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* Views expressed are those of the authors and do not necessarily reflect official positions of De Nederlandsche Bank.

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Generalized stability of monetary unions under regime switching in monetary and fiscal policies

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Abstract

Earlier studies on the equilibrium properties of standard dynamic macroeconomic models have shown that an inflation-targeting central bank imposes strict budgetary requirements on fiscal policy needed to obtain a unique and stable equilibrium. The failure of only one fiscal authority within a monetary union to meet these requirements already results in non-existence of equilibrium and an unstable monetary union. We show that such outcomes can be averted if fiscal authorities can make a credible commitment to switch to more sustainable fiscal regimes in the future. In addition, we illustrate how alternative policy measures, such as fiscal bailouts and debt monetization by the central bank, also broaden the range of policy stances under which monetary unions are stable.

Keywords: Markov switching, monetary union, equilibrium stability and uniqueness, monetary-fiscal interactions.

JEL classifications: E62, E63.

1 Introduction

The stability of monetary unions has become a topic of considerable interest in recent years. Much of the literature on the stability and uniqueness of rational expectations equilibria, and the policies required to deliver them, has assumed policy rules to be constant throughout time. However, given the sizable response of policymakers to the recent global recession, the empirical relevancy of this assumption is cast into doubt. Moreover, without autonomous control over monetary instruments, member states of a monetary union are required to adjust their fiscal stance from time to time to absorb country-specific shocks, whilst ensuring a sustainable path for sovereign debt. A more realistic characterization of policy behavior, therefore, is one that allows policymakers to alter their stance in response to changing economic conditions. In this paper, we study the implications of such regime-switching behavior for equilibrium stability in a monetary union.

Using a simple general equilibrium model for an endowment economy, we show that equilibrium stability depends, not only on the policy stance within a particular regime, but also on the *frequency* with which the economy switches between regimes. Our model economy consists of two countries who form a monetary union. Monetary policy is controlled by a supranational central bank, while fiscal policy is conducted nationally (and independently) in each country. Our main contribution is that we allow policymakers to alter their stance over time according to an exogenous Markov process. The model is solved using the ‘forward solution method’ developed by [Cho \(2015\)](#). This method assumes that all agents are aware of the Markov-switching nature of the economy and know the corresponding transition matrix. The ability to alter the policy stance gives rise to different *policy regimes*, i.e. different combinations of monetary and fiscal policies.

Our point of departure is a regime in which monetary policy aims to anchor inflation expectations by actively targeting union-wide inflation through appropriate adjustments in the nominal interest rate. Fiscal policy is conducted differently across the two countries. In one country, taxes respond endogenously to changes in government debt in such a way that

long-term debt sustainability is ensured. In contrast, taxes are kept constant in the other country. As shown in [Leeper \(1991\)](#), this constellation of policies, if held fixed, is unable to deliver a stable equilibrium. In particular, for stability to be achieved in the fixed-regime case, taxes ought to offset changes in government debt, such that the growth rate of debt is below the real interest rate. Yet, if taxes are held constant, national debt can grow without bounds. This result has been shown by [Bergin \(2000\)](#) to carry over to monetary unions, in which stability can be obtained only if *all* member states maintain a sufficient feedback between debt and taxes (provided the central bank actively targets inflation). However, the case in which a member state of a monetary union ignores (at least temporarily) the accumulation of its debt is particularly interesting, given the strong reliance on expansionary fiscal policy to ward-off adverse country-specific shocks. For such cases to be feasible, we consider three alternative regimes to which the economy can move to.

In the first alternative regime, all member states target their public debt through appropriate adjustment in the tax rate, while monetary policy remains dedicated to stabilize inflation (expectations). We show that, if the frequency with which the economy moves to this alternative regime is sufficiently high, a stable equilibrium can be obtained, even if one member state occasionally abandons its debt target in the initial regime. Furthermore, the weaker is the feedback between taxes and debt in the initial regime, the more vigorous must be the response of taxes to debt growth in the alternative regime. Hence, an intertemporal trade-off arises between weak fiscal consolidation today and aggressive consolidation in the future. The *slope* of this trade-off is determined by the fraction of time the economy resides in the initial regime. According to this result, stability of a monetary union can thus be guaranteed, even if member states temporarily divert their fiscal tools away from debt stabilization.

In the second alternative regime, the central bank temporarily abandons its inflation target. It does so to allow inflation reduce the *real* value of national debt to a level consistent with the government's intertemporal budget constraint. Again, if occurring sufficiently often,

switching between the initial regime and this alternative regime can deliver a stable and unique equilibrium. Notice that this result is a generalization of the Fiscal Theory of the Price Level (see e.g. [Leeper, 1991](#), [Sims, 1994](#), and [Woodford, 1996](#)): when the fiscal response to debt is insufficiently strong, a weak monetary response to inflation is a necessary condition for stability, yet unlike the fixed-regime case it is not a *sufficient* condition. Whether the condition is sufficient under regime-switching possibilities now also depends on the fraction of time spent in the initial regime. Although switching to either the first or second alternative regime can deliver stable solutions, the latter necessarily requires greater bouts of inflation in order to force down the debt burden in real terms. In fact, we show that a deficit-financed tax cut raises inflation under the possibility of moving to the second alternative regime, whereas inflation is entirely unresponsive if debt stabilization is achieved through fiscal measures only (due to Ricardian equivalence), as is the case under the first alternative regime.

In the third alternative regime, we allow the partner state to provide a fiscal bailout and thereby assume (part of) the debt burden of the other country. As in the first two cases, this alternative regime allows one of the member states to temporarily disregard its debt obligations without necessarily jeopardizing the stability of the monetary union. In this case, however, the onus of stabilizing debt now falls entirely upon the bailout donor and is increasing in the frequency with which the economy moves to the initial regime. Therefore, this alternative regime unavoidably entails a transfer of wealth between countries required to ensure equilibrium stability.

As a final experiment, we consider the possibility of a country defaulting on (part of) its debt. Although sovereign default relaxes the government's intertemporal budget constraint, we find that it also reduces the likelihood of obtaining a stable equilibrium. In fact, the longer the economy resides in the regime with sovereign default, the smaller will be the feasible set of fiscal and monetary policies. The adverse effects of sovereign default arise due to a crowding-out effect on consumption of a debt-elastic interest rate spread. This spread widens with the prospect of sovereign default and reduces consumption spending by

households. As consumption falls, so does inflation, which raises the real level of government debt and therefore increases the prospect of default further. To escape this vicious spiral, the defaulting country must adopt an aggressive debt consolidation policy in other regimes.

Our results on the implications of switching between various policy regimes offer guidance to policymakers in (re)shaping the institutional framework of monetary unions. Since crises of any nature can be expected to occur at the national level, with limited scope for a centralized response, the viability of a monetary union hinges on the credibility and desirability of the associated national policy responses. Although not evaluated based on a suitable welfare criterion, our results do suggest which policy response is possibly most appropriate when a monetary union faces a sovereign debt crisis. Furthermore, the benefits of credible regime-switching naturally calls into question the appropriateness of budgetary restrictions that are too rigid to permit switching.

This paper builds on [Bergin \(2000\)](#), who examines the implications of fiscal insolvency for equilibrium stability and determinacy in a monetary union under the fixed regime case. [Bergin](#) shows that a unique equilibrium can, in that case, be obtained, either through accommodating monetary policy, that allows for a sufficient rise in the common price level, or when one government's debt is offset by another's budget surplus, which is akin to a fiscal bailout. We show that, when allowing for changes in policy regimes, the conditions for equilibrium stability change markedly and depend on the future expected path and duration of each regime.¹

Likewise, [Davig and Leeper \(2007b\)](#) use a simple dynamic Fisherian model featuring regime-switching monetary policy to show that equilibrium determinacy can be ensured, even if the central bank abandons its inflation target from time to time. As in our paper, what matters for determinacy is that the monetary authority makes credible its commitment

¹The importance of recurring changes in policy regimes has been emphasized in empirical studies as well. For instance, [Favero and Monacelli \(2003\)](#), [Chung et al. \(2007\)](#) and [Bianchi and Melosi \(2014\)](#), among others, show that the Great Inflation of the '60s and '70s in the US has coincided with a weak fiscal response to debt, accommodated by monetary policy, whereas the subsequent Great Moderation was associated with a change in the monetary-fiscal policy mix characterized by more aggressive inflation and debt targeting.

to actively target inflation in the *long run*. In a related paper, [Davig and Leeper \(2007a\)](#) allow for changes in both the monetary and fiscal stance and show that non-existence of equilibrium can be averted, even when debt-targeting by fiscal authorities is weak. For such a regime to be feasible, agents are required to expect more aggressive debt-targeting in the future. Similarly, [Bianchi and Melosi \(2014\)](#) show that, following a severe recession, macroeconomic stability depends not only on the policy response in the short run, but also on agents' expectations of the fiscal stance once the economy exits the recession. We contribute to this literature by extending the analysis to a monetary union and by allowing for explicit fiscal bailout and sovereign default options.

The rest of the paper is organized as follows. Section 2 describes the model and the different policy regimes over which the economy may switch. In this section, we also describe the equilibrium properties in the absence of regime switching. The solution method and equilibrium properties under regime switching are discussed in Section 3. Then, in Section 4, we calibrate the model parameters and examine the implications of regime-switching possibilities for equilibrium stability and uniqueness. In Section 5, we discuss the effects of transitory tax cuts under regime switching. Finally, Section 6 concludes.

2 A regime-switching model for a monetary union

In this section we introduce a simple model of a monetary union that is made up of two endowment economies. The endowments both countries receive are perfectly substitutable and tradable. This means that the law of one price holds within the monetary union.

In the first part of this section, we focus on the supra-national monetary policy rule implemented by the monetary authority. Next, we consider the fiscal policy rules used by the national governments of the two member states. We then consider how these policy decisions influence the households' optimal savings decisions. We consider the no-arbitrage and market clearing conditions that need to hold in equilibrium in the fourth part of this

section. Our main contribution is to allow for the policies followed by the central bank and the national governments to vary over time. In the final part of this section, we introduce the set of policy regimes, and possible transitions between them, that we consider. Throughout, as is commonly done in Markov-switching rational expectations models, we index these regimes by s_t and allow for the policy parameters to vary across regimes.

2.1 Monetary policy rule

The member countries of the monetary union that we study are subject to one single central bank which sets the gross nominal risk-free interest rate, R_t , in order to stabilise union-wide gross inflation, $\pi_t \equiv P_t/P_{t-1}$, where P_t is the aggregate price level in the union. In particular, we assume that the central bank targets a gross inflation rate of $\bar{\pi}$ and does so by following a restricted Taylor rule (Taylor, 1993) of the form

$$\frac{R_t}{R} = \left(\frac{\pi_t}{\bar{\pi}} \right)^{\phi_{\pi, s_t}}. \quad (1)$$

Here, the coefficient $\phi_{\pi, s_t} \geq 0$ determines the degree to which the central bank responds to deviations from the inflation target. This parameter varies across policy regimes, s_t . When $\phi_{\pi, s_t} > 1$ monetary policy is *active* and the central bank raises the real interest rate when inflation increases. Monetary policy is referred to as *passive* if this is not the case. Because the Taylor-rule coefficient varies across the policy regimes, monetary policy does not always have to be active or passive, it can switch between these two stances, as in Davig and Leeper (2007c).

2.2 Fiscal policy, defaults, and bailouts

The two countries, which we index by $j \in \{1, 2\}$, that make up the monetary union each pursue their own fiscal policy. In each period the governments levy a lump-sum tax $\tau_{j,t}$ and issue one-period nominal government bonds. We denote the nominal value of the one-period

sovereign debt of country j in time t by $B_{j,t}$. The gross nominal return on this debt is given by $R_{j,t}$. These taxes and bonds are used to finance public consumption $g_{j,t}$ and payment of the principle and interest on previous period's debt. To keep the model tractable and to avoid having to solve portfolio choice decisions, we assume that all government debt is domestically held by the household sector of the same country.²

We allow for two cases where country 1 does not pay off its debt. The first is by *defaulting*. Such a default is a separate policy regime and results in a haircut on the outstanding debt that is not paid back to the domestic household sector, which are the creditors. The size of the haircut is given by $\delta_{s_t} \in [0, 1]$ and, again, depends on the policy regime. In regimes where country 1 does not default on its debt, $\delta_{s_t} = 0$, whereas in case of default, the sovereign haircut is determined by the degree of government indebtedness, i.e.

$$\delta_{s_t} = \left(\frac{b_{1,t-1}}{b_1} \right)^{\phi_{\delta, s_t}} - 1, \quad (2)$$

where $\phi_{\delta, s_t} \geq 0$ measures the sovereign default elasticity, $b_{1,t} \equiv B_{1,t}/P_t$ denotes real government debt and b_1 is the steady-state value of government debt in country 1.³ Note that, in steady state, $\delta = 0$ for any regime s_t . The default probability depends on the likelihood of ending up in a policy regime in which country 1 defaults, which is a function of the transition probabilities between different regimes. We discuss these transition probabilities in the fourth part of this section.

The second case is a *bailout* in which country 2 takes on part of the debt liabilities of country 1. This is, effectively, a union-wide redistributive fiscal policy from country 2 to country 1. We formalize it as follows. In case of a bailout, country 2 takes over a fraction $\gamma_{s_t} \in (0, 1]$ of country 1's debt. If there is no bailout, $\gamma_{s_t} = 0$. In case of bailout, the bailout

²One can think of this assumption as capturing the exposure of the domestic banking system to sovereign debt of the home country.

³In principle, one can add an intercept and scaling parameter to this equation as well. However, adding such parameters does not qualitatively change the results we present in our analysis. Therefore, we use this much more parsimonious representation.

size is determined by country 1's outstanding public debt:

$$\gamma_{st} = \left(\frac{b_{1,t-1}}{b_1} \right)^{\phi_{\gamma, st}} - 1, \quad (3)$$

with $\phi_{\gamma, st} \geq 0$ denoting the bailout elasticity. The real costs of the bailout for country 2 are given by $\Theta_{st} \equiv \gamma_{st} (1 - \delta_{st}) R_{1,t-1} B_{1,t-1} / P_t$. Just like for the default, the probability of such a bailout depends on the likelihood of ending up in a bailout regime.

The asymmetry between country 1, which potentially defaults on its debt or is the recipient of a bailout, and country 2 which finances the potential bailout of country 1, is reflected in the government budget constraints. The public flow budget constraint of country 1 can be written as

$$B_{1,t} = (1 - \gamma_{st}) (1 - \delta_{st}) R_{1,t-1} B_{1,t-1} - P_t (\tau_{1,t} - g_{1,t}). \quad (4)$$

Here, the bailout size, γ_{st} , and the default haircut, δ_{st} , reduce the (expected) repayments on the previous period's debt obligations. In addition, the path of sovereign debt outstanding is determined by the real primary surplus, $S_{1,t} \equiv \tau_{1,t} - g_{1,t}$ and the price level, P_t .

Since we assume that country 2 always fulfils its debt obligations, its budget constraint does not include a haircut or bailout term. It does, however, contain the cost of bailing out country 1. Country 2's public flow budget constraint reads

$$B_{2,t} = R_{2,t-1} B_{2,t-1} - P_t (\tau_{2,t} - g_{2,t} - \Theta_{st}). \quad (5)$$

The primary surplus for country 2 includes the possible bailout expenditures and is given by $S_{2,t} \equiv \tau_{2,t} - g_{2,t} - \Theta_{st}$.

So far, we focused on the budget constraints of countries 1 and 2. However, in the context of our model, fiscal policy is really about what determines the path of government consumption, $g_{j,t}$, and the lump-sum tax rate, $\tau_{j,t}$. As can be seen from the budget constraints, (4) and (5), what matters for the path of sovereign debt is the *primary surplus*, i.e. the

difference between $\tau_{j,t}$ and $g_{j,t}$, rather than the path of taxes and government consumption individually. With this in mind, we focus on fiscal policy through fluctuations in taxes rather than spending. Thus, we assume that real government consumption remains constant over time in both countries, i.e. $g_{j,t} = g_j$ for all t .

We assume that the government of country j sets the path of taxes according to a fiscal rule that targets a real debt level of \bar{b}_j . To achieve this goal both governments set their taxes in proportion to deviations of the stock of outstanding government debt from the real debt target. In particular, the fiscal authority follows the policy rule

$$\tau_{j,t} = \phi_{b_{j,t}} (b_{j,t-1} - \bar{b}_j) + z_{\tau_{j,t}}. \quad (6)$$

Here, $z_{\tau_{j,t}}$ is a tax shock that represents random, but possibly persistent, deviations from this policy rule. The inclusion of the tax shock allows us to consider impulse response functions, as we do in Section 5. The coefficient $\phi_{b_{j,t}} \geq 0$, which varies by policy regime s_t , characterizes the responsiveness of a country's fiscal policy with respect to debt its level.

Throughout, we use the terminology introduced by [Leeper \(1991\)](#) and call fiscal policy *passive* when $\phi_{b_{j,t}} > \frac{1-\beta}{\beta}$, where $\beta \in (0, 1)$ denotes the household's discount factor. This is the case in which the government prevents public debt from growing faster than the real interest rate. Fiscal policy is called *active* if this is not the case. The distinction between passive and active fiscal policy is important because if public debt persistently grows faster than the real interest rate, the expected future present discounted value of government debt will be infinite.

Of course, the growth of public debt in country 1 is potentially mitigated when it defaults on its debt or when it gets bailed out by country 2. To which degree defaults and bailouts stem the growth of public debt depends on the frequency with which they occur and the magnitude of the haircuts, δ_{s_t} , and the size of the bailouts, γ_{s_t} . Moreover, the growth rate of public debt in country 2 potentially increases in the frequency with which it bails out

country 1 and in the magnitude of these bailouts.

2.3 The household sector

Each country's household sector consists of a representative household. Part of the income of households in each period in both countries is a constant endowment y_j . The rest of the household sectors' income is made up of the interest and principle payments they receive on the one-period government bonds that they invested in during the previous period, i.e. $R_{j,t-1}B_{j,t-1}$. Of course, households in country 1 face the probability that these payments are subject to a haircut due to their government defaulting on its debt. Households spend this income on three different things. First of all, they buy consumption goods, $c_{j,t}$, at the price P_t . Second, they invest in government bonds, $B_{j,t}$. Finally, they pay taxes $P_t\tau_{j,t}$.

Combining these sources of income and expenditure yields the following *ex-post* flow budget constraint of the household sector in country 1

$$P_t c_{1,t} + B_{1,t} + P_t \tau_{1,t} = (1 - \delta_{s_t}) R_{1,t-1} B_{1,t-1} + P_t y_1. \quad (7)$$

Because the government in country 2 never defaults on its debt, households in country 2 are never subjected to a haircut, δ_{s_t} , on the government bonds they own. This means that the period budget constraint that households in country 2 face reads

$$P_t c_{2,t} + B_{2,t} + P_t \tau_{2,t} = R_{2,t-1} B_{2,t-1} + P_t y_2. \quad (8)$$

We assume that the representative households in each country have the same log-utility preferences. Households in country j choose their paths of consumption, $c_{j,t}$, and savings, $B_{j,t}$, to maximise expected life-time utility, given by

$$E_t \sum_{k=0}^{\infty} \beta^k \log c_{j,t+k}, \quad (9)$$

where $\beta \in (0, 1)$ denotes the discount factor. They do so subject to their respective period budget constraints, (7) and (8).

This yields the following two Euler equations, that are the intertemporal optimality conditions for the households in countries 1 and 2 respectively,

$$\frac{1}{c_{1,t}} = \beta R_{1,t} E_t \left[(1 - \delta_{s_{t+1}}) \frac{1}{\pi_{t+1}} \frac{1}{c_{1,t+1}} \right], \text{ and} \quad (10)$$

$$\frac{1}{c_{2,t}} = \beta R_{2,t} E_t \left[\frac{1}{\pi_{t+1}} \frac{1}{c_{2,t+1}} \right]. \quad (11)$$

In addition, their path of consumption and savings satisfies the transversality condition that the expected present discounted value of savings infinitely far in the future is zero.

2.4 No arbitrage and market clearing

In principle, households can also issue one-period non-state-contingent risk-free nominal bonds between each other.⁴ However, these bonds must be in zero net supply in equilibrium and we therefore dropped them in the definition of the household's problem above. The existence of these bonds is important, though, because it implies two no-arbitrage conditions that link the interest rates on public debt to the risk-free rate set by the central bank.

The simplest of these two conditions is for the rate on public debt of country 2, $R_{2,t}$. Since the government in country 2 never defaults, its debt is as risk-free as the rate set by the central bank that determines the price of inter-household borrowing. Thus, in equilibrium, it must be the case that

$$R_{2,t} = R_t. \quad (12)$$

The comparable no-arbitrage condition for country 1 is slightly more complicated. It reads

$$R_{1,t} = R_t \frac{E_t \left[\frac{1}{\pi_{t+1}} \frac{1}{c_{1,t+1}} \right]}{E_t \left[(1 - \delta_{s,t+1}) \frac{1}{\pi_{t+1}} \frac{1}{c_{1,t+1}} \right]} \Xi_t. \quad (13)$$

⁴These bonds are best thought of as the between household borrowing and lending facilitated by the banking system that is not explicitly modelled here.

Here, there are two terms that cause a spread between the interest rate paid on public debt in country 1 and the risk-free rate. The first is the ratio of the expected marginal utility of a dollar spent by the households of country 1 in the next period and the expectation of this same marginal utility corrected for the haircut, i.e. the marginal benefit of saving in the absence of the possibility of a government default and the marginal benefit of saving in the presence of government defaults. The second term that determines the interest rate spread is Ξ_t .

The first term is the direct default premium that makes sure that lenders to the government are compensated for the expected default on public debt in country 1. In our stylized model, we make the assumption that all this debt is owned by the domestic household sector. In this case, if this default premium is the only wedge between the risk-free rate and that on government debt, a default on public debt would have no real effects. This is because the default premium on the government debt, to a first order, exactly compensates households for the expected decline in their earnings due to default.

However, empirically, the spread on government debt is more sensitive to default probabilities than the direct default-premium term in (13) implies (e.g. [Borensztein and Panizza, 2009](#); [Balteanu et al., 2011](#); [Panetta et al., 2011](#); [Bofondi et al., 2013](#); [Demirgüç-Kunt and Huizinga, 2013](#); [Zoli, 2013](#); [Popov and Van Horen, 2013](#); [Albertazzi et al., 2014](#)). Because of this, we assume that there is a ‘risk premium’, Ξ_t , that creates an additional wedge between the lending rate and the risk-free rate R_t . The risk premium reflects tensions in financial markets and is a function of the expected sovereign haircut, i.e.

$$\Xi_t = \left(E_t \delta_{s_{t+1}} \right)^{\chi_{st}} . \quad (14)$$

Equation (14) establishes a link between public and private credit conditions which [Corsetti et al. \(2013\)](#) refer to as the ‘sovereign risk channel’. The sovereign risk channel is typically explained by financial intermediaries holding substantial amounts of sovereign bonds,

thereby exposing the private sector to sudden changes in the value of sovereign bonds.⁵ The extent to which the household sector is exposed to sovereign risk is captured here by the parameter $\chi_{s_t} \geq 0$; when $\chi_{s_t} = 0$, households are fully insulated from changes in sovereign risk. Another explanation is that the government gets charged a risk premium on top of the default premium.⁶

Perfect substitutability and tradability of the endowments, y_1 and y_2 , across countries implies the following goods market clearing condition for the monetary union,

$$c_{1,t} + c_{2,t} + g_1 + g_2 = y_1 + y_2. \quad (15)$$

which in turn implies household consumption in the monetary union is constant.⁷

2.5 Policy regimes and regime switches

Thus far, we have discussed the different monetary and fiscal rules that the central bank and the governments can follow. Here we focus on the likelihood of transitions between these policy rules. In particular, we consider the transitions between different policy *regimes*. Since the government of country 2 is assumed to always pursue a passive fiscal policy rule, each of these policy regimes consists of a combination of the policy pursued by the central bank and that pursued by the government of country 1.

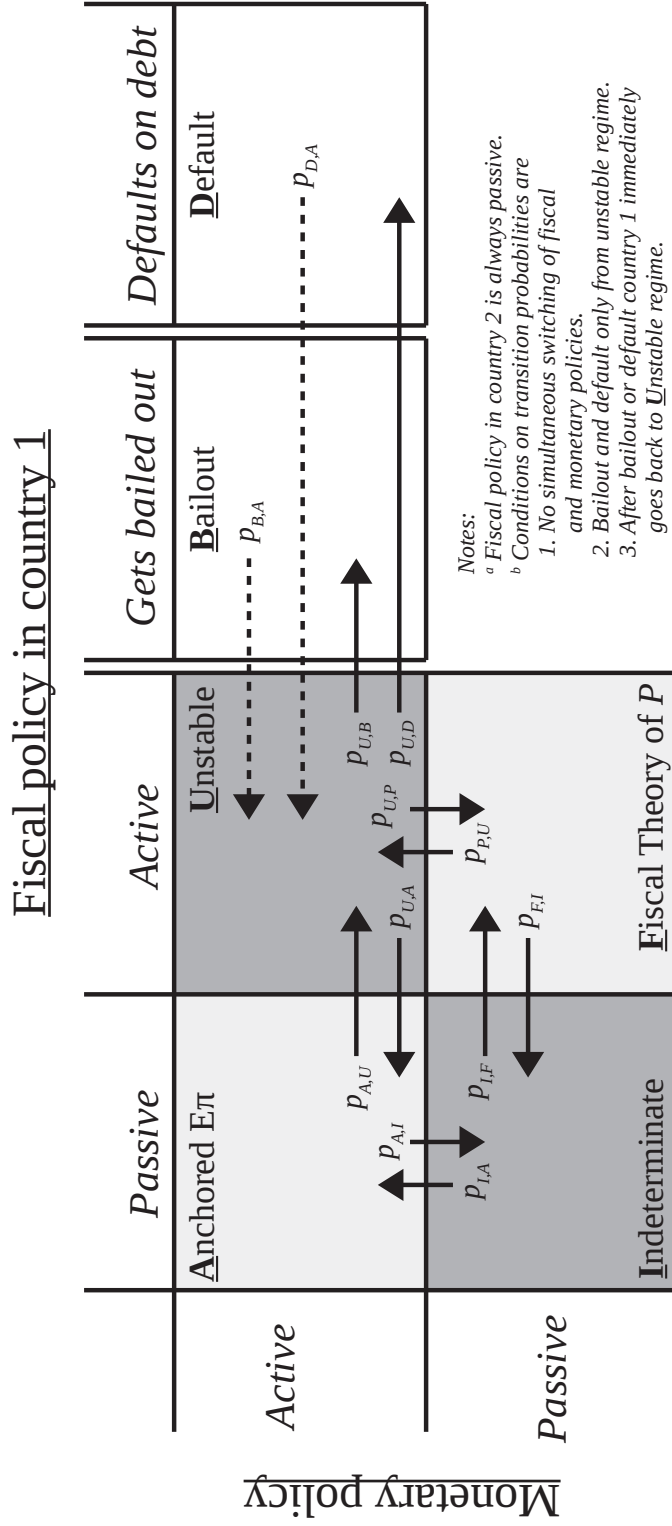
Figure 1 depicts the grid with the six different policy regimes that we consider. To make clear how these regimes are linked to previous studies, we name them after the property of the rational expectations equilibrium of the model in case of *no regime* switches. Doing

⁵According to another theory on the sovereign risk channel, heavily strained public finances may adversely affect private borrowing conditions when a country's sovereign is forced to raise future taxes significantly, or even appropriate private property, thereby limiting the ability of firms and households to honour their own debt obligations.

⁶Such a risk premium would drop out in the linearized solution method we use in this paper and one can interpret χ_{s_t} as a way to reintroduce it to illustrate its importance.

⁷Because aggregate endowments in the union are constant, there is no stimulative role of either fiscal or monetary policy at the monetary union level. All stimulus, in terms of consumption, in one country comes at the cost of consumption in the other.

Figure 1: Policy regimes, names, and transition probabilities



so allows us, later on, to emphasize how the regime switches that we introduce change the equilibrium properties of the model. We index the six regimes we distinguish by their first letter.

The six regimes can be split up into three groups of two. The first group consists of the light-grey shaded regimes in Figure 1. These are the regimes under which, in the absence of regime changes, there is a well-defined rational expectations equilibrium. They are:

- **Ancored $E\pi$.** This is the regime under which the policy stances by the central bank and the government of country 1 result in *anchored inflation expectations* and long-run fiscal stability. This is the case when monetary policy is active, and follows the [Taylor-principle](#), and fiscal policy in country 1 is passive. That is, $\phi_{\pi, s_t} > 1$ and $\phi_{b_1, s_t} > \frac{1-\beta}{\beta}$. This is the type of equilibrium that is commonly studied in New-Keynesian models ([Woodford, 2001](#)).
- **Fiscal Theory of the Price Level.** This is the regime under which fiscal policy in country 1 does not assure long-run fiscal stability for any path of inflation. In that case the price level jumps to affect real debt holdings by the government such that the present discounted value of future real government debt is finite. Such jumps in the price level mean that, in this regime, inflation expectations are unanchored. This is the case when monetary policy is passive and fiscal policy in country 1 is active. That is, $\phi_{\pi, s_t} \leq 1$ and $\phi_{b_1, s_t} \leq \frac{1-\beta}{\beta}$.

The second group is made up of the two regimes under which, if they were permanent, there is no well-defined rational expectations equilibrium. These are the dark-grey shaded regimes in Figure 1. They are:

- **Indeterminate.** This is the regime in which the fiscal authority in country 1 runs a passive policy, $\phi_{b_1, s_t} > \frac{1-\beta}{\beta}$, that assures long-run fiscal stability no matter what the path of inflation. However, the central bank runs a passive monetary policy, $\phi_{\pi, s_t} \leq 1$, that does not pin down a unique path of inflation expectations. As a consequence, for

any level of real government debt by countries 1 and 2, inflation expectations are not uniquely determined and there are multiple equilibrium paths consistent with rational expectations.

- **Unstable**. Under this regime the central bank pursues an active monetary policy, $\phi_{\pi, s_t} > 1$, that aims to uniquely pin down inflation expectations. However, the government of country 1 runs an active fiscal policy, $\phi_{b_1, s_t} \leq \frac{1-\beta}{\beta}$, which results in a non-zero expected long-run present discounted value of the country's real debt levels for the path of inflation implied by monetary policy. As [Bergin \(2000\)](#) shows, in this case there does not exist a stable rational expectations equilibrium no matter how small country 1 is in the monetary union.

The third group is made up of the two regimes in which real debt holdings of country 1 are reduced through channels different from conventional fiscal policy. These are:

- **Bailout**. This is the case in which country 1's real debt holdings are reduced through a bailout by country 2.
- **Default**. This is the case in which country 1's government defaults on a fraction of the debt it owes the households in country 1.

The inexistence of a rational expectations equilibrium in the unstable regime means that current economic theory has little to say about monetary unions where the central bank sticks to the Taylor Principle while fiscal authorities' decisions are not consistent with long-run solvency. The only thing we learn from current theory is that, if the policy stances in this regime were *permanent*, then the governments with the explosive debt path would not have access to sovereign debt markets.

However, if the policy stances are not permanent then the economy might cycle between regimes in which the debt of country 1 grows precipitously and cycles where its growth slows down or even reverses. If the economy is not in the unstable regime too frequently and

excessive debt growth in the periods in the unstable regime are offset by slowdowns in growth or reductions of real debt levels in other regimes then, even though the economy might sometimes transition through the unstable regime, the rational expectations equilibrium could still be stable and determinate.

In order to formalize this intuition, we have to be explicit about how the economy moves through the different policy regimes. Following other studies that use Markov-Switching Rational Expectations models, we assume that the transition probabilities between policy regimes only depend on the policy regime that the economy is in and that they do not depend on other equilibrium variables that determine the state of the economy we consider.⁸ We assume that these transition probabilities are constant over time. Throughout, p_{s_{t-1},s_t} denotes the probability of being in regime s_t in period t conditional on being in regime s_{t-1} in the previous period.

An unconstrained Markov transition matrix between our six policy regimes is made up of 30 transition probabilities. Though it is, in principle, possible to analyze the effect of each of these parameters on the equilibrium properties of our economy, this turns out to be rather unwieldy in practice. For tractability purposes, we impose three conditions on the Markov transition matrix. These conditions are listed as items 1 through 3 in note *b* at the bottom-right corner of Figure 1.

The first condition is that there are no concurrent switches of both the monetary and fiscal policy stances. This implies the following four restrictions on the Markov transition matrix: $p_{A,F} = p_{F,A} = p_{U,I} = p_{I,U} = 0$.

The second condition is that a bailout or a default only occurs if the economy is in the unstable regime in the previous period. That is, we assume that these unconventional measures occur when the central bank sticks to the Taylor Principle and the government of

⁸For example, the probability of a bailout depends on whether the economy is in the Anchored inflation expectations, FTPL, Indeterminate, or Unstable regimes. However, it does not depend on the real debt levels of countries 1 and 2. Current state-of-the-art solution techniques for rational expectations models with regime switching are not able to handle such state-dependence of the regime transition probabilities. This is why we abstract from it.

country 1 pursues a fiscal policy under which, if it sticks to it, its debt grows at an unsustainable rate. This simple restriction implies two assumptions. The first is that bailouts and defaults only occur when agents in the economy get concerned about the long-run sustainability of the current fiscal policy stance. The second is that there are no consecutive periods with either bailouts or defaults. This pins down the following ten transition probabilities $p_{s_{t-1},B} = p_{s_{t-1},D} = 0$ for $s_{t-1} \in \{A, I, F, B, D\}$.

Because we use the Unstable regime as our benchmark in the rest of our analysis, we impose the condition that after a bailout or default the government of country 1 resumes the active fiscal policy stance it took before. Together with the previous conditions this means that $p_{B,s_t} = p_{D,s_t} = 0$ for $s_t \in \{I, F, A\}$ and $p_{B,U} = p_{D,U} = 1$. These latter are the transitions associated with the dashed arrows in Figure 1.

Jointly, the three conditions imply that monetary policy is always active in case of a default or bailout. This is why the bottom-right two cells of the policy regime grid in Figure 1 are not plotted. In total, these three conditions impose twenty conditions on the Markov transition matrix. The solid arrows in Figure 1 show the ten regime changes that correspond to the restricted transition probabilities that we consider in the rest of our analysis.

3 Equilibrium conditions and properties

Throughout the rest of this paper we investigate the stability, determinacy, and dynamic properties of the equilibrium of the Markov Switching Rational Expectations (MSRE) model that we introduced in the previous section. In this section we discuss the solution method we use and the associated equilibrium conditions that we check for stability and determinacy. In addition, we describe the additional equilibrium properties that we focus on in the numerical results we present in the subsequent sections.

3.1 Solution method and stability conditions

Analyses of conventional Linear Rational Expectations (LRE) models without regime switching generally rely on the eigenvalue conditions described in [Blanchard and Kahn \(1980\)](#) to establish stability and determinacy of the equilibrium. However, as [Farmer et al. \(2009\)](#) point out, these conditions are necessary but not sufficient in the context of models with regime changes. For this reason, we instead use the solution method introduced by [Cho \(2015\)](#).^{9,10} Cho’s method is a generalization of the application of the method of undetermined coefficients in LRE models developed by [McCallum \(2007\)](#) and [Cho and McCallum \(2012\)](#) to MSRE models. It allows for both investigating the necessary and sufficient conditions for stability and determinacy as well as the dynamics of the equilibrium through Impulse Response Functions.

The main insight of the method is that the solution of the MSRE model can be written in a specific matrix form. The elements of these matrices are the undetermined coefficients that need to be solved for to obtain the solution. Unfortunately, the system of matrix equations can not be solved directly. It turns out, though, that a solution can be obtained by iterating over the equations until convergence. This is what [Cho and Moreno \(2011\)](#) call the “Forward Method.”

Necessary and sufficient conditions for determinacy, indeterminacy, and instability can then be derived in terms of the solution matrices. In our numerical analysis it is these conditions that we check to see whether our model yields a mean-square stable Rational Expectations Equilibrium and to establish whether this equilibrium is either determinate or indeterminate.

Because the solution method yields a complete representation of the solution of the MSRE model, it also allows for the calculation of impulse response functions, which differ depending

⁹In particular, [Cho \(2015\)](#) derives these necessary and sufficient conditions for the case where stability, determinacy, and indeterminacy in the MSRE model are defined in terms of mean-square stable solutions.

¹⁰The mathematical details of how we linearize our model and write it in the representation used by [Cho \(2015\)](#) as well as the main results from [Cho’s](#) paper that we utilize can be found in the appendix.

on the policy regime in place when shocks occur. We use these impulse response functions for two main purposes.

3.2 Quantifying how unanchored inflation expectations are

In Figure 1 we distinguished between the two determinate policy regimes as one being “Anchored Inflation Expectations”, A, and the other being the “Fiscal Theory of the Price Level,” F. In terms of the solution method by Blanchard and Kahn (1980), this distinction stems from the stable nodes in (b_t, π_t) -space. When there are no policy changes in the A regime, fiscal policy is always passive in all member countries of the monetary union. This assures that, no matter what the path of inflation, real government debt grows at a rate at which the transversality conditions on the governments’ budget constraints hold. Together with the active monetary policy, this pins down a unique path of inflation. In the absence of any shocks, this path corresponds to inflation being anchored at its steady-state level.¹¹ This means that the stable node of the model in the A regime, obtained using the solution method of Blanchard and Kahn (1980), is $(b_t, 0)$.¹² That is, no matter what the level of real government debt, in the absence of any shocks inflation is equal to zero.

Thus, in the A regime, inflation does not respond to shocks to government debt. That is, if one assumes that fiscal policy is passive and monetary policy is active, then one can analyze monetary policy without making any specific assumptions about fiscal policy. This is the reason that monetary policy is often analyzed in models that do not even include a government sector, fiscal policy, and government debt.

In the F regime, fiscal policy is not passive and does not assure that the transversality condition on real government debt obligations is satisfied. Instead, because monetary policy is passive, it is the path of inflation that adjusts to assure long-run solvency of the government. As Kocherlakota and Phelan (1999) discuss, in this case it is the fiscal policy choice of the government, in our model the government of country 1, that acts as a selection

¹¹This steady-state level is zero in the case of our stylized model.

¹²See Davig and Leeper (2007a) for an example of this result.

mechanism among the many paths of inflation that are consistent with the passive monetary policy stance of the central bank. This implies that, in this regime, inflation expectations are unanchored and are a function of the level of real government debt, i.e. the stable node is of the form $(b_t, a\pi_t)$, where $a > 0$.

The solution method that we apply does not yield specific stable nodes. However, the equivalent implications can be checked by considering the impulse response of inflation to a shock to real government debt holdings. In case of *anchored* inflation expectations, inflation does not respond at all to a shock to the level of real government debt. In case of *unanchored* inflation expectations, inflation adjusts to the level of government debt. Hence, the first purpose for which we use the impulse response functions is to quantify the magnitude of the impulse response of inflation to a shock to government debt, which we interpret as a measure of how unanchored inflation expectations are.

3.3 Dynamic response to shocks in the unstable regime

The second way in which we use the impulse response functions is to document the response of the economy to a fiscal shock for parameter combinations for which the model with regime switching yields a determinate equilibrium in which there is a positive probability of the economy ending up passing through the Unstable policy regime. In particular, we consider the response of the economy in case the fiscal shock occurs when the economy is in the U regime. The reason we focus on this is because, in the absence of regime switching, an equilibrium path does not even exist. Thus, we focus on the dynamics of a monetary union that starts off in a situation in which the central bank pursues an active monetary policy and in which some of its members, as captured here by country 1, pursue a fiscal policy that, if unaltered, does not assure long-run fiscal sustainability. This is a very relevant real-life scenario for which current theory has little more to offer than the observation that the monetary union is not stable in that case.

We focus on how the stability and determinacy properties of the equilibrium change in

case of regime switching in the next section. We document the dynamic response of the economy to a fiscal shock in Section 5.

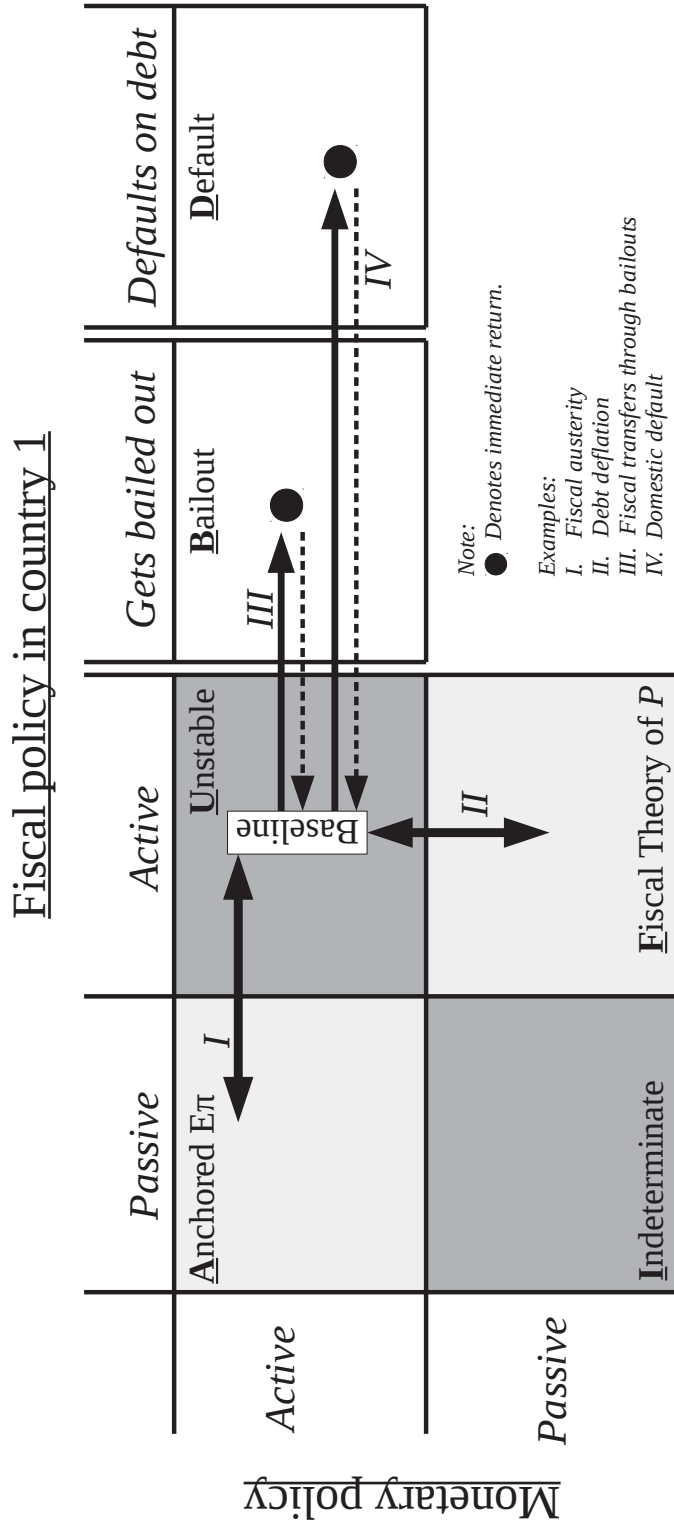
4 Generalized stability conditions

In this section, we illustrate that episodes of combined active monetary policy and active fiscal policy do not necessarily lead to instability of a monetary union. In such a case, the stability of the union in our model can potentially be established by offsetting such episodes through four different channels. The first is through periods of *fiscal austerity* in which the government of country 1 temporarily pursues a passive fiscal policy. The second channel is through *debt deflation* when the central bank sometimes abandons its active monetary policy and allows the path of inflation to adjust to reduce the real sovereign debt burden of country 1. The third is through *fiscal transfers from country 2 to country 1 in the form of bailouts*. The last potential channel is through the government of country 1 defaulting on its obligations towards its own domestic household sector.

We illustrate our main point for each of these four channels using four examples. Each of the examples isolates one of the four respective channels. The most important thing they have in common is that they all involve the economy being in the Unstable regime periodically. In fact, our baseline case is the one in which the economy is *always* in the Unstable regime and there, thus, does not exist a rational expectations equilibrium. Figure 2 depicts the four examples we consider in the regime-grid introduced in Figure 1. The four examples, numbered I through IV in the figure, correspond to the respective channels discussed above. The baseline regime is highlighted as the origin of the arrows for each of the examples.

Beside the periodic episodes in the Unstable regime, the examples also have a set of benchmark parameter values in common. These are the values of the parameters that are relevant in all examples. Keeping these parameters fixed enhances the comparability across

Figure 2: Four examples of generalized stability conditions



examples and allows us to isolate the importance of the parameters we change for the stability conditions of the monetary union in our model. The benchmark parameter values across examples are listed in Table 1 and are based on a quarterly frequency for t . To clarify our parameter choices, we added a column with the interpretation of the parameter values chosen. Lines 4 and 5 of the table together with the goods market clearing condition (15) imply that consumption makes up 60% of output, i.e. $c_1 + c_2 = 0.6$.

4.1 Fiscal austerity (I)

The first example that we consider is the one in which periods of *fiscal austerity* in country 1 potentially offset the explosive growth of real government debt under the active fiscal policy stance the government of country 1 pursues when in the Unstable regime that is our baseline. Such episodes of *fiscal austerity* are periods during which the government of country 1 implements a passive fiscal policy rule focused on stabilizing its real debt level to sustainable levels. We use this example to point out three main insights. The first is that, for a given frequency of switching between fiscal policy regimes in country 1, the more explosive real debt levels are in the baseline U regime the more austere fiscal policy needs to be in the other regime to assure stability of real debt levels of country 1.

To illustrate this, we consider variation in $\phi_{b_1,A}$ versus $\phi_{b_1,U}$ for a given pair of transition probabilities, $p_{U,A}$ and $p_{A,U}$. Because, as you can see from Figure 2, we assume that the other transition probabilities in this example are zero, the choice of the pair of transition probabilities also pins down $p_{UU} = 1 - p_{UA}$ and $p_{AA} = 1 - p_{AU}$. Solving for the steady state of the two-state Markov process implied by these transition probabilities yields that the fraction of time the economy spends in the Unstable regime, f_U , in this case equals

$$f_U = \frac{1}{1 + \frac{p_{UA}}{p_{AU}}}.$$

So, what really matters for the stability of the economy in this case is the *ratio* of the

Table 1: Benchmark parameter values

No.	Parameter	Description	Value	Interpretation
1.	β	Discount factor	0.99	4% annual real interest rate
2.	ρ_τ	Tax-smoothing parameter	0.9	High persistence of tax shocks (Traum and Yang, 2013)
3.	b_1, b_2	Steady-state debt-to-output ratio	2.4	60% debt to annualized GDP ratio
4.	y_1, y_2	Output levels	$y_1 = y_2 = 0.5$	Monetary union of “equals”
5.	g_1, g_2	Steady-state government spending ratio	0.2	20% government spending to GDP ratio
6.	$\phi_{\pi,A}, \phi_{\pi,U}$	Active monetary policy stance	1.5	Consistent with Taylor rule (Taylor, 1993)
7.	ϕ_{b_2}	Fiscal policy in country 2	0.02	Ensures passive fiscal policy

transition probabilities. We report the results of our experiment in terms of how the stability of the monetary union depends on f_U .¹³ We consider which combinations of $(\phi_{b_1,U}, \phi_{b_1,A})$ where $\phi_{b_1,U} \leq \frac{1-\beta}{\beta} < \phi_{b_1,A}$ result in a stable equilibrium for a given fraction of time the economy spends in the Unstable regime.

The theoretical benchmark, based on [Bergin \(2000\)](#), is the case in which $f_U = 1$ and the unstable regime is an absorbing state. In that case the monetary union is unstable no matter what the fiscal policy stances $(\phi_{b_1,U}, \phi_{b_1,A})$. [Figure 3](#) shows the combinations of $(\phi_{b_1,U}, \phi_{b_1,A})$ that result in a stable equilibrium for different levels of $f_U < 1$. In particular, it plots the boundary of the stable parameter set for different values of f_U . For each of these boundaries, the points on the lower-left side are the policy stances under which there is no stable equilibrium while those on the upper-right are part of the stable policy parameter space for the given level of f_U that the boundary corresponds with.

As this figure shows, the more active fiscal policy is in the unstable regime, i.e. the lower $\phi_{b_1,U}$, the more austere fiscal policy needs to be when it is passive, i.e. the higher $\phi_{b_1,A}$, to assure stability of the monetary union. This can be seen from the fact that the boundary of the stable parameter set is downward sloping in the Figure for all the values $f_U < 1$.

The second main insight is that there is a trade-off between the degree and frequency of austerity. That is, for a given active fiscal policy in country 1 in the baseline regime, the more frequently the government of country 1 temporarily takes a passive fiscal policy stance the less severe the degree of austerity during such periods has to be to assure long-run fiscal solvency. This can also be seen from the Figure when one considers the evolution of the stable parameter set as a function of f_U for a given $\phi_{b_1,U}$. Namely, the more time the economy spends in the unstable regime, i.e. the higher f_U , the more passive fiscal policy needs to be in periods of austerity to assure stability. That is, the higher $\phi_{b_1,A}$. Thus, there is a direct trade-off between the frequency and severity of austerity measures needed to stabilize the monetary union.

¹³Since f_U only depends on the ratio p_{UA}/p_{AU} , we normalize p_{UA} to equal 0.05 such that the expected duration of an episode in the Unstable regime is twenty quarters (5 years).

The final point we make with this example is that if monetary policy does not have any real effects on economic outcomes in country 1 and is always active, then stability is purely a fiscal policy issue. That is, the fiscal policy stances that result in stability of the monetary union do not depend on the monetary policy rule, i.e. on $\phi_{\pi, s_t} > 1$. In terms of Figure 3, as long as $\phi_{\pi, A} > 1$ and $\phi_{\pi, U} > 1$ then the boundaries of the stable parameter sets plotted in the figure are invariant to the actual values of $\phi_{\pi, A}$ and $\phi_{\pi, U}$.

The reason for this is that the dynamics of the evolution of inflation expectations under active monetary policy in this example do not depend on fiscal policy. In particular, this evolution is given by

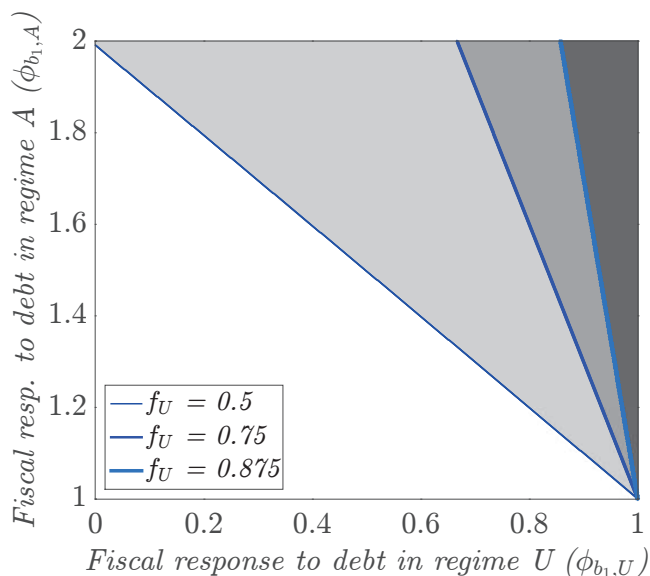
$$E_{t|s_t} [\pi_{t+1}] = E_{t|s_t} [\phi_{\pi, s_t}] \pi_t \text{ for } s_t \in \{U, A\}. \quad (16)$$

Thus, because $\phi_{\pi, A} > 1$ and $\phi_{\pi, U} > 1$

$$E_{t|s_t} [\phi_{\pi, s_t}] > 1 \text{ for } s_t \in \{A, U\}.$$

Thus, the path of inflation expectations is always an unstable node and inflation expectations are anchored, no matter what the fiscal policy stances, $\phi_{b_1, A}$ and $\phi_{b_1, U}$, and the frequency of occurrence of regimes, f_U . Conditional on these anchored inflation expectations, stability then is a matter of fiscal policy assuring a zero expected long-run discounted value of real debt burdens. This long-run stability of the real debt burden depends on the weighted average of the fiscal policy stances in country 1 under the A and U regimes, where the weight of the latter equals the frequency with which the economy is in the Unstable regime, i.e. f_U . This is what drives the trade offs in the first two points of this example and the shape of the stable parameter sets in Figure 3.

Figure 3: Stable parameter sets $(\phi_{b_1,U}, \phi_{b_1,A})$ under Fiscal Austerity experiment



Notes: The figure shows the stable parameter sets in $(\phi_{b_1,U}, \phi_{b_1,A})$ -space for three different frequencies of occurrence of the Unstable regime, f_U . The areas to the lower-left of the boundaries of the sets for the different f_U reflect parameter combinations where there is no stable equilibrium. The gray-shaded areas depict parameter combinations that result in a stable equilibrium.

4.2 Debt deflation (II)

Where in the previous example stability and determinacy of the equilibrium was the result of the fiscal authority in country 1 frequently stepping on the break during periods of austerity to stem real debt growth, in this example we consider the case in which stability and determinacy are achieved through the reduction of the real value of country 1's nominal debt obligations as a consequence of increased inflation. Thus, this example is a classic case of debt deflation, as first described by Fisher (1933). In the context of the interaction of fiscal and monetary policy that we consider here, of course, this is more often referred to as the Fiscal Theory of the Price Level. So, in this example, we consider the stability and determinacy of the equilibrium in this economy when it switches between the baseline Unstable regime, in which both the central bank and the government of country 1 take an active policy stance, and the Fiscal Theory of the Price Level regime, in which the central bank abandons its active monetary policy stance. The stable and determinate equilibrium outcomes in this

case involve unanchored inflation expectations, such that the level of inflation depends on the deviation of country 1's real debt level from its target, \bar{b}_1 .

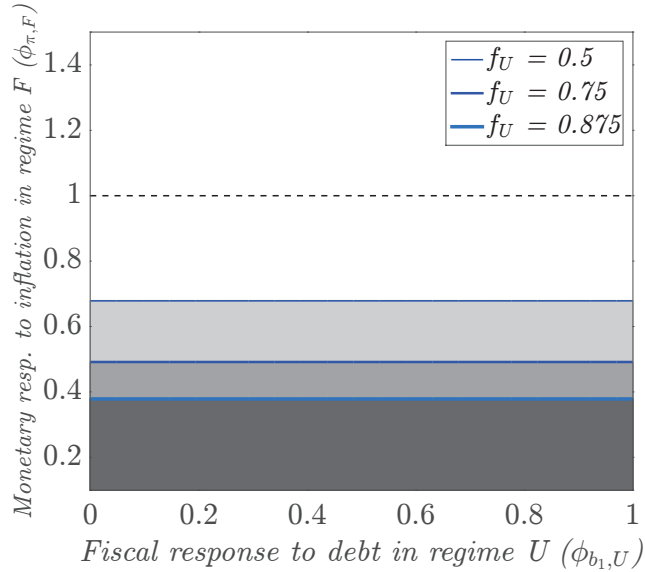
This experiment is thus about the interaction of the monetary policy and fiscal policy responses in the Unstable and the Fiscal-Theory-of-the-Price-Level regimes. These policies, in principle, involve four parameters, namely $\phi_{\pi,U}$, $\phi_{\pi,F}$, $\phi_{b_1,U}$, and $\phi_{b_1,F}$. In our example we reduce this to two parameters. This is because we keep $\phi_{\pi,U} = 1.5$ at its benchmark value. Moreover, we assume that the stance of fiscal policy does not vary across the Unstable and the Fiscal Theory of the Price Level regimes, such that $\phi_{b_1,U} = \phi_{b_1,F}$. Given these two restrictions, we can consider the stability of equilibrium in $(\phi_{b_1,U}, \phi_{\pi,F})$ -space. Just like in the previous example, we consider these stability sets for different frequencies with which the economy is in the unstable regime, i.e. f_U . In this case, this frequency is given by

$$f_U = \frac{1}{1 + \frac{p_{UF}}{p_{FU}}}.$$

The resulting stability sets for the parameters are plotted in Figure 4. The most striking feature of the figure is that the boundaries of the stable parameter sets are horizontal. This indicates that, conditional on persistent active fiscal policy in country 1, the only thing that matters for stability of the equilibrium is whether the central bank pursues a passive enough monetary policy such that inflation expectations are unanchored. This unanchoredness allows inflation to adjust to stabilize country 1's real debt levels, no matter how active that country's fiscal policy stance.

The reason for this is that, as we illustrated in (16), whether inflation expectations are anchored or not does not depend on the fiscal policy stances of countries 1 and 2. So, whether inflation expectations are able to adjust to stabilize real debt levels only depends on the stance of monetary policy across the regimes and the frequency with which the regimes occur. Again, this is the result of the assumption that monetary policy does not have any real effects and thus does not affect real debt growth in country 1 in that way.

Figure 4: Stable parameter sets $(\phi_{b_1,U}, \phi_{\pi,F})$ under Debt Deflation experiment



Notes: The figure shows the stable parameter sets in $(\phi_{b_1,U}, \phi_{\pi,F})$ -space for three different frequencies of occurrence of the Unstable regime, f_U . The areas above the boundaries of the sets for the different f_U reflect parameter combinations where there is no stable equilibrium. The gray-shaded areas depict parameter combinations that result in a stable equilibrium.

The second feature of Figure 4 is that the more frequently the economy is in the Unstable regime, in which fiscal policy is active, the more passive monetary policy needs to be in the F regime. In this figure we keep $\phi_{\pi,U} = 1.5$. However, if we let $\phi_{\pi,U}$ vary then we would also find that the more active monetary policy is in the U regime, the more passive it has to be when the central bank temporarily abandons the Taylor principle.

4.3 Fiscal transfers (III)

This example illustrates the third channel through which stability of the monetary union can be achieved in case the fiscal authority of part of the union tends to pursue an active policy rule that results in a potentially explosive real debt burden. This channel involves fiscal transfers in the form of bailouts from country 2 to country 1.

Similar to the first example of fiscal austerity, what emerges from this example are two main trade-offs. The first is that, conditional on a certain frequency of bailouts, the size of

the bailout necessary for stability of the monetary union is increasing in how active fiscal policy of country 1 is in the Unstable regime (decreasing in $\phi_{b_1,U}$).

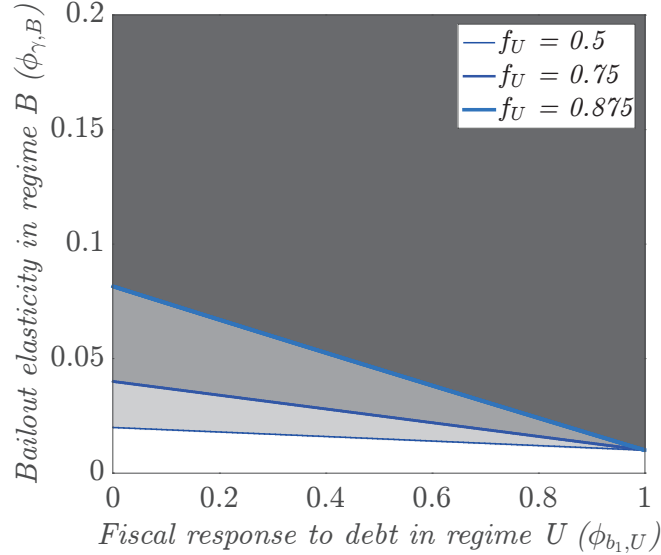
We illustrate this in Figure 5. This figure shows the stable parameter sets for the combinations of the fiscal policy parameter in the Unstable regime, $\phi_{b_1,U}$, and the elasticity of the fraction of debt of country 1 that is transferred to country 2 in each bailout, i.e. $\phi_{\gamma,B}$. The former parameter is plotted on the horizontal axis while the latter, which is our proxy for the size of the bailouts, is on the vertical axis. The figure contains the sets of stable parameter combinations for different frequencies of bailout, p_{UB} . For comparison purposes we translate this bailout probability again into the fraction of time the economy is in the Unstable regime, in this case, $f_U = \frac{1}{1 + \frac{p_{UB}}{p_{BU}}}$.

What you can see from the figure is that, for a given frequency of bailouts, i.e. for a given f_U , a more active fiscal policy of country 1 in the passive regime, i.e. the lower $\phi_{b_1,U}$, the larger the size of the bailouts, as captured by $\phi_{\gamma,B}$, that is needed to assure stability of the debt in country 1 and of the monetary union. This is reflected in the boundaries of the stable parameter sets being downward sloping. Note that, because taxes are lump-sum in our model there is no limit to tax revenue in country 2 due to the existence of a Laffer curve. This means that the relative sizes of countries 1 and 2 do not affect the stable parameter sets in Figure 5. The only limitation on the relative size of country 1 compared to country 2 is that the required bailouts don't violate the monetary union's budget constraint. If they do then the bailouts implied by our parameterization are simply infeasible.

The flipside of the trade-off above is that, conditional on the size of the bailouts, the necessary frequency of bailouts, p_{UB} , is increasing in the rate of growth of real debt in country 1, i.e. decreasing in $\phi_{b_1,U}$. This can also be seen from Figure 5 in that for a given level of $\phi_{\gamma,B}$ on the vertical axis, the minimum frequency of time the economy spends in the Unstable regime, f_U , necessary for stability is decreasing in the fiscal policy parameter $\phi_{b_1,U}$.

Just like in the first example, because our stylized model does not include real effects of monetary policy, as long as the central bank always pursues an active monetary policy

Figure 5: Stable parameter sets $(\phi_{b_1,U}, \phi_{\gamma,B})$ under Fiscal Transfers experiment



Notes: The figure shows the stable parameter sets in $(\phi_{b_1,U}, \phi_{\gamma,B})$ -space for three different frequencies of occurrence of the Unstable regime, f_U . The areas below the boundaries of the sets for the different f_U reflect parameter combinations where there is no stable equilibrium. The gray-shaded areas depict parameter combinations that result in a stable equilibrium.

and anchors inflation expectations, i.e. $\phi_{\pi,U} > 1$ and $\phi_{\pi,B} > 1$, then the set of fiscal policy parameters that lead to stability and determinacy of equilibrium in the model does *not* depend on the monetary policy parameter. Thus, as long as this is the case, the shape of the stability sets plotted in Figure 5 does not change.

4.4 Default on domestic debt obligations (IV)

A fourth potential way to alleviate the debt burden of the government of country 1 is for it to default on its debt obligations. This, however, does not help reduce the growth of country 1's real government debt. The reason is that the expected reduction in debt service payments due to the default gets priced into the interest rate in terms of the default premium. Hence, the potential of a default actually increases country 1's government's debt service payments when it does not default.

To a first order, the increase in the debt service payments due to the default premium

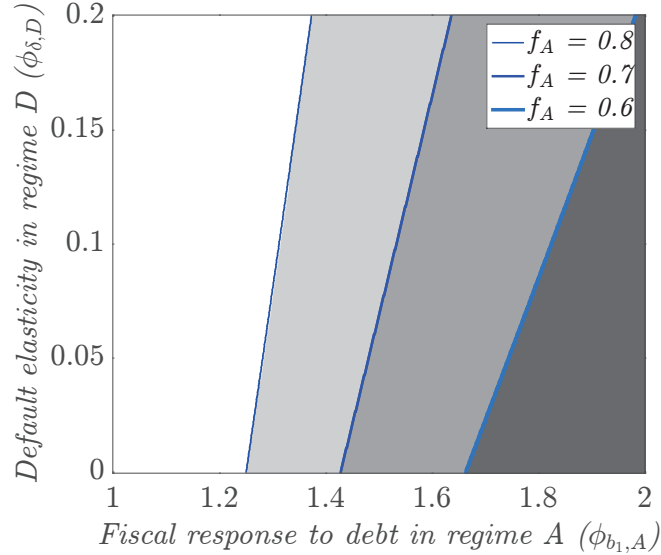
is equal to the expected reduction in payments because of the default. In that case, which is the one captured by our linearized solution method, the potential default of country 1 does not affect the stability properties of the model. This means that the monetary union remains unstable even if the government of country 1 has the option to default when this default is priced into interest rates.

Even more importantly, the possibility of a default can be a destabilizing factor even if the economy is in the, normally stable and determinate, Anchored $E\pi$ regime. This can happen if, in addition to the default premium, the government gets charged an additional premium, i.e. $\Xi_t > 1$ in (14), due to the 'sovereign risk channel' or because investors in government bonds charge a risk premium. We illustrate this in Figure 6.

This figure plots the stable parameter sets that correspond to the Anchored $E\pi$ regime with a certain probability of defaults. To starkly make our point we consider the cases in which the economy is in the Anchored $E\pi$ regime 80, 70, and 60 percent of the time and country 1 defaults in the other periods. This is not because we believe defaults are that frequent, but because it allows us to clearly illustrate our point. Conditional on the central bank following an active monetary policy where $\phi_{\pi,A} = 1.5$, the figure plots the stable parameter sets for the combinations of the default size elasticity, $\phi_{\delta,D}$, and country 1's fiscal policy rule parameter, $\phi_{b_1,A}$. These sets are calculated assuming that $\phi_{b_1,D} = 0$.

To interpret this figure, it is useful to use the case where $\phi_{\delta,D} = 0$ as the benchmark. The cut offs of the stable parameter sets in that case are due to fiscal policy in country 1 being active when it ends up in the default regime. The default itself is actually of size zero. Now, when $\phi_{\delta,D} > 0$, the default regime involves a positive haircut and the larger $\phi_{\delta,D}$ the larger the size of the default for a given level of excess debt in country 1. As you can see from the upward sloping boundaries of the stability sets, the larger $\phi_{\delta,D}$ the smaller the set of parameters for which there is a stable rational expectations equilibrium. The higher $\phi_{\delta,D}$, the more passive fiscal policy needs to be in the Anchored $E\pi$ regime in order to establish stability.

Figure 6: Stable parameter sets $(\phi_{b_1,U}, \phi_{\delta,D})$ under Default experiment



Notes: The figure shows the stable parameter sets in $(\phi_{b_1,U}, \phi_{\delta,D})$ -space for three different frequencies of occurrence of the Unstable regime, f_U . The areas to the left of the boundaries of the sets for the different f_U reflect parameter combinations where there is no stable equilibrium. The gray-shaded areas depict parameter combinations that result in a stable equilibrium.

Thus, in our stylized model, defaults on debt do not help in stabilizing the monetary union when the default risk is fully priced into bond rates. If bond rates respond beyond just the default risk, then potential defaults can actually reduce the stability of the monetary union because the excess adjustment in bond rates in response to the default risk results in an increased growth rate of real government debt. This can result in instability of the monetary union even in the standard stable and determinate case of active monetary policy and passive fiscal policy.

4.5 Generalizations and implications

Our model and examples are necessarily stylized. They do make an important point, however. Namely, the conclusion of traditional macroeconomic models of fiscal and monetary policies that a monetary union in which both the monetary authority and (part of) the fiscal authority pursue an active policy stance is unstable (Bergin, 2000) is predicated on the assumption that

these active policy stances are permanent. If these policy stances are not, then a monetary union that goes through periods in which both monetary and (part of) fiscal policy are active can still be stable and equilibrium in that case determinate.

In both the *Fiscal Austerity* as well as the *Fiscal Transfers* examples we found that if the central bank commits to permanently satisfying the Taylor (1993) Principle and the classical dichotomy holds, then stability of a monetary union is solely dependent on fiscal policy choices. We deliberately chose these examples to illustrate that under active monetary policy that has limited real effects, especially on countries that accumulate an explosive real debt burden, the stability of a monetary union mainly depends on fiscal policy choices. To assure stability, such choices necessarily need to include mechanisms to alleviate the debt burden of countries that tend to pursue active fiscal policies (country 1 in our case). Our results indicate that there are really only two options for such a fiscal relieve valve. Either have the taxpayers in country 1 pay off the real debt burden their government accumulates by implementing frequent periods of *fiscal austerity* in country 1. Or have the taxpayers in the rest of the monetary union pay for the real debt burden of country 1 through *fiscal transfers*, in our model in the form of bailouts.

As far as the interaction of monetary and fiscal policy choices is concerned, our results can be interpreted as a generalization of those in Leeper (1991) to monetary unions with Markov regime switching in policies. Leeper (1991) shows that, for a stable monetary-fiscal-policy equilibrium to exist, one of the policy stances needs to be active while the other is passive. Moreover, what makes fiscal policy active or passive only depends on the fiscal policy parameters and not on the monetary policy rule. The reverse is true for monetary policy. As we showed in our *Fiscal Austerity*, *Fiscal Transfers*, and *Debt Deflation* examples, our model shares this property with Leeper (1991). The only difference is that ‘active’ and ‘passive’ are not defined in terms of permanent policy stances. Instead they are defined in terms of a weighted average of the parameters across policy regimes, where the weights are determined by the regime transition probabilities. This means that, even in a monetary

union that we consider, monetary and fiscal policy actions need to be coordinated in the sense that one of them is, on average, active and the other passive. However, the degree to which one is active does not affect the conditions under which the other one is passive. Thus, in many ways our results are to Bergin (2000) what Davig and Leeper (2007c) are to the Taylor (1993) Principle.

What is important is that, when one realizes that a monetary union is not necessarily unstable when it faces episodes in which both the central bank and (part of) its fiscal authority pursue active policies, it is possible to analyze the economy's response to shocks and policy decisions even though it is in the Unstable regime when they occur. In the next section, we present such impulse responses.

5 Fiscal shocks in the unstable regime

In this section, we examine the effects of expansionary fiscal policy in country 1 on the monetary union. In particular, we simulate an exogenous tax cut of 1% (i.e. a fiscal policy shock) from steady state in country 1 that occurs when the economy is in the Unstable regime. We then study the effects of the tax cut on union-wide inflation and the debt positions of the two member states. As in the examples in the previous section, we allow the economy to switch between the Unstable and one other regime. Corresponding to the first three examples in the previous section, these other regimes are the Anchored E π , Fiscal Theory of the Price Level, and the Bailout regimes. The impulse responses we consider are the *expected* outcomes, where the expectations are taken over the path of possible future policy regimes conditional on starting in the Unstable one. Each row of the three panels in Figure 7 corresponds to the respective case, I-III, from the previous section.

The impulse responses are plotted for the benchmark parameters listed in Table 1. In addition, we chose specific values of the policy parameters, ϕ_{b_1, s_t} , ϕ_{π, s_t} , and ϕ_{γ, s_t} , in the different regimes that clearly illustrate the qualitative properties of the IRFs. The values of

these policy parameters are listed in Table 2.

The final column of this table contains specific transition probabilities for the three examples of impulse responses we consider. The particular shape of the IRFs depends on these parameters because they determine the expected path of future policy regimes *conditional* on currently being in the Unstable one. The qualitative properties of the IRFs that is our focus in this section however do not depend on these parameters and are best captured by the unconditional probability of being in the Unstable regime, f_U , that we focused on before. This is why we label the IRFs in terms of this unconditional probability.

As we discussed in the previous section, stability of the equilibrium in the Fiscal Austerity (I) and Bailout (III) scenarios required inflation expectations to be anchored. Such anchored inflation expectations mean that inflation does not budge in response to a fiscal policy shock. This can be seen in the first and third rows of column 1 of Figure 7. They show that inflation remains at its target level in response to the tax cut in country 1 in the Fiscal Austerity (I) and Bailout (III) examples.

In the Fiscal Austerity (I) example, the tax cut also does not affect the public finances of country 2. It just results in country 1 running up excess (above target) government debt. This debt growth is restrained by passive fiscal policy once the economy is in regime A. Thus, as you can see from the second panel in the first row of the figure, the longer the episodes in the Unstable regime last, i.e. the higher f_U , the more pronounced the run up of country 1's debt in response to the tax cut. In fact, in the case where the economy is in the Unstable regime 87.5% of the time the austerity measures are not enough to offset country 1's debt growth and the equilibrium is unstable. This is why there is no IRF with the solid (blue) line plotted in the top row of Figure 7.

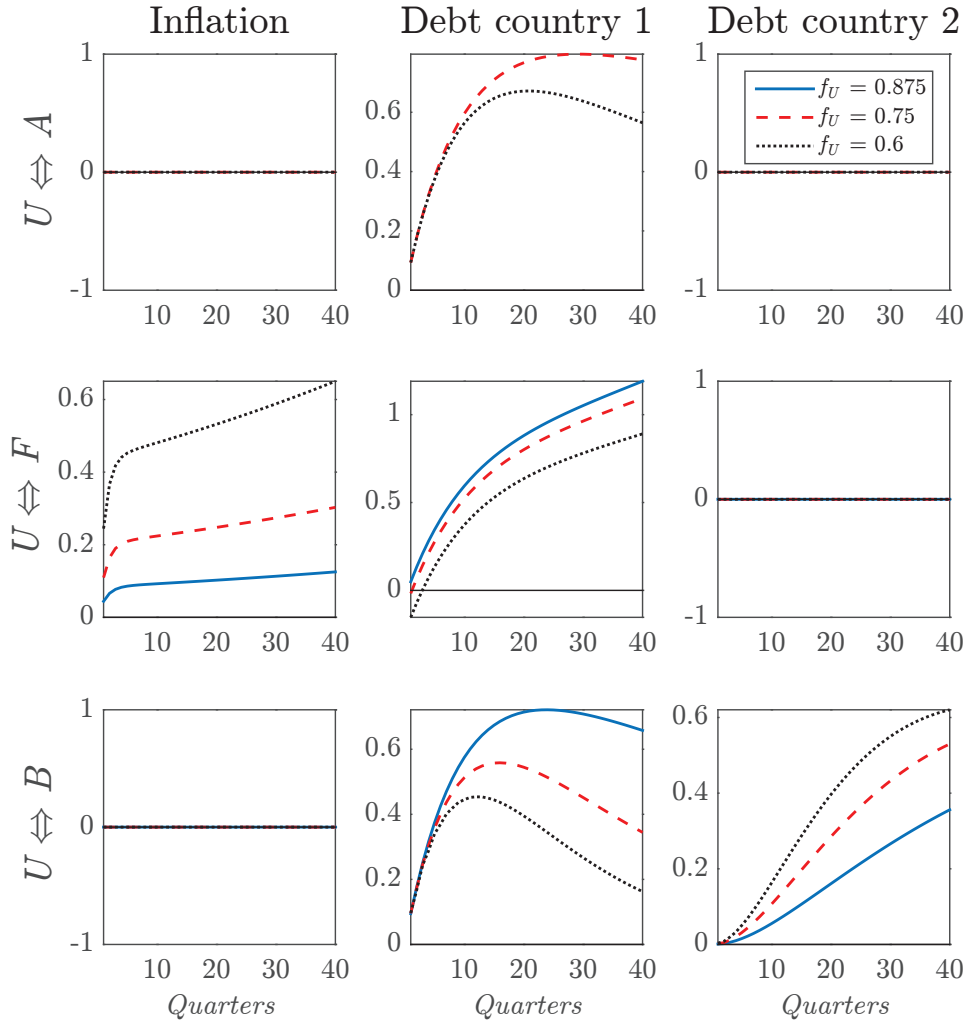
In the Bailout (III) example, the public finances of country 2 are affected by the tax cut in country 1. This is because country 2 takes over some of country 1's debt obligations to stem the explosive debt growth in country 1 and assure stability of the monetary union. As can be seen from the middle and right panels in the bottom row of Figure 7, the more

Table 2: Regime-specific parameters used for impulse response functions

Regime	Fiscal policy, $\phi_{b_1, st}$	Monetary policy, $\phi_{\pi, st}$	Bailout, $\phi_{\gamma, st}$	Transition probabilities
U	0	1.5	0	$p_{UU} = 1 - \frac{1-f_U}{f_U} p_{RU}$, $\mathcal{R} = \{A, F, B\}$
A	0.07	1.5	0	$p_{AU} = 0.25$
F	0	0.5	0	$p_{FU} = 0.25$
B	0	1.5	0.15	$p_{BU} = 1$

Notes: Parameters in four policy regimes considered in impulse response functions, as well as the three transition probabilities associated with the respective rows of the impulse response function Figure 7.

Figure 7: Responses to a temporary tax cut in country 1 under regime switching



Notes: The figure shows the responses of inflation (first column) and government debt in country 1 and 2 (second and third columns), following an exogenous reduction in taxes in country 1 of 1% while in steady state in the Unstable regime. The axis-labels on the left-hand side show between U and which other regime the economy switches. IRFs are plotted as log-deviations from steady state. Each of the three lines plotted corresponds to a different frequency, f_U , at which the economy is in regime U. The actual shape of the impulse responses does not only depend on f_U but also on the specific transition probabilities underlying it, listed in Table 2. This is because impulse responses are plotted conditional on the shock occurring in the Unstable regime.

frequent the bailouts, i.e. the less time the economy is in the unstable regime and the lower is f_U , the more the tax cut in country 1 affects real debt balances in country 2.

Finally, the three panels in the middle row of Figure 7 show the response of inflation and government debt to a tax cut in country 1 in the Unstable regime when stability of the monetary union is accomplished through debt deflation. This is the case where Markov regime switching has the most profound impact on the impulse response functions.

In the absence of policy regime switches, when the economy is always in the Fiscal Theory of the Price Level regime, a tax cut in country 1 would have no effect on real debt holdings and would be immediately offset by a jump in the price level, as in the one-country case discussed in [Kocherlakota and Phelan \(1999\)](#). The middle row of Figure 7 reveals that this is not the case when the economy switches between the U and F regimes. In that case, the expected response is a joint run up in the debt level of country 1 as well as an increase in inflation that stems country 1's excessive debt growth. The more the economy is in the unstable regime, i.e. the higher f_U , the more pronounced the debt cycle in country 1 after the tax cut and the less *expected* inflation increases as a result of the tax cut.

Of course, the examples in this section are deliberately stylized to isolate the impact of Markov Regime Switching on the impulse responses in our model. They are more realistic in one dimension than those of existing models of fiscal and monetary policy in a monetary union. Namely, they consider the real-life relevant case of an unexpected fiscal policy move by a fiscal authority in a monetary union that pursues an active policy while the central bank adheres to the Taylor Principle.

In addition to the direct takeaways from the examples, there is one more thing one should realize from our analysis. In order to understand the effect of fiscal policy shocks in a monetary union with episodes of jointly active fiscal and monetary policy, one has to take a stand on what types of shifts in policy regimes will occur in order to assure stability of the monetary union as well as how frequently such shifts occur.

6 Conclusion

Current macroeconomic models of joint monetary and fiscal policies in monetary unions have little to say about the case in which both the central bank as well as (part of) the fiscal authority take active policy stances. In fact, [Bergin \(2000\)](#) showed that, if these stances are permanent, then no stable rational expectations equilibrium even exists. This result is true no matter how small a part of the fiscal authority pursues this active policy. This is rather unsatisfactory, because the reality is that in almost all monetary systems of the world where the central bank sticks to the Taylor Principle there is at least some fiscal authority that pursues an active fiscal policy.¹⁴

In this paper we illustrated that the result in [Bergin \(2000\)](#) hinges on the assumption of the permanence of the policy regime. If one allows for the fiscal and monetary authorities to switch their policy stances between active and passive, as well as allow for potential fiscal transfers within the monetary union, then the monetary union can be stable in spite of exhibiting episodes of jointly active monetary and fiscal policies. In a sense, our paper is to [Bergin \(2000\)](#) what [Davig and Leeper \(2007c\)](#) is to the [Taylor \(1993\)](#) Principle.

We explored four potential ways to alleviate the explosive debt growth that occurs under active fiscal policies. The first involved frequent episodes of fiscal austerity where the fiscal authority with the active policy reigns in debt growth by pursuing passive policy aimed at reducing government debt levels. The second involved episodes where the central bank abandons the Taylor Principle and allows inflation expectations to become unanchored. This results in a run up of inflation that deflates the value of the explosive real government debt holdings and in stability of the monetary union. The third involved bailouts in which the part of the fiscal authority that pursues a passive policy takes on some of the growing debt burden of the active fiscal policy authority. Such fiscal redistributions can also stem government debt growth and stabilize the equilibrium. Surprisingly, the fourth potential way of assuring stability of the monetary union, by allowing countries to default on their

¹⁴For example, Greece before 2011 in the Euro Area and Puerto Rico in the U.S. Dollar system.

debt, does not work. If markets properly price government bonds and include an appropriate default premium, then the threat of a default results in an increase in debt service payments that offsets the *expected* reduction in payments on the debt due to future defaults. On net, this does not reduce the payments the fiscal authorities need to make on their debt and, thus, does not stabilize the monetary union.

The main theoretical insight of our analysis is that allowing for policy regime switches enables one to think about economic dynamics in the empirically relevant case of a central bank following the Taylor principle and part of the fiscal authority of the monetary union pursuing fiscal policies that, if permanent, would violate transversality conditions. Once one realizes this, it also becomes apparent that such policy switches are inevitable in monetary unions where fiscal standards can not always be credibly enforced. In such monetary unions it is imperative that periods of austerity, bailouts and fiscal transfers, as well as episodes of unconventional monetary policy that ignores the Taylor Principle, are not just one-off events. They are part of the policy fabric and institutions necessary to assure stability of the monetary union.

References

- Albertazzi, U., Ropele, T., Sene, G., and Signoretti, F. M. (2014). The impact of the sovereign debt crisis on the activity of italian banks. *Journal of Banking & Finance*, 46:387–402.
- Balteanu, I., Erce, A., and Fernandez, L. (2011). Bank crises and sovereign defaults: Exploring the links. *Bank of Spain*.
- Bergin, P. R. (2000). Fiscal solvency and price level determination in a monetary union. *Journal of Monetary Economics*, 45(1):37–53.
- Bianchi, F. and Melosi, L. (2014). Escaping the great recession. Technical Report w20238, National Bureau of Economic Research.
- Blanchard, O. J. and Kahn, C. M. (1980). The solution of linear difference models under rational expectations. *Econometrica: Journal of the Econometric Society*, pages 1305–1311.
- Bofondi, M., Carpinelli, L., and Sette, E. (2013). Credit supply during a sovereign debt crisis. Temi di Discussione (Working Paper) 909, Bank of Italy.
- Borensztein, E. and Panizza, U. (2009). The costs of sovereign default. *IMF Staff Papers*, 56(4):683–741.
- Cho, S. (2015). Sufficient conditions for determinacy in a class of markov-switching rational expectations models: A technical guide.
- Cho, S. and McCallum, B. T. (2012). Refining linear rational expectations models and equilibria. NBER Working Paper 18348, National Bureau of Economic Research.
- Cho, S. and Moreno, A. (2011). The forward method as a solution refinement in rational expectations models. *Journal of Economic Dynamics and Control*, 35(3):257–272.

- Chung, H., Davig, T., and Leeper, E. M. (2007). Monetary and fiscal policy switching. *Journal of Money, Credit and Banking*, 39(4):809–842.
- Corsetti, G., Kuester, K., Meier, A., and Müller, G. J. (2013). Sovereign risk, fiscal policy, and macroeconomic stability. *The Economic Journal*, 123(566):F99–F132.
- Davig, T. and Leeper, E. M. (2007a). Fluctuating macro policies and the fiscal theory. In *NBER Macroeconomics Annual 2006, Volume 21*, pages 247–316. MIT Press.
- Davig, T. and Leeper, E. M. (2007b). Generalizing the Taylor principle. *American Economic Review*, 97(3):607–635.
- Davig, T. and Leeper, E. M. (2007c). Generalizing the taylor principle. *American Economic Review*, 97(3):607–635.
- Demirgüç-Kunt, A. and Huizinga, H. (2013). Are banks too big to fail or too big to save? International evidence from equity prices and CDS spreads. *Journal of Banking and Finance*, 37(3):875–894.
- Farmer, R. E., Waggoner, D. F., and Zha, T. (2009). Understanding markov-switching rational expectations models. *Journal of Economic Theory*, 144(5):1849–1867.
- Favero, C. and Monacelli, T. (2003). Monetary-fiscal mix and inflation performance: Evidence from the us. IGIER Working Paper Series No. 234.
- Fisher, I. (1933). The debt-deflation theory of great depressions. *Econometrica*, 1(4):337–357.
- Kocherlakota, N. R. and Phelan, C. (1999). Explaining the fiscal theory of the price level. *Quarterly Review*, (Fall):14–23.
- Leeper, E. M. (1991). Equilibria under ‘active’ and ‘passive’ monetary and fiscal policies. *Journal of Monetary Economics*, 27(1):129–147.

- McCallum, B. T. (2007). E-stability vis-a-vis determinacy results for a broad class of linear rational expectations models. *Journal of Economic Dynamics and Control*, 31(4):1376–1391.
- Panetta, F., Correa, R., Davies, M., Di Cesare, A., Marques, J.-M., Nadal de Simone, F., Signoretti, F., Vespro, C., Vildo, S., and Wieland, M. (2011). The impact of sovereign credit risk on bank funding conditions. CGFS Papers 43, Bank of International Settlements.
- Popov, A. and Van Horen, N. (2013). The impact of sovereign debt exposure on bank lending: Evidence from the European debt crisis. De Nederlandsche Bank Working Paper 382, De Nederlandsche Bank.
- Sims, C. A. (1994). A simple model for study of the determination of the price level and the interaction of monetary and fiscal policy. *Economic Theory*, 4(3):381–399.
- Taylor, J. B. (1993). Discretion versus policy rules in practice. In *Carnegie-Rochester conference series on public policy*, volume 39, pages 195–214. Elsevier.
- Traum, N. and Yang, S.-C. S. (2013). When does government debt crowd out investment? *Journal of Applied Econometrics*.
- Woodford, M. (1996). Control of the public debt: A requirement for price stability? National Bureau of Economic Research Working Paper 5684.
- Woodford, M. (2001). The Taylor rule and optimal monetary policy. *The American Economic Review*, 91(2):232–237.
- Zoli, M. E. (2013). Italian sovereign spreads: Their determinants and pass-through to bank funding costs and lending conditions. IMF Working Paper 13-84, International Monetary Fund.

A Details on application of MSRE methods in our model

A.1 Linearization and MSRE representation of model

In order to analyze the equilibrium properties of the model, we linearize the model around a known steady state. The full linearized model is given by

$$\hat{c}_{1,t} = E_t \hat{c}_{1,t+1} - (\hat{R}_{1,t} - E_t \hat{\pi}_{t+1}) + \phi_{\delta,s_t} \hat{b}_{1,t}, \quad (17)$$

$$\hat{R}_{1,t} = \hat{\Xi}_t + \hat{R}_t + \phi_{\delta,s_t} \hat{b}_{1,t}, \quad (18)$$

$$\hat{\Xi}_t = \chi_{s_t} \phi_{\delta,s_t} \hat{b}_{1,t}, \quad (19)$$

$$\hat{c}_{2,t} = E_t \hat{c}_{2,t+1} - (\hat{R}_{2,t} - E_t \hat{\pi}_{t+1}), \quad (20)$$

$$\hat{R}_{2,t} = \hat{R}_t, \quad (21)$$

$$\hat{\tau}_{1,t} = \frac{b_1}{\tau_1} \phi_{b_1,s_t} \hat{b}_{1,t-1} + z_{\tau_1,t}, \quad (22)$$

$$\hat{\tau}_{2,t} = \frac{b_2}{\tau_2} \phi_{b_2,s_t} \hat{b}_{2,t-1} + z_{\tau_2,t}, \quad (23)$$

$$\hat{R}_t = \phi_{\pi,s_t} \hat{\pi}_t, \quad (24)$$

$$\hat{b}_{1,t} = \frac{1}{\beta} (\hat{R}_{1,t-1} - \hat{\pi}_t) + \frac{1}{\beta} (1 - \phi_{\gamma,s_t} - \phi_{\delta,s_t}) \hat{b}_{1,t-1} - \frac{\tau_1}{b_1} \hat{\tau}_{1,t}, \quad (25)$$

$$\hat{b}_{2,t} = \frac{1}{\beta} (\hat{R}_{2,t-1} - \hat{\pi}_t + \hat{b}_{2,t-1}) - \frac{\tau_2}{b_2} \hat{\tau}_{2,t} + \phi_{\gamma,s_t} \frac{1}{\beta} \frac{y_1}{y_2} \frac{b_1}{b_2} \hat{b}_{1,t-1}, \quad (26)$$

$$z_{\tau_1,t} = \rho_\tau z_{\tau_1,t-1} + \epsilon_{\tau_1,t}, \quad \epsilon_{\tau_1,t} \sim \mathcal{N}(0, \sigma_{\tau_1}^2), \quad (27)$$

$$z_{\tau_2,t} = \rho_\tau z_{\tau_2,t-1} + \epsilon_{\tau_2,t}, \quad \epsilon_{\tau_2,t} \sim \mathcal{N}(0, \sigma_{\tau_2}^2), \quad (28)$$

where a variable with a hat denotes the percentage deviation from its corresponding steady-state value, i.e. $\hat{X}_t \equiv (X_t - X) / X$, for any generic variable X_t . After appropriate substitutions, the system (17)-(28) can be reduced to a system in one forward-looking variable, $\hat{\pi}_t$,

two pre-determined state variables, $\hat{b}_{1,t}$ and $\hat{b}_{2,t}$, and two exogenous variables, $\hat{z}_{\tau_1,t}$ and $\hat{z}_{\tau_2,t}$:

$$\phi_{\pi,s_t} \hat{\pi}_t = E_t \hat{\pi}_{t+1} - \frac{c_1}{c_1 + c_2} \chi_{s_t} \phi_{\delta,s_t} \hat{b}_{1,t} \quad (29)$$

$$+ \Lambda_{s_t} \left[E_t \hat{b}_{1,t+1} - \frac{1}{\beta} (\hat{R}_t - E_t \hat{\pi}_{t+1}) - \frac{1}{\beta} (1 + \chi_{s_t} \phi_{\delta,s_t} - \phi_{\gamma,s_t}) \hat{b}_{1,t} + \frac{\tau_1}{b_1} E_t \hat{\tau}_{1,t+1} \right],$$

$$\hat{b}_{1,t} = \frac{1}{\beta} \left[\phi_{\pi,s_t} \hat{\pi}_{t-1} - \hat{\pi}_t + (1 + \chi_{s_t} \phi_{\delta,s_t} - \phi_{\gamma,s_t}) \hat{b}_{1,t-1} \right] - \frac{\tau_1}{b_1} \hat{\tau}_{1,t}, \quad (30)$$

$$\hat{b}_{2,t} = \frac{1}{\beta} \left(\phi_{\pi,s_t} \hat{\pi}_{t-1} - \hat{\pi}_t + \phi_{\gamma,s_t} \frac{y_1}{y_2} \frac{b_1}{b_2} \hat{b}_{1,t-1} + \hat{b}_{2,t-1} \right) - \frac{\tau_2}{b_2} \hat{\tau}_{2,t}, \quad (31)$$

$$\hat{\tau}_{1,t} = \frac{b_1}{\tau_1} \phi_{b_1,s_t} \hat{b}_{1,t-1} + z_{\tau_1,t}, \quad (32)$$

$$\hat{\tau}_{2,t} = \frac{b_2}{\tau_2} \phi_{b_2,s_t} \hat{b}_{2,t-1} + z_{\tau_2,t}, \quad (33)$$

$$\hat{z}_{\tau_1,t} = \rho_{\tau} \hat{z}_{\tau_1,t-1} + \epsilon_{\tau_1,t}, \quad (34)$$

$$\hat{z}_{\tau_2,t} = \rho_{\tau} \hat{z}_{\tau_2,t-1} + \epsilon_{\tau_2,t}, \quad (35)$$

where the term in square brackets on the right-hand side of (29) is used to make the model non-block recursive and ensures informational consistency between agent's expectations and the solution method.¹⁵ In matrix form, with $x_t \equiv [\hat{\pi}_t \ \hat{b}_{1,t} \ \hat{b}_{2,t} \ \hat{\tau}_{1,t} \ \hat{\tau}_{2,t}]'$, $z_t \equiv [\hat{z}_{\tau_1,t} \ \hat{z}_{\tau_2,t}]'$ and $\epsilon_t \equiv [\epsilon_{\tau_1,t} \ \epsilon_{\tau_2,t}]'$, we can write the MSRE representation of the model as follows:

$$\mathcal{B}_1(s_t) x_t = E_t [\mathcal{A}(s_t, s_{t+1}) x_{t+1}] + \mathcal{B}_2(s_t) x_{t-1} + \mathcal{C} z_t, \quad (36)$$

$$z_t = \rho z_{t-1} + \epsilon_t, \quad (37)$$

¹⁵See Cho (2015). Specifically, recall that, under regime $\mathcal{R} = \{A, U, B, D\}$, inflation expectations are fully anchored by monetary policy and household's intertemporal decisions are independent from fiscal policy. The latter implies that households form expectations using the information set covering only Equation (29). However, under regime F , sovereign debt in country 1 determines the price level such that intertemporal allocations depend on fiscal policy as well. Hence, agents require a larger information set that includes Equation (30). Since the forward solution method does not automatically expand the information set, we set $\Lambda_{s_t} \neq 0$ if $s_t = F$ and $\Lambda_{s_t} = 0$ otherwise.

where $E_{t-1}[\epsilon_t] = 0$ and

$$\mathcal{B}_1(s_t) \equiv \begin{bmatrix} \frac{\beta + \Lambda_{s_t}}{\beta} \phi_{\pi, s_t} & \frac{c_1}{c_1 + c_2} \chi_{s_t} \phi_{\delta, s_t} + \frac{\Lambda_{s_t}}{\beta} (1 + \chi_{s_t} \phi_{\delta, s_t} - \phi_{\gamma, s_t}) & 0 & 0 & 0 \\ \frac{1}{\beta} & 1 & 0 & \frac{\tau_1}{b_1} & 0 \\ \frac{1}{\beta} & 0 & 1 & 0 & \frac{\tau_2}{b_2} \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad (38)$$

$$\mathcal{A}_1(s_t, s_{t+1}) \equiv \begin{bmatrix} \frac{\beta + \Lambda_{s_t}}{\beta} & \Lambda_{s_t} & 0 & \Lambda_{s_t} \frac{\tau_1}{b_1} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (39)$$

$$\mathcal{B}_2(s_t) \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ \frac{1}{\beta} \phi_{\pi, s_t} & \frac{1}{\beta} (1 + \chi_{s_t} \phi_{\delta, s_t} - \phi_{\gamma, s_t}) & 0 & 0 & 0 \\ \frac{1}{\beta} \phi_{\pi, s_t} & \phi_{\gamma, s_t} \frac{1}{\beta} \frac{y_1}{y_2} \frac{b_1}{b_2} & \frac{1}{\beta} & 0 & 0 \\ 0 & \frac{b_1}{\tau_1} \phi_{b_1, s_t} & 0 & 0 & 0 \\ 0 & 0 & \frac{b_2}{\tau_2} \phi_{b_2, s_t} & 0 & 0 \end{bmatrix}, \quad (40)$$

$$\mathcal{C} \equiv \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad (41)$$

$$\varrho \equiv \begin{bmatrix} \rho_\tau & 0 \\ 0 & \rho_\tau \end{bmatrix}. \quad (42)$$

A.2 Solution and stability and determinacy conditions

As explained in Section 3, we apply the ‘forward method’ suggested by [Cho \(2015\)](#) to derive the conditions for equilibrium stability and uniqueness for the class of MSRE models given by (36). The main idea is to find a (particular fundamental) solution for this model that yields two matrices: one governing the uniqueness of a stable (fundamental) solution, and one governing the non-existence of (stable) sunspot components (see also [Cho and Moreno, 2011](#); [Cho and McCallum, 2012](#)). Whether the model is determinate or not then depends on the maximum absolute eigenvalues of those two matrices. The forward method is used because MSRE models lack an eigensystem, which implies the Schur decomposition cannot be applied. In this section, we briefly describe the forward method.

First, left-multiply (36) by $\mathcal{B}_1(s_t)^{-1}$ to obtain

$$x_t = E_t [A(s_t, s_{t+1}) x_{t+1}] + B(s_t) x_{t-1} + C(s_t) z_t, \quad (43)$$

where $A(s_t, s_{t+1}) \equiv \mathcal{B}_1(s_t)^{-1} \mathcal{A}(s_t, s_{t+1})$, $B(s_t) \equiv \mathcal{B}_1(s_t)^{-1} \mathcal{B}_2(s_t)$ and $C(s_t) \equiv \mathcal{B}_1(s_t)^{-1} \mathcal{C}$. Then, define $F_1(s_t, s_{t+1}) = M_1(s_t, s_{t+1}) = A(s_t, s_{t+1})$, $\Omega_1(s_t) = B(s_t)$ and $\Gamma_1(s_t) = C(s_t)$, such that

$$x_t = E_t [M_1(s_t, s_{t+1}) x_{t+1}] + \Omega_1(s_t) x_{t-1} + \Gamma_1(s_t) z_t.$$

By updating this equation forward recursively, we obtain the ‘forward representation’ of the model:

$$x_t = E_t [M_k(s_t, s_{t+1}, \dots, s_{t+k}) x_{t+k}] + \Omega_k(s_t) x_{t-1} + \Gamma_k(s_t) z_t, \quad (44)$$

where, for all $k = 2, 3, 4, \dots$,

$$M_k(s_t, s_{t+1}, \dots, s_{t+k}) = \Upsilon_k(s_t)^{-1} A(s_t, s_{t+1}) M_{k-1}(s_{t+1}), \quad (45)$$

$$\Omega_k(s_t) = \Upsilon_k(s_t)^{-1} B(s_t), \quad (46)$$

$$\Gamma_k(s_t) = \Upsilon_k(s_t)^{-1} [C(s_t) + A(s_t, s_{t+1}) \Gamma_{k-1}(s_t) \varrho], \quad (47)$$

$$F_k(s_t, s_{t+1}) = \Upsilon_k(s_t)^{-1} A(s_t, s_{t+1}), \quad (48)$$

$$\Upsilon_k(s_t) \equiv I_n - E_t[A(s_t, s_{t+1}) \Omega_{k-1}(s_{t+1})], \quad (49)$$

with n the number of endogenous variables in the model. For the forward representation to exist, the regularity condition $|I_n - E_t[A(s_t, s_{t+1}) \Omega_{k-1}(s_{t+1})]| \neq 0$ must be satisfied for all $k > 1$.

According to [Cho](#), the MSRE model (43) satisfies the so-called ‘forward convergence condition’ if the matrices $\Omega_k(s_t)$ and $\Gamma_k(s_t)$ in the forward representation (44) converge, for every regime s_t , as k tends to infinity. Under this condition, the model implies the following ‘forward solution’:

$$x_t = \Omega^*(s_t) x_{t-1} + \Gamma^*(s_t) z_t, \quad (50)$$

where $\Omega^*(s_t) = \lim_{k \rightarrow \infty} \Omega_k(s_t)$ and $\Gamma^*(s_t) = \lim_{k \rightarrow \infty} \Gamma_k(s_t)$, for every s_t . Note that the forward solution (50) implies $\lim_{k \rightarrow \infty} E_t[M_k(s_t, s_{t+1}, \dots, s_{t+k}) x_{t+k}] = 0_{n \times 1}$, i.e. the ‘no-bubble condition’, which is akin to the standard transversality condition and ensures that expectations far in the future do not affect the endogenous variables contemporaneously.

Having found $\Omega^*(s_t)$, if it exists, one can obtain $F^*(s_t, s_{t+1}) = \lim_{k \rightarrow \infty} F_k(s_t, s_{t+1})$ using

(48). Then, construct the following ‘probability weighted matrices’:

$$\bar{\Psi}_M = [p_{ji}M_j] = \begin{bmatrix} p_{11}M_1 & \cdots & p_{S1}M_S \\ \vdots & \ddots & \vdots \\ p_{1S}M_1 & \cdots & p_{SS}M_S \end{bmatrix},$$

$$\bar{\Psi}_{M \otimes M} = [p_{ji}M_j \otimes M_j] = \begin{bmatrix} p_{11}M_1 \otimes M_1 & \cdots & p_{S1}M_S \otimes M_1 \\ \vdots & \ddots & \vdots \\ p_{1S}M_S \otimes M_S & \cdots & p_{SS}M_S \otimes M_S \end{bmatrix},$$

with $M = \{\Omega^*, F^*\}$, and where S denotes the total number of regimes and \otimes the Kronecker product. The MSRE model (43), then, is determinate in the mean-square stability sense if¹⁶

$$r_\sigma(\Psi_{\Omega^* \otimes \Omega^*}) < 1 \quad \text{and} \quad r_\sigma(\Psi_{F^* \otimes F^*}) \leq 1, \quad (51)$$

where $r_\sigma(M) = \max_{i \leq i \leq n} (|\lambda_i|)$ is the spectral radius of a $n \times n$ matrix M whose eigenvalues are given by λ_i . The first condition in (51), i.e. $r_\sigma(\Psi_{\Omega^* \otimes \Omega^*}) < 1$, ensures stability of the forward solution, whereas the second condition, $r_\sigma(\Psi_{F^* \otimes F^*}) \leq 1$, ensures that the forward solution is the only stable solution and the non-existence of stable sunspot components associated with the forward solution. Thus, if $r_\sigma(\Psi_{\Omega^* \otimes \Omega^*}) \geq 1$, there is no stable solution, while if $r_\sigma(\Psi_{F^* \otimes F^*}) > 1$, the MSRE model (43) is indeterminate. Finally, the case where $r_\sigma(\Psi_{\Omega^* \otimes \Omega^*}) \geq 1$ and $r_\sigma(\Psi_{F^* \otimes F^*}) > 1$ may arise in our case if fiscal policy is active and monetary policy passive. As under the Fiscal Theory of the Price Level, we consider this case to yield a stable and unique solution as well.

¹⁶A $n \times 1$ process w_t is mean-square stable if there exists a $n \times 1$ vector \bar{w} and a $n \times n$ matrix Q , such that $\lim_{t \rightarrow \infty} (E[w_t] - \bar{w}) = 0_{n \times 1}$ and $\lim_{t \rightarrow \infty} (E[w_t w_t'] - Q) = 0_{n \times n}$.

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