φ while the former also depends on lenders' matching probability Γ^L. Hence the spread between R^L and R^B increases. This spread is due to the fact that the penalty rate R^{DF} paid by the central bank on excess reserves acts like a tax on capital returns. As such, this spread distorts the savings decision by the household. This leads to lower levels of output and welfare. However, as the upper two panels demonstrate, these effects are quantitatively rather small, in fact second order.

In proposition 10 in the previous section we showed that the central bank can obtain the allocation associated with a certain level of public bond purchases by alternatively issuing a smaller level of loans to banks. Figure 4 shows this relationship in steady state. To obtain the same steady state allocation as that associated to a balance sheet, which measures 15% of GDP and where the only assets are bonds, the central bank would need to extend loans to banks in the amount of about 13% of GDP, which is 12%

[28] We have seen the relationship between $R^{IB} - R^{DF}$ and excess reserves before in Figure 2.

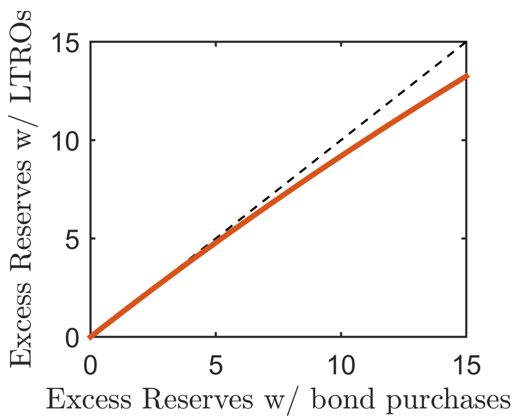

Figure 4: This figure shows by how much the central bank needs to expand its balance sheet by issuing bonds in order to obtain the allocation associated with the amount of governemnet bond purchases on the x axis. On both axis, the excess reserves are expressed as fraction of steady state GDP. The dashed line is the 45 degree line.

less. As the figure shows, the larger the balance sheet is, the larger is the difference between the amounts required in the two cases.

4.2.2 Balance sheet policies in financial crises

Next we consider the dynamic implications of different balance sheet policies when the economy is hit by a financial crisis. We model the financial crisis as an unexpected, temporary reduction in the bank leverage ratio, ϕ. We consider three different scenarios, depending on the central bank balance sheet configuration, in terms of initial size and response to the crisis:

(i) A *lean* initial balance sheet with no unconventional policy response to the crisis: $b_t^{G,CB} = 0$ for all t.

(ii) Starting from the same *lean* balance sheet as in scenario (i), the central bank implements a temporary bond purchase program.

(iii) A *large* initial balance sheet but no unconventional policy response to the crisis.

In all three cases the interest rate policy follows the same Taylor rule, and the economy is assumed to rest at the corresponding steady state before the shock (assumed to take place at time $t = 1$). The reduction in ϕ is assumed to last for 12 periods and is then fully reversed; once the shock arrives, the future path of ϕ is perfectly foreseen thereafter.[29] We choose the size of the shock –common to all scenarios– such that the ELB constraint binds for a number of periods under scenario (i). In scenario (ii), we assume

[29] We solve for the paths of the endogenous variables using the Newton-based perfect forsight solver for the non-linear model implemented in DYNARE. Notice that the future path of the exogenous variables (bank leverage, and the unconventional policy response, if any) is revealed on the impact period.

Computing the nonlinear solution has two advantages. As we will see the relationship between the central bank balance sheet size and the real variables is highly nonlinear.

the central bank purchases bonds for 2 periods at a speed of 2% of steady state GDP per quarter, which is roughly in line with the ECB's quarterly purchases the asset purchasing programme initiated in 2015. After that period, we assume that reinvestment continues for one more period; following that, the central bank stops reinvestment and the balance sheet size falls gradually as the government bonds mature.[30] This unconventional policy path is revealed at the same time as the deleveraging shock.[31] Finally, in scenario (iii) we consider an initial balance sheet size of 15% of GDP. This number corresponds roughly to the amount of excess reserves that will be reached at the end of the ECB's asset purchase programme as it is currently announced.

The blue solid lines Figure 5 displays the economy's response in *scenario (i)*, i.e. starting from a lean balance sheet and in the absence of balance sheet policies. For given aggregate bank equity, the shock reduces banks' investment capacity. This produces a fall in investment, aggregate demand, labor, and inflation. Also, since more bank equity is necessary to sustain a similar level of investment, previously active banks need to reduce their balance sheets, which crowds in some banks that were previously not investing. Since these banks are less efficient, the average capital efficiency Ω_t falls. Moreover, since banks' balance sheets feature a degree of maturity mismatch and exposure to surprise deflation –through holdings of long-term nominal bonds and real capital, as well as short-term nominal liabilities– the direct effects produce losses to bank equity N_t, which reinforces the contractionary impact. The central bank responds to the fall in inflation by lowering its two policy rates –holding the corridor width constant–, but soon the deposit facility rate hits the ELB, preventing further (conventional) monetary accommodation for some periods. With inflation falling far more than nominal rates, the real rate actually increases initially, which reduces household consumption and worsens the contractionary effects of the shock.[32]

The dashed yellow lines in Figure 5 display responses in *scenario (ii)*, where the economy starts from the same lean balance sheet as in scenario (i) but the central bank engages in temporary asset purchases. This scenario highlights how the matching friction in the interbank market gives rise to a *new transmission channel* for temporary QE policies. By purchasing government bonds from investment banks, the central bank liberates balance sheet capacity for those banks that choose not to invest in the real economy (b_t^G falls in equation 16). This leads to an increase in lending orders in the interbank market (higher Φ_t^L), while leaving borrowing orders largely unaffected. This reduces the *tightness* in the interbank market (lower Φ_t^B/Φ_t^L), as more lenders face the same amount of borrowers. As a result, the matching probability for lending banks Γ_t^L falls, so these banks end up holding more excess reserves, which in turn finance the central bank's asset purchases. At the same time, the reduction in interbank market tightness increases borrowers' effective bargaining power φ_t. *Ceteris paribus*, this depresses the interbank rate R_t^{IB} for given

[30] In terms of the rule in equation (32) for central bank's bond holdings, we thus assume $np_t = 0.02 \times Y^{annual}$ for $t = 1, 2$ and $ri_t = 1$ for $t = 1, 2, 3$.

[31] The ECB purchased assets over a much longer horizon. However, the ECB also faced a much longer duration of the ZLB. Unfortunatley, the nonlinear perfect foresight approximation we use does not allow us to consider shocks strong enough to yield longer periods of binding ZLB, which is a wellknown limitiation of this approach. Hence, we consider a shorter purchase program.

[32] In this scenario, the interbank rate R_t^{IB} stays in the middle of the corridor. However, given that the calibrated matching technology is essentially match-efficient, the search friction in the interbank market has no effect.

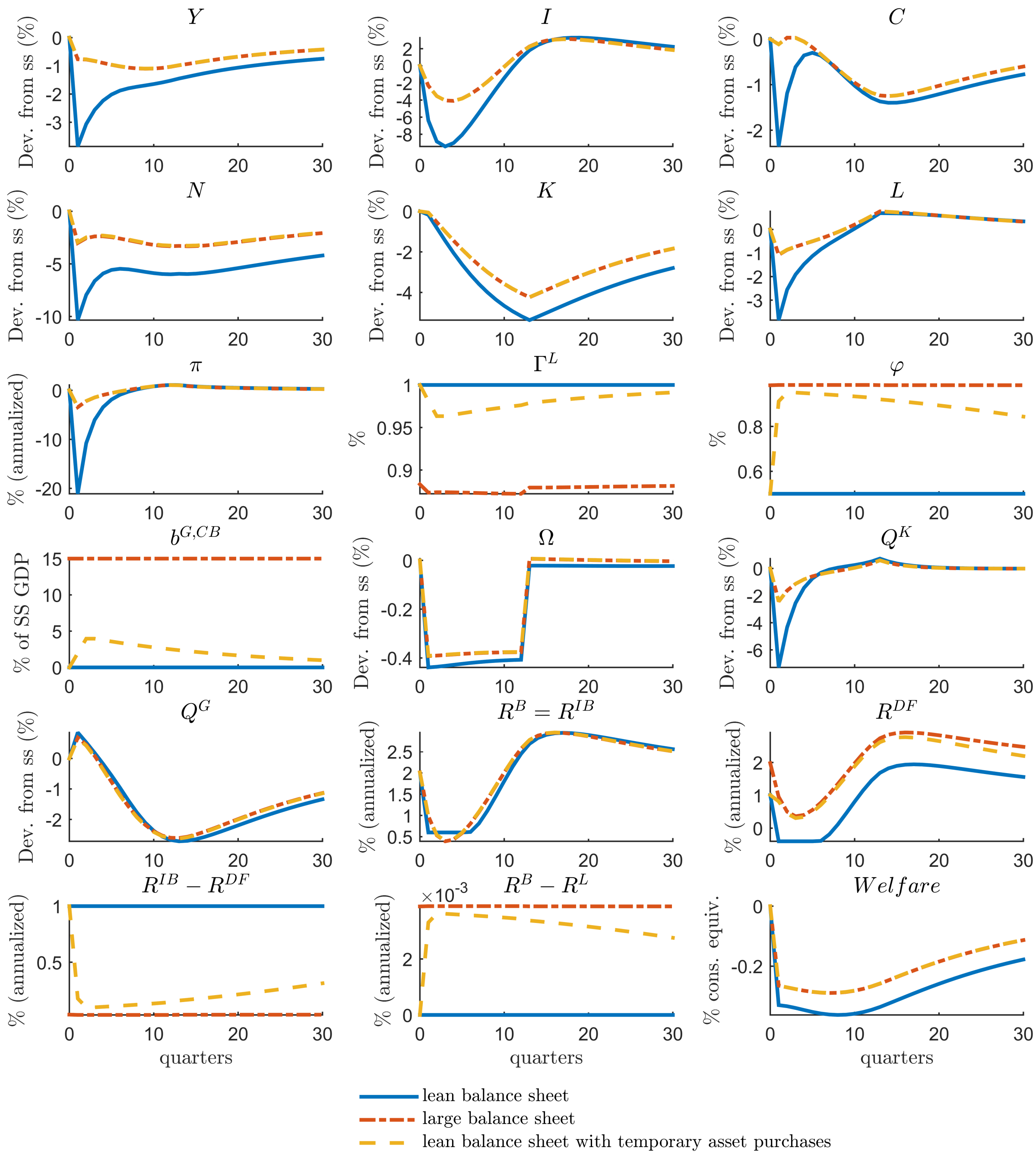

Figure 5: *This figure shows the responses of key variables to the shock under the different assumptions about unconventional monetary policy. Interest rates and infaltion are expressed in annualized percentage points. The size of the central banks balance sheet is expressed as ratio over steady state GDP. Welfare is expressed in deviations from the steady state in units of consumption equivalent. All other variables are expressed as deviations from the steady state. Note that as we saw in the previous figure, the steady state of scenario (ii) is only marginally different from the steay state of the other two scenarios.*

deposit facility rate (DFR). Hence the spread between the interbank rate and the DFR decreases, in line with the empirical relationship between the latter spread and excess reserves documented in Figure 2. And as the interbank-DFR spread narrows, so does the gap between the effective lending rate R_t^L –and hence households' deposit rate R_t^D– and the DFR. Thus, a temporary QE allows the central bank to add further stimulus during a binding-ZLB episode by pushing interbank interest rates, and interest rates across the economy in general, against the bottom of the interest rate corridor. This reduction in nominal rates has the usual expansionary effect of interest rate policy in New Keynesian models, by encouraging households consumption, etc. As a result, aggregate demand and inflation improve relative to the no-QE scenario. Furthermore, in the context of our model with heterogenous banks, the stimulus also leads to a relaxation of the leverage constraint by dampening the equity (N_t) losses of banks associated with the deleveraging shock, which in turn increases the average capital efficiency Ω_t. For the calibration considered here, the expansionary effect is so strong that, once all general equilibrium effects are considered, the DFR actually fails to hit the ZLB in the first place.[33]

Finally, the red dashed-dotted lines in Figure 5 show responses in *scenario (iii)*, where the central bank permanently operates a large balance sheet but does not implement QE in response to the same financial crisis. Notably, the responses are very similar to those of scenario (ii). This results from the fact that, with a *large* balance sheet, the central bank has more policy space for conventional interest-rate policy, since the associated steady-state level of the DFR is *higher* than under a lean balance sheet (Propositions 4 and 8). Even absent further stimulative unconventional measures, this enlarged policy space implies that the ZLB is not reached and hence the negative effects associated with it do not materialize.[34]

The last two scenarios highlight an important result in our model: temporary QE –i.e. temporary excess liquidity provision– and a permanently large balance sheet –i.e. permanent excess liquidity provision– are close substitutes. Both are able to deliver the same relaxation of the ELB constraint vis-à-vis the lean-balance-sheet/no-QE scenario, namely by compressing the interbank-DFR spread.[35]

[33] Notice that QE is only really effective at the ELB, when conventional monetary policy looses control over the interbank rate R^{IB}. During normal times conventional monetary policy (as described by the taylor rule) undoes the effect of quantitative measures.

[34] Notice that the fact that the cental bank keeps its balance sheets constant in real terms is similar to a small temporary balance sheet extension, because the supply of excess reserves becomes relatively more important as the economy, and in particular the capital stock, shrinks in real terms. This reduces the matching probability of lenders Γ_t^L and drives up the market power of the borrower φ_t a (hardly visible) bit. Ceteris paribus this would have effects on R^{IB}, however the response of conventional monetary policy through the taylor rule undoes this effect, since the ELB is avoided by a large margin.

[35] The reduction in the interbank-DFR spread however goes hand in hand with an increase in the $R^B - R^L$ spread: Whenever banks hold excess liquidity at the central bank they forgo the slightly higher rate paid on sovereign bonds R^G or interbank loans R^L. The flipside of this is that the central bank pays $R^G = R^L$ on its assets while paying R^{DF} on its liabilities, and hence makes a profit. This is similar to a tax on capital and distorts the savings choice in the economy. In scenario (ii) this distortion arises temporarily and is clearly overcompensated in terms of welfare. In scenario (iii) this distortion arises permanently. However, quantitatively this effect is negligible.

4.2.3 On the relative effectiveness of different balance sheet policies

So far we have shown that balance sheet policies can be effective through the *interbank transmission channel* and can thus help to overcome the ELB. In this subsection we compare different implementations of such policies. First, we compare the purchase of government bonds with the extension of loans to banks through facilities such as the ECB's long term refinancing operations (LTROs). Second, we analyze the marginal effectiveness of balance sheet policies. To this end we compare the response of the economy to a financial crisis in scenarios (ii) and (iii) from above with three further scenarios:

(iv) Starting from the same *lean* balance sheet as in scenario (ii), the central bank implements a temporary program though which it provides loans to banks (LTROs). The amount of loans is assumed to be such that the balance sheet sizes in (ii) and (iv) are identical.

(v) As in scenario (ii) the central bank starts from a *lean* balance sheet and purchases bonds. However it stops purchases one period *earlier* than in scenario (ii).

(vi) As in scenario (ii) the central bank starts from a *lean* balance sheet and purchases bonds. However it stops purchases one period *later* than in scenario (ii).

Notice that this time we simulated a stronger crisis so that none of the policies is able to avoid hitting the ELB. Figure 6 compares the responses of key variables. The variables in the first row are expressed in % deviations from scenario (iii).

Comparing scenario (ii) and scenario (iv), we find that the effect of the LTRO program is almost identical to that of an equally sized public bond purchase program, with the latter being marginally less expansionary. This finding complements proposition 10. There we had shown theoretically that the same effect can be obtained through both public bond purchases as well as through extension of loans to banks. However the magnitude of the balance sheet expansions required to obtain the same effect varied across the two instruments. Here we show numerically that even if the magnitude of the balance sheet extension is kept fixed, the two instruments are almost perfect substitutes.

Alternatively one may ask how the central bank balance sheet would need to evolve, if the path of the real variables in the bond purchase scenario (ii) was to be replicated by an LTRO program. From proposition 10 we know that this would require loans to be equal to a the fraction Γ_t^L of the amount of bonds purchased. While this alternative path for the balance sheet is not plotted, we can deduce it from the panel in row 2, column 3 of figure 6 : The balance sheet would need to grow by slightly less, with the peak being 4% lower.

Comparing scenarios (ii), (v) and (vi) we observe how the marginal effect of one more quarter of net purchases declines. While extending purchases from two to three quarters increases GDP by 4% at the peak, adding another quarter of purchases only results in an additional increase of roughly 1%. This result complements proposition 7 and 5, which showed that the marginal effect of QE decreases and converges towards 0 as the balance sheet size is expanded. The intuition is straightforward: As the central bank

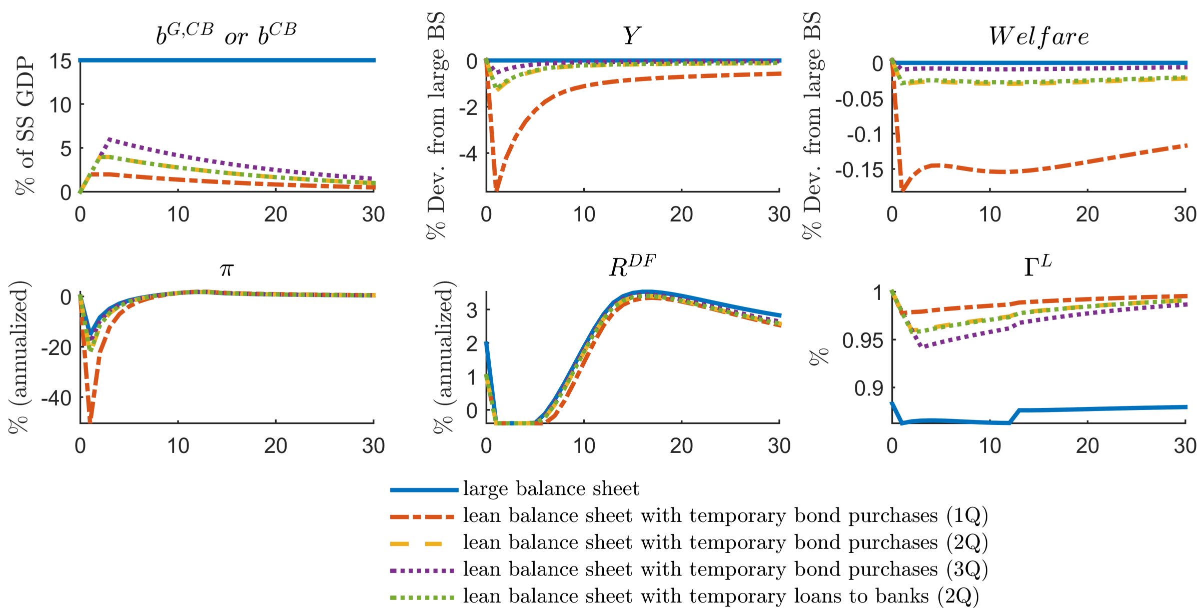

Figure 6: *This figure compares the responses of key variables to a delevaraging shock under the different assumptions about unconventional monetary policy specified in scenarios (ii)-(v). Interest rates and infaltion are expressed in annualized percentage points. The size of the central banks balance sheet is expressed as ratio over steady state GDP. GDP is expressed as deviation from the path associated with the large balance sheet scenario (iii) (blue). Welfare is expressed as deviations from welfare in scenario (iii) % in units of consumption equivalent.*

withdraws bonds from the market, lending banks invest more of their funds in the interbank market, which increases the borrowers effective bargaining weight and depresses the spread between the DFR and the interbank rate. However, the effective bargaining weight can not exceed one, hence the marginal effect of bond purchases on the latter needs to diminish as the amount of bonds purchased increases.[36]

This feature of the model explains the empirical finding by Reis (2016) that only the first QE announcement in the US had a significant effect on inflation, while for later QE announcements no such effect can be found in the data. According to the model, each round of QE at the ELB has a non-negative effect, yet this effect diminishes rapidly, which is why it would soon be impossible to detect empirically.[37]

5 Conclusion

TBC.

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[36] Note that for LTROs a similar holds: As the central bank lends to borrowing banks, they reliy less on funding from the interbank market, which increases the borrowers effective bragaining weight in this market and depresses the spread between the DFR and the interbank rate.

[37] Reis (2016) explains his empicial findings with a (postulated) satiation point in the demand for reserves. Our model features only asymptotic satiation, however from a practical or empirical point of view this difference is largely irrelevant.

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Appendix: For online publication

A Proofs

A.1 Proof of Lemma 1 (Banker's problem)

The bank's maximization problem can be expressed conveniently as a 2 stage problem, where first the banker decides how much dividends to consume and how much equity to keep on the balance sheet, before subsequently choosing on his portfolio.

$$V_t\left(E_t^j,\omega_t^j\right) = \max_{\Pi_t^j, N_t^j} \log\left(\Pi_t^j\right) + \bar{V}_t\left(N_t^j,\omega_t^j\right) \tag{41}$$

$$s.t. \tag{42}$$

$$N_t^j + \Pi_t^j = E_t^j$$

and

$$\bar{V}_t\left(N_t^j,\omega_t^j\right) = \widehat{\beta} \max_{A_t^j\geq 0, b_t^{j,G}\geq 0, B_t^j, E_{t+1}} \mathbb{E}\left[V_{t+1}\left(E_{t+1}^j,\omega_{t+1}^j\right)\right] \tag{43}$$

$$s.t. \tag{44}$$

$$Q_t^K A_t^j \leq \phi N_t^j \tag{45}$$

$$Q_t^K A_t^j + b_t^{j,G} = N_t^j + B_t^j \tag{46}$$

$$E_{t+1}^j = R_{t+1}^A \omega_t^j Q_t^K A_t^j - \frac{B_t^j}{1+\pi_{t+1}}\left(\mathbf{1}_{B_t^j>0} R_t^B + \mathbf{1}_{B_t^j<0} R_t^L\right) + \frac{R_{t+1}^G}{1+\pi_{t+1}} b_t^{j,G}$$

To solve this problem we start by making the following guess (which we later verify)

$$\bar{V}_t\left(N_t^j,\omega_t^j\right) = \varkappa \log(N_t^j) + \Psi_{t,t+1},$$

where $\varkappa$ is a constant and $\Psi_{t,t+1}$ is a function of time independent of state N_t^j .

The first order condition of problem (41) with respect to Π_t^j yields

$$1/\Pi_t^j = \varkappa/N_t^j \implies \Pi_t^j = N_t^j/\varkappa = (1+1/\varkappa)\, E_t^j,$$

Plugging the solution for dividends back into (41) we get

$$V_t\left(E_t^j,\omega_t^j\right) = -\log(\varkappa) + (\varkappa+1)\log\left(N_t^j\right) + \Psi_{t,t+1}.$$

Plugging this and the solution for dividends into the second stage problem (43) we obtain

$$\bar{V}_t\left(N_t^j\right) = \widehat{\beta} \max_{A_t^j \geq 0, b_t^{j,G} \geq 0, b_t^j \geq 0, N_{t+1}^j} \left[(\varkappa+1)\log\left(N_{t+1}^j\right) - \log(\varkappa) + \Psi_{t+1,t+2}\right] \tag{47}$$

$$Q_t^K A_t^j \leq \phi N_t^j \tag{48}$$

$$Q_t^K A_t^j + b_t^{j,G} = N_t^j + B_t^j \tag{49}$$

$$N_{t+1}^j(1+\varkappa)/\varkappa = R_{t+1}^A \omega_t^j Q_t^K A_t^j - \frac{B_t^j}{1+\pi_{t+1}}\left(\mathbf{1}_{B_t^j>0} R_t^B + \mathbf{1}_{B_t^j<0} R_t^L\right) + \frac{R_{t+1}^G}{1+\pi_{t+1}} b_t^{j,G}$$

We can simplify rewrite this problem by eliminating N_{t+1}^j and B_t^j using the two equality constraints to get

$$\bar{V}_t\left(N_t^j, \omega_t^j\right) = \widehat{\beta} \max_{A_t^j \geq 0, b_t^{j,G} \geq 0} \left[(\varkappa+1)\log\left(\varkappa/(1+\varkappa)\left[R_{t+1}^A \omega_t^j Q_t^K A_t^j - \frac{Q_t^K A_t^j + b_t^{j,G} - N_t^j}{1+\pi_{t+1}}\left(\mathbf{1}_{N_t^j < Q_t^K A_t^j + b_t^{j,G}} R_t^B + \mathbf{1}_{N_t^j \geq} \right.\right.\right.$$

$$Q_t^K A_t^j \leq \phi N_t^j \tag{50}$$

The first order condition with respect to A_t^j is

$$\frac{\varkappa}{N_{t+1}^j}\left[R_{t+1}^A \omega_t^j Q_t^K - \left(\mathbf{1}_{N_t^j \geq Q_t^K A_t^j + b_t^{j,G}} \frac{R_t^L}{1+\pi_{t+1}} + \mathbf{1}_{N_t^j < Q_t^K A_t^j + b_t^{j,G}} \frac{R_t^B}{1+\pi_{t+1}}\right) Q_t^K\right] - \lambda_{\phi t}^j Q_t^K + \lambda_{At}^j = 0,$$

where $\lambda_{\phi t}^j$, and λ_{At}^j are the Lagrange multipliers associated to the maximum leverage condition and to the non-negativity constraint on real assets, respectively.

The first order condition with respect to $b_t^{j,G}$ is

$$\frac{\varkappa}{N_{t+1}^j}\left[\frac{R_{t+1}^G}{1+\pi_{t+1}} - \left(\mathbf{1}_{N_t^j \geq Q_t^K A_t^j + b_t^{j,G}} \frac{R_t^L}{1+\pi_{t+1}} + \mathbf{1}_{N_t^j < Q_t^K A_t^j + b_t^{j,G}} \frac{R_t^B}{1+\pi_{t+1}}\right)\right] + \lambda_{b_t^{j,G}}^j = 0,$$

where $\lambda_{b_t^{j,G}}^j$ is the Lagrange multipliers associated to the non-negativity constraint on public debt.

Taking into account the positive spread between the borrowing and lending rates $R_t^B \geq R_t^L = R_{t+1}^G$, we may separate three regions:

1. If $R_{t+1}^A \omega_t^j \geq \frac{R_t^B}{1+\pi_{t+1}}$ then $b_t^{j,G} = 0, \lambda_{At}^j = 0$ and banks lever up until they hit the constraint $B_t^j = (\phi - 1) N_t^j$ to finance as much investment as possible : $Q_t^K A_t^j = \phi N_t^j$. In this case next periods pre-dividend equity is given by:

$$E_{t+1}^j = R_{t+1}^A \omega_t^j \phi N_t^j - \frac{R_t^B}{1+\pi_{t+1}}(\phi - 1) N_t^j,$$

2. If $\frac{R_t^L}{1+\pi_{t+1}} \leq R_{t+1}^A \omega_t^j < \frac{R_t^B}{1+\pi_{t+1}}$ then $b_t^{j,G} = 0$, $\lambda_{\phi t}^j = \lambda_{At}^j = 0$ and the maximum is given $Q_t^K A_t^j = N_t^j$.

In this case

$$E_{t+1}^{j} = R_{t+1}^{A}\omega_t^j N_t^j.$$

3. If $R_{t+1}^{A}\omega_t^j < \frac{R_t^L}{1+\pi_{t+1}}$ then $\lambda_{b_t^{j,G}}^{j} = \lambda_{\phi t}^{j} = 0$ the maximum is $A_t^j = 0$.In this case

$$E_{t+1}^{j} = \frac{R_t^L}{1+\pi_{t+1}} N_t^j,$$

and the optimal amount of public debt is undetermined in the range $b_t^{j,G} \in \left[0, N_t^j\right]$.

Plugging our conditional solution for N_{t+1}^{j} into (47) and rearranging we verify our initial guess

$$\bar{V}_t\left(N_t^j\right) = \beta\left(\varkappa+1\right)\log\left(N_t^j\right) + \Psi\left(R_{t+t}^{G}, R_t^L, R_t^B, R_{t+1}^{A}, \pi_{t+1}, \omega_t^j\right),$$

Equating this expression with our initial guess we finally get

$$\varkappa = \frac{\widehat{\beta}}{1-\widehat{\beta}}.$$

A.2 Proof of Proposition 1

Description of the bargaining problem The interbank market is modelled as an OTC market with matching frictions. In particular, we assume that after making their investment choices, banks place orders of size $1/\Delta$ at the interbank market. We will consider the limit case of $\Delta \to \infty$. Since all orders will be carried out, the sum of the orders of each bank is determined by their shortfall/excess of funds given a bank's investment choice and equity. Φ^L and Φ^B denote the total amounts of all banks' lending and borrowing orders.

After orders are placed, lenders and borrowers match. This happens in a multi-round system with initially $N = 2$ rounds of infinitessimal duration (i.e. the interbank market closes within the same period t). All orders outstanding in the final round are automatically redirected to the central bank, who carries them out at the deposit and lending facility rates R_t^{DF} and R_t^{LF}. In each of the previous round, borrowers and lenders are randomly matched according to a matching function $\Upsilon(\Phi^L, \Phi^B)$, giving rise to the matching probabilities Γ^L and Γ^B for lenders and borrowers. If an order finds no match it directly goes to the next round. If a borrowing and a lending order are matched, then the two parties engage in a Nash bargain, in which they may agree upon a certain interest rate for this transaction. ξ denotes the bargaining power of the borrower in these negotiations. If no agreement is reached, this has two consequences. First, the match is dissolved and the two orders go to the next round with probability ϑ and with probability $1-\vartheta$ they are redirected to the central bank. Second, if at any round at least one match fails to reach an agreement, then an additional round of bargaining is added after the current round ($N = N+1$) and all orders, apart from the ones including the non-agreeing banks, proceed to this "additional" next round, independently of whether they found a match and whether they reached an agreement.

The latter "veto power" assumption is assumed to maintain tractability. It allows us to focus on a symmetric equilibrium, in which all matches reach the same agreement R^{IB} in the first round.

To proof the proposition we proceed in 2 steps. First we characterize the Nash bargaining solution for infinitesimal orders for round 1 assuming that no matches fail to reach an agreement in round 2 (if this round is reached). Second we show that an equilibrium exists where all matches agree in the first round.

Nash bargaining round 1 conditional on universal agreement in round 2 Recall the definitions of the value functions from the previous proof. Furthermore, consider two investment banks, j and $\hat{\jmath}$, that are respectively a borrower and a lender in the interbank market trading round in period t (i.e. $B_t^j < 0 < B_t^{\hat{\jmath}}$). Let

$$\eta\left(B_t^j, \Delta\right) \equiv \frac{\left|B_t^j\right|}{\Delta}$$

denote the number of orders placed by bank j (similarly for bank $\hat{\jmath}$).[38] Let R_0 denote the gross interest rate paid on the o-th order, for $o = 1, 2, ..., \eta(B_t^j, \Delta)$, such that total interest payments for bank j are

$$\Delta \sum_{o=1}^{\eta(B_t^j,\Delta)} R_o = \Delta \sum_{o=1}^{\eta(B_t^j,\Delta)-1} R_o + \Delta R_{\eta(B_t^j,\Delta)}.$$

Assume without loss of generality that both banks bargaining over their respective last (η-th) order. Let $\bar{V}_t^{\bar{\jmath}}(R, \Delta, N_t^j)$ denote the value function for a generic bank $\bar{\jmath}$ that bargains over the rate $R \equiv R_{\eta(B_t^j,\Delta)}$ of its last order (taking as given the outcome on all other orders) when orders are of size Δ. At this point, the bank has already chosen its optimal investment policy; we denote by $A_t^{\bar{\jmath}*} \equiv \mathbf{A}_t(N_t^{\bar{\jmath}}, \omega_t^{\bar{\jmath}})$ and $B_t^{\bar{\jmath}*} \equiv \mathbf{B}_t(N_t^{\bar{\jmath}}, \omega_t^{\bar{\jmath}})$ and $b_t^{G,\bar{\jmath}*} = \mathbf{b}_t^G(N_t^{\bar{\jmath}}, \omega_t^{\bar{\jmath}})$ such optimal choices, where $\mathbf{A}_t(\cdot), \mathbf{B}_t(\cdot)$ and $\mathbf{b}_t^G(\cdot)$ are the corresponding policy functions. The function $\bar{V}_t^{\bar{\jmath}}(R, \Delta, N_t^{\bar{\jmath}}, \omega_t^{\bar{\jmath}})$ is given by

$$\begin{aligned} \bar{V}_t^{\bar{\jmath}}(R, \Delta, N_t^{\bar{\jmath}}, \omega_t^{\bar{\jmath}}) &= \widehat{\beta}\mathbb{E}V_{t+1}\left(E_{t+1}^j, \omega_{t+1}^j\right) && (51) \\ s.t.\ E_{t+1}^{\bar{\jmath}} &= R_{t+1}^A \omega_t^{\bar{\jmath}} Q_t^K A_t^{\bar{\jmath}*} - \frac{1}{1+\pi_{t+1}}\left(\Delta \sum_{o=1}^{\eta(B_t^{\bar{\jmath}*},\Delta)-1} R_o + \Delta R_{\eta(B_t^{\bar{\jmath}*},\Delta)}\right) + \frac{R_{t+1}^G}{1+\pi_{t+1}} b_t^{G,\bar{\jmath}*} && (52) \end{aligned}$$

Furthermore assume that all matches agree no later than in the second round. We verify this assumption below.[39] Then the *surplus* for (borrowing) bank j from agreeing on a gross interest rate R, as opposed to dissolving the match and returning to the matching stage is

$$\bar{V}_t^j(R, \Delta, \cdot) - \left[\vartheta \Gamma_t^{B2} \bar{V}_t^j\left(R_t^{IB2}, \Delta, \cdot\right) + \left(\vartheta - \vartheta\Gamma_t^{B2} + 1 - \vartheta\right) \bar{V}_t^j\left(R_t^{LF}, \Delta, \cdot\right)\right].$$

[38] We ignore for simplicity the issues raised in Appendix E in Bianchi & Bigio (2014) regarding the fact that $\eta\left(B_t^j, \Delta\right)$ must be an integer number with $\eta\left(B_t^j, \Delta\right)\Delta < |x|$. This gives rise to residuals and discontinuities that disappear as Δ goes to zero.

[39] There may be other equilibria, but we focus on equlibria where no more than one round of negotiations fails.

Here Γ_t^{B2} is the probability that the order finds a match in the second round, conditional on not being redirected to the central bank and R_t^{IB2} is the corresponding interbank rate.

Similarly, the surplus for (lending) bank $\hat{\jmath}$ from such an agreement is

$$\bar{V}_t^{\hat{\jmath}}\left(-R,\Delta,\cdot\right)-\left[\vartheta\Gamma_t^{L2}\bar{V}_t^{\hat{\jmath}}\left(-R_t^{L2},\Delta,\cdot\right)+\left(\vartheta-\vartheta\Gamma_t^{L2}+1-\vartheta\right)\bar{V}_t^{\hat{\jmath}}\left(-R_t^{DF},\Delta,\cdot\right)\right].$$

where Γ_t^{L2} is the probability that the order finds a match in the second round, conditional on not being redirected to the central bank.

Therefore, the Nash bargaining protocol solves

$$\begin{aligned}&\max_R\left\{\bar{V}_t^{j}\left(R,\Delta,\cdot\right)-\left[\vartheta\Gamma_t^{B2}\bar{V}_t^{j}\left(R_t^{IB2},\Delta,\cdot\right)+\left(\vartheta-\vartheta\Gamma_t^{B2}+1-\vartheta\right)\bar{V}_t^{j}\left(R_t^{LF},\Delta,\cdot\right)\right]\right\}^{\xi}\\&\cdot\left\{\bar{V}_t^{\hat{\jmath}}\left(-R,\Delta,\cdot\right)-\left[\vartheta\Gamma_t^{L2}\bar{V}_t^{\hat{\jmath}}\left(-R_t^{L2},\Delta,\cdot\right)+\left(\vartheta-\vartheta\Gamma_t^{L2}+1-\vartheta\right)\bar{V}_t^{\hat{\jmath}}\left(-R_t^{DF},\Delta,\cdot\right)\right]\right\}^{1-\xi}.\end{aligned}$$

The corresponding FOC is

$$\begin{aligned}&\xi\frac{\bar{V}_t^{\hat{\jmath}}\left(-R,\Delta,\cdot\right)-\left[\vartheta\Gamma_t^{L2}\bar{V}_t^{\hat{\jmath}}\left(-R_t^{L2},\Delta,\cdot\right)+\left(\vartheta-\vartheta\Gamma_t^{L2}+1-\vartheta\right)\bar{V}_t^{\hat{\jmath}}\left(-R_t^{DF},\Delta,\cdot\right)\right]}{m_t^{\hat{\jmath}}\left(\Delta\right)\Delta}\\=\;&\left(1-\xi\right)\frac{\bar{V}_t^{j}\left(R,\Delta,\cdot\right)-\left[\vartheta\Gamma_t^{B2}\bar{V}_t^{j}\left(R_t^{IB2},\Delta,\cdot\right)+\left(\vartheta-\vartheta\Gamma_t^{B2}+1-\vartheta\right)\bar{V}_t^{j}\left(R_t^{LF},\Delta,\cdot\right)\right]}{m_t^{j}\left(\Delta\right)\Delta},\end{aligned}\tag{53}$$

where

$$m_t^j\left(\Delta\right)\equiv\widehat{\beta}\mathbb{E}\frac{\partial V_{t+1}}{\partial E_{t+1}^j}\left(E_{t+1}^j,\omega_{t+1}^j\right)$$

is the marginal value of deposits for bank $\bar{\jmath}=j,\hat{\jmath}$ when orders are of size Δ (with E_{t+1}^j given by 52).

Now consider the limit case that the order size goes towards zero. Since the surplus of both firms is differentiable in Δ and goes to zero as $\Delta\to0$,[40] we can apply L'Hôpital's rule to get

$$\begin{aligned}&\lim_{\Delta\to0}\frac{\bar{V}_t^{j}\left(R,\Delta,\cdot\right)-\left[\vartheta\Gamma_t^{B2}\bar{V}_t^{j}\left(R_t^{IB2},\Delta,\cdot\right)+\left(\vartheta-\vartheta\Gamma_t^{B2}+1-\vartheta\right)\bar{V}_t^{j}\left(R_t^{LF},\Delta,\cdot\right)\right]}{\Delta}\\=\;&\lim_{\Delta\to0}\frac{\partial}{\partial\Delta}\left\{\bar{V}_t^{j}\left(R,\Delta,\cdot\right)-\left[\vartheta\Gamma_t^{B2}\bar{V}_t^{j}\left(R_t^{IB2},\Delta,\cdot\right)+\left(\vartheta-\vartheta\Gamma_t^{B2}+1-\vartheta\right)\bar{V}_t^{j}\left(R_t^{LF},\Delta,\cdot\right)\right]\right\}\\=\;&m_t^j\left(\Delta\right)\left[\left(\sum_{o=1}^{\eta\left(B_t^j,\Delta\right)-1}R_o+R\right)-\vartheta\Gamma_t^{B2}\left(\sum_{o=1}^{\eta\left(B_t^j,\Delta\right)-1}R_o+R_t^{IB2}\right)-\left(\vartheta-\vartheta\Gamma_t^{B2}+1-\vartheta\right)\left(\sum_{o=1}^{\eta\left(B_t^j,\Delta\right)-1}R_o+R_t^{LF}\right)\right]\\=\;&m_t^j\left(\Delta\right)\left(R-\vartheta\Gamma_t^{B2}R_t^{IB2}-\left(\vartheta-\vartheta\Gamma_t^{B2}+1-\vartheta\right)R_t^{LF}\right),\end{aligned}$$

[40] Notice that, from (51), $\bar{V}_t^{\bar{\jmath}}$ depends on Δ and R only through the dependence of V_{t+1} on $E_{t+1}^{\bar{\jmath}}$, which in turn is given by (52). Since $\lim_{\Delta\to0}E_{t+1}^{\bar{\jmath}}$ is independent of whether an agreement is reached or not, we have that the surplus from reaching an agreement converges to zero as $\Delta\to0$.

and similarly for bank $\hat{\jmath}$. We can thus take the limit as $\Delta \to 0$ in (53) to obtain

$$\begin{aligned}
\xi \frac{m_t^j(\Delta)\left(\vartheta \Gamma_t^{L2} R_t^{IB2} + \left(\vartheta - \vartheta \Gamma_t^{L2} + 1 - \vartheta\right) R_t^{DF} - R\right)}{m_t^j(\Delta)} &= (1-\xi) \frac{m_t^j(\Delta)\left(R - \vartheta \Gamma_t^{B2} R_t^{IB2} - \left(\vartheta - \vartheta \Gamma_t^{B2} + 1 - \vartheta\right) R_t^{LF}\right)}{m_t^j(\Delta)} \\
&\Leftrightarrow \\
\xi\left(\vartheta \Gamma_t^{L2} R_t^{IB2} + \left(\vartheta - \vartheta \Gamma_t^{L2} + 1 - \vartheta\right) R_t^{DF} - R\right) &= (1-\xi)\left(R - \vartheta \Gamma_t^{B2} R_t^{IB2} - \left(\vartheta - \vartheta \Gamma_t^{B2} + 1 - \vartheta\right) R_t^{LF}\right).
\end{aligned}$$

Defining the expected interest rate conditional on reaching the matching stage of round 2 as $R_t^{B2} \equiv \Gamma_t^{L2} R_t^{IB2} + \left(1 - \Gamma_t^{L2}\right) R_t^{LF}$ this condition reads

$$\begin{aligned}
\xi\left(\vartheta R_t^{B2} + (1-\vartheta) R_t^{DF} - R\right) &= (1-\xi)\left(R - \vartheta R_t^{B2} - (1-\vartheta) R_t^{LF}\right) \\
&\Leftrightarrow \\
R &= \xi\left(\vartheta R_t^{B2} + (1-\vartheta) R_t^{DF}\right) + (1-\xi)\left(\vartheta R_t^{B2} + (1-\vartheta) R_t^{LF}\right)
\end{aligned}$$

A corresponding logic applies to a match between a borrowing investment bank and a lending retail bank.

Equilibrium We just saw that conditional on every match reaching an agreement in round 2, every match reaches an agreement in round 1. Furthermore notice that round 2 after a veto in round 1 is identical to round 1. Hence we focus on a symmetric equilibrium where all agents behave the same (across agents and rounds).[41] In such an equilibrium it must be that $R_t^{L2} = R_t^L$ and $R_t^{B2} = R_t^B$, where R_t^L and R_t^B are the expected interest rates the banks will obtain at round 1. Considering this and substituting in the expressions (19,20) we obtain

$$R_t^{IB} = \xi\left\{\vartheta\left[\Gamma_t^L R_t^{IB} + \left(1 - \Gamma_t^L\right) R_t^{DF}\right] + (1-\vartheta) R_t^{DF}\right\} + (1-\xi)\left\{\vartheta\left[\Gamma_t^B R_t^{IB} + \left(1 - \Gamma_t^B\right) R_t^{LF}\right] + (1-\vartheta) R_t^{LF}\right\},$$

and

$$R_t^{IB} = \frac{(1-\xi)\left(1 - \vartheta \Gamma_t^B\right)}{\left[1 - \xi \vartheta \Gamma_t^L - (1-\xi) \vartheta \Gamma_t^B\right]} R_t^{LF} + \frac{\xi\left(1 - \vartheta \Gamma_t^L\right)}{\left[1 - \xi \vartheta \Gamma_t^L - (1-\xi) \vartheta \Gamma_t^B\right]} R_t^{DF},$$

Defining

$$\varphi_t = \frac{\xi\left(1 - \vartheta \Gamma_t^L\right)}{\left[1 - \xi \vartheta \Gamma_t^L - (1-\xi) \vartheta \Gamma_t^B\right]}$$

we can express the interbank rate as

$$R_t^{IB} = \varphi_t R_t^{DF} + (1 - \varphi_t) R_t^{LF}$$

as in the proposition. Such an equilibrium exits as long as $1 - \xi \vartheta \Gamma_t^L - (1-\xi) \vartheta \Gamma_t^B \neq 0$.Given that $\xi, \Gamma_t^L, \Gamma_t^B \in (0,1]$ and $\vartheta \in (0,1)$ we have that $1 - \xi \vartheta \Gamma_t^L - (1-\xi) \vartheta \Gamma_t^B > 0$ such that existence is guaranteed.

[41] Note that we are not ruling out that other equilibria exist.

5.0.4 A.3 Proof of Proposition 6

For $\vartheta = 1$ and $\Gamma_t^L = \Gamma_t^B = 1$ the interbank rate is no more pinned down uniquely any more (see the last part of the proof of proposition 1). Any interbank rate can be an equilibrium. We therefore focus on the rate that arises in the limit as $\vartheta \nearrow 1$. $.R_{t|\vartheta=1}^{IB} \equiv \lim_{\vartheta \nearrow 1} R_t^{IB}$. To determine this limit we need to determine the limit $\lim_{\vartheta -> 1} \varphi_t$, since $R_t^{IB} = \varphi_t R_t^{DF} + (1 - \varphi_t) R_t^{LF}$.

$$\lim_{\vartheta \nearrow 1} \varphi_t = \lim_{\vartheta \nearrow 1} \frac{\xi \left(1 - \vartheta \Gamma_t^L\right)}{1 - \xi \vartheta \Gamma_t^L - (1 - \xi) \vartheta \Gamma_t^B}$$

To evaluate this expression for $\Gamma_t^L = \Gamma_t^B = 1$, we need to apply L'Hôpital's rule. Hence

$$\lim_{\vartheta \nearrow 1} \varphi_t = \begin{cases} \xi & \text{if } \Gamma_t^L = \Gamma_t^B = 1 \\ \frac{\xi \left(1 - \Gamma_t^L\right)}{1 - \xi \Gamma_t^L - (1 - \xi) \Gamma_t^B} & \text{else} \end{cases}$$

Given match efficiency we know that $\Gamma_t^B = 1$. Using this the above expression simplifies to

$$\lim_{\vartheta \nearrow 1} \varphi_t = \begin{cases} 1 \text{ if } \Gamma_t^L < 1 \\ \xi \text{ if } \Gamma_t^L = 1 \end{cases}$$

Since $b_t^{G,CB} > 0 \Leftrightarrow \Gamma_t^L < 1$ this can be expressed as a non-continuous function of $b_t^{G,CB}$

$$\lim_{\vartheta \nearrow 1} \varphi_t(b_t^{G,CB}) = \begin{cases} 1 \text{ if } b_t^{G,CB} > 0 \\ \xi \text{ if } b_t^{G,CB} = 0 \end{cases}$$

Hence

$$\lim_{\vartheta \nearrow 1} R_t^{IB} = \begin{cases} R_t^{DF} & \text{if } b_t^{G,CB} > 0 \\ \xi R_t^{DF} + (1 - \xi) R_t^{LF} & \text{if } b_t^{G,CB} = 0 \end{cases}$$

(Alternatively we might focus on the limit $\Gamma_t^L \nearrow 1$. *In that case any balance sheet size, including 0, would imply a corridor system)*

5.0.5 A.4 Proof of Proposition 7

To proof this proposition we start by considering the set of equilibrium conditions defined in appendix B1, but considering the path of the deposit facility rate as exogenous (instead of determined by the TR). Now we use equations (91), (92), (98), (95), to solve for the variables $\Gamma_t^B, \Gamma_t^L, \Phi_t^L, \varphi_t$ in the order of mentioned (notice that for the first 2 variables we used the match efficiency, and that Γ_t^L and Φ_t^L need to be solved for simultaneously)

$$\begin{aligned}
\Gamma_t^B &= 1 & (54)\\
\Gamma_t^L &= \frac{\Phi_t^B}{b_t^{G,CB} + \Phi_t^B} & (55)\\
\Phi_t^L &= b_t^{G,CB} + \Phi_t^B & (56)\\
\varphi_t &= \frac{\xi\left(1-\vartheta\frac{\Phi_t^B}{b_t^{G,CB}+\Phi_t^B}\right)}{\left[1-\xi\vartheta\frac{\Phi_t^B}{b_t^{G,CB}+\Phi_t^B}-(1-\xi)\,\vartheta\right]} & (57)
\end{aligned}$$

Notice that this equation contains 2 endogenous variables (Φ_t^B and φ_t) and 1 exogenous policy variable ($b_t^{G,CB}$). This equation is derived solely from the interbank market and the central bank's balance sheet. The rest of the equilbrium conditions from appendix B1 (after elimination of $\Gamma_t^B, \Gamma_t^L, \Phi_t^L$) defines another realtionship between these two variables. This relationship is much more complex as it depends on the full set of dynamic equilibrium conditions. This relationship can be used to derive an implicit function $\Phi^B(b_t^{G,CB})$.

If we plug this function into the above equation we get the following equation

$$\varphi_t = \frac{\xi\left(1-\vartheta\frac{\Phi^B(R_t^B)}{b_t^{G,CB}+\Phi^B(R_t^B)}\right)}{1-\xi\vartheta\frac{\Phi_t^B(R_t^B)}{b_t^{G,CB}+\Phi^B(R_t^B)}-(1-\xi)\,\vartheta}$$

which shows us the direct effect of $b_t^{G,CB}$ on φ_t as well as the indirect general eequilibrium effect through $\Phi^B(R_t^B)$. Deriving this expression we get

$$\frac{\partial\varphi_t}{\partial b_t^{G,CB}} = \frac{\overbrace{(1-\vartheta)\,\vartheta\,(1-\xi)\,\xi}^{+}\left(\Phi_t^B - b_t^{G,CB}\Phi_t^{B\prime}\right)}{\underbrace{\left(b_t^{G,CB}(1-\vartheta)+b_t^{G,CB}\vartheta\xi+\Phi_t^B(1-\vartheta)\right)^2}_{+}}$$

which is positive if $\Phi_t^B > b_t^{G,CB}\Phi_t^{B\prime}$,.as we shall assume. Deriving again we get

$$\frac{\partial^2\varphi_t}{\left(\partial b_t^{G,CB}\right)^2} = \frac{\overbrace{(1-\vartheta)\,\vartheta\,(1-\xi)\,\xi}^{+}\left\{2\left[\vartheta(1-\xi)-\Phi_t^{B\prime}(1-\vartheta)-1\right]\overbrace{\left(\Phi_t^B-b_t^{G,CB}\Phi_t^{B\prime}\right)}^{+}-b_t^{G,CB}\Phi_t^{B\prime\prime}\overbrace{\left(b_t^{G,CB}(1-\vartheta)+b_t^{G,C\ldots}\right.}^{+}\right.}{\underbrace{\left(b_t^{G,CB}(1-\vartheta)+b_t^{G,CB}\vartheta\xi+\Phi_t^B(1-\vartheta)\right)^3}_{+}}$$

One can show that $\frac{\partial^2\varphi_t}{\left(\partial b_t^{G,CB}\right)^2} < 0$, given that $\xi, \vartheta \in (0,1)$, $\chi \in (0,\infty)$ and $b_t^{G,CB} \geq 0$ this second derivative can be shown to be positive under the condition that

$$2\left[\vartheta\left(1-\xi\right)-\Phi_t^{B\prime}\left(1-\vartheta\right)-1\right]\left(\Phi_t^B-b_t^{G,CB}\Phi_t^{B\prime}\right)<b_t^{G,CB}\Phi_t^{B\prime\prime}\left(b_t^{G,CB}\left(1-\vartheta\right)+b_t^{G,CB}\vartheta\xi+\Phi_t^B\left(1-\vartheta\right)\right)$$

$$\underbrace{\frac{2\left[\vartheta\left(1-\xi\right)-\Phi_t^{B\prime}\left(1-\vartheta\right)-1\right]}{b_t^{G,CB}\left(1-\vartheta\right)+b_t^{G,CB}\vartheta\xi+\Phi_t^B\left(1-\vartheta\right)}\frac{\Phi_t^B-b_t^{G,CB}\Phi_t^{B\prime}}{b_t^{G,CB}}}_{-\text{ if }\Phi_t^{B\prime}\text{ not too negative}}<\Phi_t^{B\prime\prime}$$

It is not possible to prove this condition holds in general equilibrium. However, it does hold in partial equilibrium ($\Phi_t^{B\prime}=\Phi_t^{B\prime\prime}=0$), when the LHS is a strictly negative number and the RHS is 0. Hence it must also hold and when the general equilirbium effects are not too big, i.e. when $\left|\Phi_t^{B\prime\prime}\right|$ and $\left|\Phi_t^{B\prime}\right|$ small enough.

Since $R_t^B=R_t^{IB}=R_t^{DF}+\left[1-\varphi_t\right]\chi$ we have that $\frac{\partial^2\varphi_t}{\left(\partial b_t^{G,CB}\right)^2}<0$ implies $\frac{\partial^2\varphi_t}{\left(\partial b_t^{G,CB}\right)^2}>0$, given we consider R_t^{DF} exogenously fixed.

5.0.6 A.5 Proof of Proposition 10

Consider a particular set of sequence of prices and quantities and exogenous processes, which - together with some initial conditions, constitute an equilibrium. Denote this set of sequences by $\left\{\{x_t^*\}_{x_t\epsilon X_t}\right\}_{t=0}^{\infty}$, where X is the vector of all the quantities and prices and exogenous processes. The sequences $b_t^{G,CB*}$, N_t^*, ω_t^{B*}, Γ_t^{L*}... are elements of this set. Similarly, denote the initial conditions by $\left\{x_{-1}^*\right\}_{x_{-1}\epsilon X_{-1}}$ .Assume that in this particular equilibrium the central bank holds some government bonds $b_t^{G,CB*}>0$ for some t but never holds any loans to banks through direct lending programs $B_t^{CB*}=0$, $\forall t\geq 0$. Note that, for later use, the ratio of lending and borrowing orders in this equilibrium is given by

$$\Phi_t^{L*}/\Phi_t^{B*}=\frac{N_t^*F\left(\omega_t^{L*}\right)-\overbrace{\left(\bar{b}_t^{G*}-b_t^{G,CB*}\right)}^{b_t^{G*}}+B_t^{L*}}{N_t^*\left[1-F\left(\omega_t^{B*}\right)\right]\left(\phi-1\right)},\tag{58}$$

Now consider an alternative set of sequences denoted by $\left\{\{x_t^+\}_{x_t\epsilon X_t}\right\}_{t=0}^{\infty}$, where all sequences are the same as above with a few exceptions: Assume that in this set of sequences the central bank never holds some government bonds, $b_t^{G,CB+}=0$ $\forall t\geq 0$, but sometimes may extend loans to banks through direct lending programs $B_t^{CB+}\geq 0$ for some t . Also allow Φ_t^{B+}, Φ_t^{L+} and b_t^{G+} to differ from their $*-$counterparts. We will now show (by construction) that a path for B_t^{CB+},$b_t^{G,CB+}$, Φ_t^{B+}, b_t^{G+} and Φ_t^{L+} exists such that, given the same initial conditions X_{-1}^*, $\left\{X_t^+\right\}_{t=0}^{\infty}$ constitutes an equilibrium.

For $\left\{X_t^+\right\}_{t=0}^{\infty}$ to be an equilibrium, the equilibrium conditions explicitly stated in appendix B1 (augmented by B_t^{CB}) must hold together with certain implicit non-negativity constraints on certain variables. However, since most sequences in $\left\{X_t^+\right\}_{t=0}^{\infty}$ are equal to $\{X_t^*\}_{t=0}^{\infty}$, all those equilibrium conditions that only include sequences that are equal for $\left\{X_t^+\right\}_{t=0}^{\infty}$ and $\{X_t^*\}_{t=0}^{\infty}$ are obviously satisfied. There are only

5 equilibrium conditions, where the non-identical sequences B_t^{CB+},$b_t^{G,CB+}$, Φ_t^{B+}, b_t^{G+} and Φ_t^{L+} show up. Replacing those $+-$sequences, which are identical to their $*-$counterparts, by the latter, these 5 conditions read:

$$\begin{aligned}
\Phi_t^{B+} &= N_t^* \left[1 - F\left(\omega_t^{B*}\right)\right](\phi - 1) - B_t^{CB+} &(59)\\
\Phi_t^{L+} &= N_t^* F\left(\omega_t^{L*}\right) - b_t^{G+} + B_t^{L*} &(60)\\
\Phi_t^{L+}\left(1 - \Gamma_t^{L*}\right) &= B_t^{CB+} + \Phi_t^{B+}\left(1 - \Gamma_t^{B*}\right) &(61)\\
N_t^* &= \widehat{\beta}\begin{bmatrix} R_t^{A*} Q_{t-1}^{K*} \Omega_{t-1}^* K_{t-1}^* - \frac{(\phi-1)R_{t-1}^{B*}}{1+\pi_t^*}\left[1 - F\left(\omega_{t-1}^{B*}\right)\right] N_{t-1}^* + \\ + \frac{R_{t-1}^{L*}}{1+\pi_t^*}\left(F\left(\omega_{t-1}^{L*}\right) N_{t-1}^* - b_{t-1}^{G+}\right) + \frac{R_t^{G*}}{(1+\pi_t^*)} b_{t-1}^{G+} \end{bmatrix} &(62)\\
\bar{b}_t^{G*} &= b_t^{G,CB+} + b_t^{G+} &(63)
\end{aligned}$$

Notice first that as $b_t^{G,CB+} = 0$, equation (63) requires that $b_t^{G+} = \bar{b}_t^{G*}$. Furthermore, notice that equation (62) is satisfied for all $t > 0$ since in equilibrium $R_{t-1}^{L*} = R_t^{G*}$, as explained in the model section. For $t = 0$ equation (62) holds because $R_{t-1}^{L*} = R_{t-1}^{L+}$ and $b_{t-1}^{G+} = b_{t-1}^{G*}$ (since we start from the same initial conditions) and $R_t^{G+} = R_t^{G*}$ (by construction).

Next, we make the guess that Φ_t^{L+} and Φ_t^{B+} are such that $\Phi_t^{L*}/\Phi_t^{B*} = \Phi_t^{L+}/\Phi_t^{B+}$. Using equations (59,60,63) to replace Φ_t^{L+} and Φ_t^{B+} and equation 58 to replace Φ_t^{L*}/Φ_t^{B*} we get:

$$\frac{N_t^* F\left(\omega_t^{L*}\right) - \left(\bar{b}_t^{G*} - b_t^{G,CB*}\right) + B_t^{L*}}{N_t^*\left[1 - F\left(\omega_t^{B*}\right)\right](\phi - 1)} = \frac{N_t^* F\left(\omega_t^{L*}\right) - \bar{b}_t^{G*} + B_t^{L*}}{N_t^*\left[1 - F\left(\omega_t^{B*}\right)\right](\phi - 1) - B_t^{CB+}}.$$

After rearranging terms, we obtain:

$$B_t^{CB+} = b_t^{G,CB*}\left(\frac{N_t^*\left[1 - F\left(\omega_t^{B*}\right)\right](\phi - 1)}{N_t^* F\left(\omega_t^{L*}\right) - \bar{b}_t^{G*} + b_t^{G,CB*} + B_t^{L*}}\right) = b_t^{G,CB*}\frac{\Phi_t^{B*}}{\Phi_t^{L*}}. \tag{64}$$

Plugging (59) and (60) into (61) we get

$$B_t^{CB+} + \left[N_t^*\left[1 - F\left(\omega_t^{B*}\right)\right](\phi - 1) - B_t^{CB+}\right]\left(1 - \Gamma_t^{B*}\right) = \left[N_t^* F\left(\omega_t^{L*}\right) - b_t^{G+} + B_t^{L*}\right]\left(1 - \Gamma_t^{L*}\right),$$

and then taking into account the value of B_t^{CB+} in (64)

$$b_t^{G,CB*}\frac{\Phi_t^{B*}}{\Phi_t^{L*}}\Gamma_t^{B*} + \left(N_t^*\left[1 - F\left(\omega_t^{B*}\right)\right](\phi - 1)\right)\left(1 - \Gamma_t^{B*}\right) = \left(N_t^* F\left(\omega_t^{L*}\right) - \bar{b}_t^{G*} + B_t^{L*}\right)\left(1 - \Gamma_t^{L*}\right),$$

Using the fact that by interbank market clearing $\Gamma_t^{L*} = \frac{\Phi_t^{B*}}{\Phi_t^{L*}}\Gamma_t^{B*}$ we can write:

$$b_t^{G,CB*}\Gamma_t^{L*} + \left(N_t^*\left[1 - F\left(\omega_t^{B*}\right)\right](\phi - 1)\right)\left(1 - \Gamma_t^{B*}\right) = \left(N_t^* F\left(\omega_t^{L*}\right) - \bar{b}_t^{G*} + B_t^{L*}\right)\left(1 - \Gamma_t^{L*}\right), \tag{65}$$

Since the $*-$sequence constituted an equilibrium, the $*-$counterpart of equations (59-63) must hold. If we again plug the values of Φ_t^{L*} and Φ_t^{B*} in (the $*-$counterpart of) equations (59-60) into (the $*-$counterpart of) equation (61) we get

$$\begin{aligned} \overbrace{B_t^{CB*}}^{=0} + b_t^{G,CB*} + \Phi_t^{B*}\left(1-\Gamma_t^{B*}\right) &= \Phi_t^{L*}\left(1-\Gamma_t^{L*}\right), \\ b_t^{G,CB*} + \left(N_t^*\left[1-F\left(\omega_t^{B*}\right)\right](\phi-1)\right)\left(1-\Gamma_t^{B*}\right) &= \left(N_t^*F\left(\omega_t^{L*}\right) - b_t^{G*} + B_t^{L*}\right)\left(1-\Gamma_t^{L*}\right), \\ b_t^{G,CB*}\Gamma_t^{L*} + \left(N_t^*\left[1-F\left(\omega_t^{B*}\right)\right](\phi-1)\right)\left(1-\Gamma_t^{B*}\right) &= \left(N_t^*F\left(\omega_t^{L*}\right) - b_t^{G*} + B_t^{L*}\right)\left(1-\Gamma_t^{L*}\right), \quad (66) \end{aligned}$$

where in the last equation we have applied the market clearing condition (63).

Since equations (65) and (66) are the same we have shown that all of the five equations above hold for $\left\{\left\{x_t^+\right\}_{x_t \epsilon X_t}\right\}_{t=0}^{\infty}$. Hence, under the condition that the paths Φ_t^{B+}, b_t^{G+} and Φ_t^{L+} are feasible, i.e. non-negative, any equilibrium with bond purchases can be reproduced by an adequate path of lending to banks. This path satisfies the condition that $b_t^{CB+}=0$ if $b_t^{G,CB*}=0$ and $b_t^{G,CB*} \geq b_t^{CB+} = b_t^{G,CB*}\Gamma_t^{L*} > 0$ if $b_t^{G,CB*}>0$. That is a smaller total balance sheet is needed under the loan than under the bond purchases.

Feasibility requires that B_t^{CB+}, $b_t^{G,CB+}$, b_t^{G+}, Φ_t^{B+} and Φ_t^{L+} are all non-negative. This condition holds for the former three sequences as B_t^{CB+} is a exogenous positive variable, $b_t^{G,CB+}=0$ and $b_t^{G+}=\bar{b}_t^{G*}>0$, which again is an exogenous process. For the latter two this means that the following must hold:

$$\Phi_t^{L+} = N_t^*F\left(\omega_t^{L*}\right) - \bar{b}_t^{G*} + B_t^{L*} \geq 0 \quad (67)$$

$$\Phi_t^{B+} = N_t^*\left[1-F\left(\omega_t^{B*}\right)\right](\phi-1) - B_t^{CB+} \geq 0 \quad (68)$$

Substituting for B_t^{CB+} in (64), the latter condition (68) reduces to

$$\begin{aligned} N_t^*\left[1-F\left(\omega_t^{B*}\right)\right](\phi-1) - b_t^{G,CB*}\left(\frac{N_t^*\left[1-F\left(\omega_t^{B*}\right)\right](\phi-1)}{N_t^*F\left(\omega_t^{L*}\right) - \bar{b}_t^{G*} + b_t^{G,CB*} + B_t^{L*}}\right) &\geq 0, \\ N_t^*F\left(\omega_t^{L*}\right) - \bar{b}_t^{G*} + b_t^{G,CB*} + B_t^{L*} &\geq b_t^{G,CB*}, \\ N_t^*F\left(\omega_t^{L*}\right) - \bar{b}_t^{G*} + B_t^{L*} &\geq 0, \quad (69) \end{aligned}$$

which is identical to (67).

Using the $*-$counterpart of equation (61)

$$b_t^{G,CB*} + \Phi_t^{B*}\left(1-\Gamma_t^{B*}\right) = \Phi_t^{L*}\left(1-\Gamma_t^{L*}\right)$$

and using the fact that by interbank market clearing $\Phi_t^{B+}\Gamma_t^{B*} = \Phi_t^{L+}\Gamma_t^{L*}$ we obtain

$$\Phi_t^{B*} = \Phi_t^{L*} - b_t^{G,CB*}$$

Furthermore, using the $*-$counterpart of equation (60)

$$\Phi_t^{B*} = N_t^* F\left(\omega_t^{L*}\right) - \overline{b}_t^{G*} + B_t^{L*}$$

Since Φ_t^{B*} forms part of an equilibrium it must be that $\Phi_t^{B*} \geq 0$. Hence feasibility is always guaranteed.

B. Complete set of equations

B.1 Transitional dynamics

- Households

$$
\begin{aligned}
1 &= \Lambda_{t,t+1} \frac{R_t^L}{1+\pi_{t+1}}, && (70)\\
W_t &= \frac{v'(L_t)}{u'(C_t)}, && (71)\\
\Lambda_{t,t+1} &= \beta \frac{u'(C_{t+1})}{u'(C_t)} && (72)\\
1 &= Q_t^K \left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right] + \Lambda_{t,t+1} Q_{t+1}^K S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2, && (73)\\
K_t &= \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t + (1-\delta)\,\Omega_{t-1} K_{t-1}. && (74)
\end{aligned}
$$

- Firms

$$
\begin{aligned}
Y_t &= \frac{Z_t}{\Delta_t} L_t^{1-\alpha} \left(\Omega_{t-1} K_{t-1}\right)^{\alpha}, && (75)\\
1 &= \theta\,(1+\pi_t)^{\epsilon-1} + (1-\theta)\,(p_t^*)^{1-\epsilon}, && (76)\\
p_t^* &= \frac{\Xi_t^1}{\Xi_t^2}, && (77)\\
\Xi_t^1 &= \frac{\epsilon}{\epsilon-1} M_t Y_t + \theta \mathbb{E}_t \Lambda_{t,t+1} \left(1+\pi_{t+1}\right)^{\epsilon} \Xi_{t+1}^1, && (78)\\
\Xi_t^2 &= Y_t + \theta \mathbb{E}_t \Lambda_{t,t+1} \left(1+\pi_{t+1}\right)^{\epsilon-1} \Xi_{t+1}^2, && (79)\\
\Delta_t &= (1-\theta)\,(p_t^*)^{-\epsilon} + \theta\,(1+\pi_t)^{\epsilon}\,\Delta_{t-1}, && (80)\\
R_t^A &= \frac{R_t^k + (1-\delta)\,Q_t^K}{Q_{t-1}^K}, && (81)\\
R_t^k &= \alpha M_t Z_t \left[\frac{(1-\alpha)\,M_t Z_t}{W_t}\right]^{(1-\alpha)/\alpha}, && (82)\\
L_t &= \left(\frac{(1-\alpha)\,Z_t M_t}{W_t}\right)^{1/\alpha} \Omega_{t-1} K_{t-1}. && (83)
\end{aligned}
$$

- Banks

$$
\begin{aligned}
K_t &= \frac{N_t}{Q_t^K}\left\{\phi\left[1-F\left(\omega_t^B\right)\right]+\left[F\left(\omega_t^B\right)-F\left(\omega_t^L\right)\right]\right\}, \\
N_t &= \widehat{\beta}\Bigg[R_t^A Q_{t-1}^K \Omega_{t-1} K_{t-1}-\frac{(\phi-1)\,R_{t-1}^B}{1+\pi_t}\left[1-F\left(\omega_{t-1}^B\right)\right]N_{t-1}+\frac{R_{t-1}^L}{1+\pi_t}\left(F\left(\omega_{t-1}^L\right)N_{t-1}-b_{t-1}^G\right)+\frac{R_t^G}{(1+\pi_t)} \\
\omega_t^B &= \frac{R_t^B}{R_{t+1}^A\left(1+\pi_{t+1}\right)}, \\
\omega_t^L &= \frac{R_t^L}{R_{t+1}^A\left(1+\pi_{t+1}\right)} \\
R_{t+1}^G &= R_t^L.
\end{aligned}
$$

- Interbank market

$$
\begin{aligned}
\Phi_t^B &= N_t\left[1-F\left(\omega_t^B\right)\right](\phi-1), && (89)\\
\Phi_t^L &= N_t F\left(\omega_t^L\right)-b_t^G+B_t^L && (90)\\
\Gamma_t^B &= \Upsilon\left(\frac{\Phi_t^L}{\Phi_t^B},1\right), && (91)\\
\Gamma_t^L &= \Upsilon\left(1,\frac{\Phi_t^B}{\Phi_t^L}\right), && (92)\\
R_t^B &= \varphi_t\Gamma_t^B R_t^{DF}+\left[1-\varphi_t\Gamma_t^B\right]R_t^{LF}, && (93)\\
R_t^L &= \left(1-\varphi_t\right)\Gamma_t^L R_t^{LF}+\left(1-(1-\varphi_t)\Gamma_t^L\right)R_t^{DF}, && (94)\\
\varphi_t &= \frac{\xi\left(1-\vartheta\Gamma_t^L\right)}{\left[1-\xi\vartheta\Gamma_t^L-(1-\xi)\,\vartheta\Gamma_t^B\right]} && (95)
\end{aligned}
$$

- Central bank

$$
\begin{aligned}
R_t^{LF} &= R_t^{DF}+\chi && (96)\\
R_t^{DF} &= \max\left\{\rho(R_{t-1}^{DF})+(1-\rho)\left[\bar{R}+\upsilon\left(\pi_t-\bar{\pi}\right)\right],1-\kappa\right\}, && (97)\\
b_t^{G,CB}+\Phi_t^B\left(1-\Gamma_t^B\right) &= \Phi_t^L\left(1-\Gamma_t^L\right), && (98)\\
b_t^{G,CB} &= \bar{b}^{G,CB}. && (99)
\end{aligned}
$$

- Government

$$
\begin{aligned}
\bar{b} &= b_t^{G,CB}+b_t^G, && (100)\\
R_t^G &= \frac{\zeta+(1-\zeta)Q_t^G}{Q_{t-1}^G}. && (101)
\end{aligned}
$$

- Aggregate constraint

$$\Omega_{t-1} \equiv \frac{\phi\left[1-F\left(\omega_{t-1}^{B}\right)\right]\mathbb{E}\left(\omega \mid \omega \geq \omega_{t-1}^{B}\right)}{\phi\left[1-F\left(\omega_{t-1}^{B}\right)\right]+\left[F\left(\omega_{t-1}^{B}\right)-F\left(\omega_{t-1}^{L}\right)\right]}+\frac{\left[F\left(\omega_{t-1}^{B}\right)-F\left(\omega_{t-1}^{L}\right)\right]\mathbb{E}\left(\omega \mid \omega_{t-1}^{L} \leq \omega < \omega_{t-1}^{B}\right)}{\phi\left[1-F\left(\omega_{t-1}^{B}\right)\right]+\left[F\left(\omega_{t-1}^{B}\right)-F\left(\omega_{t-1}^{L}\right)\right]} \quad (102)$$

$$Y_t = C_t + I_t + \frac{1-\widehat{\beta}}{\widehat{\beta}}N_t. \quad (103)$$

There are 34 equations and 34 endogenous variables: Y_t, Q_t^K, I_t, C_t, K_t, N_t, W_t, $L_t, \Lambda_{t,t+1}$, M_t, π_t, p_t^*, Ξ_t^1, Ξ_t^2, Δ_t, R_t^A, R_t^k, R_t^L, R_t^B, R_t^{DF}, R_t^{LF}, R_t^G, Γ_t^B, Γ_t^L, Φ_t^L, Φ_t^B, φ_t, ω_t^B, ω_t^L, $b_t^{G,CB}$, b_t^G, B_t^L, Ω_t, Q_t^G.

B.2 Steady-state with zero inflation

- Households

$$\begin{aligned}
R^L &= \frac{1}{\beta}, \\
\Lambda &= \beta, \\
W &= \frac{v'(L)}{u'(C)}, \\
Q &= 1, \\
I &= K\left[1-(1-\delta)\,\Omega\right].
\end{aligned}$$

- Firms

$$\begin{aligned}
Y_t &= (\Omega K)^{\alpha} L^{1-\alpha}, \\
\Delta &= 1, \\
p^* &= 1, \\
\Xi^1 &= \frac{\epsilon}{(\epsilon-1)(1-\theta\beta)}MY, \\
\Xi^2 &= \frac{Y}{(1-\theta\beta)}, \\
M &= \frac{(\epsilon-1)}{\epsilon}, \\
R^k &= \alpha M Z\left[\frac{(1-\alpha)(\epsilon-1)Z}{W\epsilon}\right]^{(1-\alpha)/\alpha}, \\
R^A &= R^k + (1-\delta), \\
L &= \left(\frac{(1-\alpha)Z(\epsilon-1)}{W\epsilon}\right)^{1/\alpha}\Omega K.
\end{aligned}$$

- Banks

$$
\begin{aligned}
K &= N\left\{\phi\left[1-F\left(\omega_{t-1}^{B}\right)\right]+\left[F\left(\omega^{B}\right)-F\left(\omega^{L}\right)\right]\right\}, \\
N &= \beta\left(R^{A}\Omega K-N\left[\left(\phi-1\right)R^{B}\left[1-F\left(\omega^{B}\right)\right]-R^{L}F\left(\omega^{L}\right)\right]\right), \\
\omega^{B} &= \frac{R^{B}}{R^{A}}, \\
\omega^{L} &= \frac{1}{\beta R^{A}}.
\end{aligned}
$$

Interbank market

$$
\begin{aligned}
\Phi^{B} &= N\left(1-F\left(\omega^{B}\right)\right)\left(\phi-1\right), \\
\Phi^{L} &= NF\left(\omega^{L}\right)-b^{G}+B^{L} \\
\Gamma^{B} &= \Upsilon\left(\frac{\Phi^{L}}{\Phi^{B}},1\right), \\
\Gamma^{L} &= \Upsilon\left(1,\frac{\Phi^{B}}{\Phi^{L}}\right), \\
\varphi &= \frac{\xi\left(1-\vartheta\Gamma^{L}\right)}{\left[1-\xi\vartheta\Gamma^{L}-\left(1-\xi\right)\vartheta\Gamma^{B}\right]} \\
R^{B} &= \bar{R}-\Gamma^{B}\varphi\chi, \\
R^{L} &= \bar{R}-\left(1-(1-\varphi)\Gamma^{L}\right)\chi, \\
\varphi &= \frac{\xi\left(1-\vartheta\Gamma^{L}\right)}{\left[1-\xi\vartheta\Gamma^{L}-\left(1-\xi\right)\vartheta\Gamma^{B}\right]}
\end{aligned}
$$

- Central bank

$$
\begin{aligned}
R^{LF} &= \bar{R}, \\
R^{DF} &= \bar{R}-\chi, \\
b^{G,CB}+b^{CB}+\Phi^{B}\left(1-\Gamma^{B}\right) &= \Phi^{L}\left(1-\Gamma^{L}\right), \\
b^{G,CB} &= \bar{b}^{G,CB}.
\end{aligned}
$$

- Government

$$
\begin{aligned}
\bar{b} &= b^{G,CB}+b^{G}, \\
Q^{G} &= \frac{\zeta}{\left(\frac{1}{\beta}+\zeta-1\right)}.
\end{aligned}
$$

- Aggregate constraint

$$\begin{aligned}
\Omega &= \frac{\phi\left[1-F\left(\omega^{B}\right)\right]\mathbb{E}\left(\omega \mid \omega \geq \omega^{B}\right)}{\phi\left[1-F\left(\omega^{B}\right)\right]+\left[F\left(\omega^{B}\right)-F\left(\omega^{L}\right)\right]}+\frac{\left[F\left(\omega^{B}\right)-F\left(\omega^{L}\right)\right]\mathbb{E}\left(\omega \mid \omega^{L} \leq \omega < \omega^{B}\right)}{\phi\left[1-F\left(\omega^{B}\right)\right]+\left[F\left(\omega^{B}\right)-F\left(\omega^{L}\right)\right]}, \\
Y &= C+I+\frac{1-\widehat{\beta}}{\widehat{\beta}}N.
\end{aligned}$$