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# Optimal Conventional and Unconventional Monetary Policy Mix*

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**Abstract**

This paper examines the optimal coordination of conventional and unconventional monetary policy tools in an environment characterized by household heterogeneity and mortgage debt. We develop a dynamic stochastic general equilibrium (DSGE) model with three types of households—savers, borrowers, and renters—and incorporate housing investment, fixed-rate long-term mortgages, and a housing production sector. The central bank controls both the short-term interest rate and the long-term rate via the relative supply of long-term bonds. We show that household heterogeneity significantly alters the optimal policy response to macroeconomic shocks. In particular, following a cost-push shock, the optimal policy involves raising the short-term rate to combat inflation while lowering the long-term rate to alleviate financial burdens on indebted households and renters. This policy mix accelerates investment recovery but increases consumption inequality. In contrast, in a representative-agent economy, both rates are raised. Our findings highlight the importance of accounting for distributional effects in monetary policy design and suggest that yield curve control can be a valuable tool in heterogeneous economies.

*Keywords*: Monetary policy, household heterogeneity, yield curve control.
*JEL Classification*: E40, E43, E52.

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# 1 Introduction

The recent surge in inflation across advanced economies has reignited debates over the appropriate mix of conventional and unconventional monetary policy tools. In both the United States and the Euro Area, central banks responded to rising inflation by first halting asset purchase programs and subsequently raising short-term policy rates. This sequence of actions led to a sharp increase in long-term interest rates and mortgage rates, amplifying the financial burden on indebted households. These developments raise a fundamental question: should conventional and unconventional monetary policy instruments move in the same direction in response to inflationary pressures, particularly in economies where household debt is widespread?

This paper addresses this question by studying the optimal design of monetary policy in an environment characterized by household heterogeneity and long-term mortgage debt. We develop a dynamic stochastic general equilibrium (DSGE) model that features three types of households—patient savers, impatient borrowers, and hand-to-mouth renters—and incorporates housing investment, fixed-rate long-term mortgages, and a housing production sector. The central bank in our model controls both the short-term nominal interest rate and the long-term rate, through adjustments in the relative supply of long-term government bonds. This framework allows us to analyze the joint role of conventional interest rate policy and unconventional tools, such as quantitative easing, or yield curve control in stabilizing the economy.

Our analysis reveals that household heterogeneity significantly alters the optimal monetary policy response to macroeconomic shocks. In particular, we show that following an adverse cost-push shock, the central bank finds it optimal to raise the short-term interest rate to counter inflationary pressures, while simultaneously reducing the long-term rate to mitigate the negative wealth effects on indebted households and the rental burden on renters. This policy mix supports a faster recovery in housing investment and total investment, but it also leads to a widening of consumption inequality. In contrast, when household heterogeneity is removed from the model—by eliminating mortgage debt and assuming a representative agent—the optimal policy response becomes more conventional: both short- and long-term rates are raised in response to inflationary shocks.

The paper contributes to the literature on optimal monetary policy in several ways. First, it extends the standard New Keynesian framework by incorporating long-term fixed-rate mortgage contracts and a housing production sector, allowing for a richer analysis of the transmission of monetary policy through the housing market. Second, it introduces a welfare-based criterion that accounts for the distributional consequences of monetary policy across heterogeneous households. Third, it provides new insights into the role of unconventional monetary policy tools in environments with high household indebtedness.

This paper builds on and extends several strands of the literature. It relates closely to the work of Iacoviello (2005), who introduced housing and collateral constraints into DSGE models, and to Kydland et al. (2016), who emphasized the importance of long-term mortgage contracts. Unlike these studies, we incorporate both fixed-rate mortgages and a housing production sector, allowing us to distinguish between the flow and stock of household debt. Our model also shares features with Chen et al. (2012) and Gertler and Karadi (2011, 2013), who analyze the macroeconomic effects of large-scale asset purchases. However, we depart from these frameworks by explicitly modeling the welfare implications of monetary policy in a heterogeneous-agent setting. In doing so, we contribute to the growing literature on the distributional effects of monetary policy (e.g. Cloyne et al., 2019), and provide new insights into how central banks can optimally balance inflation stabilization with inequality concerns.

Methodologically, we derive a second-order approximation to the utility of each household type and construct a social welfare function that aggregates these utilities using Pareto weights. We then solve for the optimal policy under commitment, allowing the central bank to choose both the short-term interest rate and the relative supply of long-term bonds. We show that in the presence of heterogeneity, the optimal policy involves a trade-off between inflation stabilization and the mitigation of consumption inequality. This trade-off is absent in representative-agent models, where the central bank's objective reduces to minimizing inflation and output volatility.

The remainder of the paper is organized as follows. Section 2 presents the model, detailing the behavior of households, firms, and the government. Section 3 analyzes the transmission mechanisms of monetary policy in the presence of household heterogeneity. Section 4 derives the optimal policy rules and discusses the implications of heterogeneity for the design of monetary policy. Section 5 describes the calibration strategy. Section 6 presents the quantitative results, including impulse response functions under different policy regimes. Section 7 concludes with policy implications and directions for future research.

# 2 Model

The model is a closed-economy DSGE model with housing and household debt. There are three types of households in the economy: patient households (savers), impatient households (borrowers), and renters (hand-to-mouth agents), similar to Cloyne et al. (2019). We consider long-term fixed-rate mortgages and differentiate between the flow and the stock of household debt, following Kydland et al. (2016), Garriga et al. (2017), and Alpanda and Zubairy (2016). The government issues both short- and long-term bonds to finance its deficits, and the long rate is relevant for aggregate demand through its impact on mortgage-related interest burden of borrower households. The central bank controls both the short and the long rate simultaneously, targeting the short rate by a Taylor rule and targeting the long rate by adjusting the stock of outstanding long-term government bonds. The production side of the model is standard.

## 2.1 Households

The economy is populated by three types of infinitely-lived agents $a \in \{P, I, R\}$, whose intertemporal preferences over consumption, $c_{a,t}$, housing, $h_{a,t}$, and labor supply, $n_{a,t}$, are described by the following expected utility function:

$$E_t \sum_{\tau=t}^{\infty} \beta_a^{\tau-t} e^{\upsilon_t} \left( \log c_{a,\tau} + \xi \log h_{a,\tau} - \frac{n_{a,\tau}^{1+\vartheta}}{1+\vartheta} \right), \tag{1}$$

where $t$ indexes time, $\xi$ determines the relative importance of housing in the utility function, $\vartheta$ is the inverse of the Frisch-elasticity of labor supply, and $\beta_a$ is the type-specific time-discount parameter with $\beta_I < \beta_P < 1$.[1] Variable $\upsilon_t$ is a preference shock common to all households that follows a stationary AR(1) process.

### 2.1.1 Patient households (savers)

The patient households (P) are the savers in the economy; they accumulate owner-occupied and rental housing, capital, short- and long-term government bonds, and extend mortgage loans to borrowers. Their period budget constraint is given by

$$c_{P,t} + q_{h,t}\left(i_{hP,t} + i_{hR,t}\right) + q_{k,t} i_{k,t} + \frac{Q_{S,t}}{P_t} b_{S,t} + \frac{Q_{L,t}}{P_t}\left(b_{L,t} - \frac{\kappa}{\pi_t} b_{L,t-1}\right) + \frac{L_t}{P_t} \leq w_{P,t} n_{P,t} \tag{2}$$
$$+ r_{h,t} h_{R,t} + r_{k,t} k_{t-1} + \frac{P_{t-1}}{P_t}\left(b_{S,t-1} + b_{L,t-1}\right) + \left(R_{t-1}^{d} + \kappa_d\right) \frac{D_{t-1}}{P_t} + \frac{\Xi_t}{P_t} - tax_t - \Gamma_t \left( \frac{Q_{L,t}}{P_t} b_{L,t} + \frac{D_t}{P_t} \right)$$

where $P_t$ denotes the aggregate price level, while $\Xi_t$ refers to nominal profits of intermediate-goods producers received and $tax_t$ denotes real taxes paid to the government, both in lump-sum fashion. $w_{P,t}$ is the real wage rate on the labor services of patient households, $q_{h,t}$ and $q_{k,t}$ denote the (real) relative prices of housing and capital, respectively, while $i_{hP,t}$, $i_{hR,t}$ and $i_{k,t}$ are the patient households' investment purchases in owner-occupied housing, rental housing, and capital. The related laws of motion for the stock of these real assets are given by

$$h_{P,t} = \left(1 - \delta_h\right) h_{P,t-1} + i_{hP,t}, \tag{3}$$
$$h_{R,t} = \left(1 - \delta_h\right) h_{R,t-1} + i_{hR,t}, \tag{4}$$
$$k_t = \left(1 - \delta_k\right) k_{t-1} + i_{k,t}, \tag{5}$$

where $\delta_h$ and $\delta_k$ are depreciation rates of housing and capital. Similarly, $r_{h,t}$ and $r_{k,t}$ denote the rental income patient households receive from their rental housing and capital holdings.

$Q_{S,t}$ and $Q_{L,t}$ denote the nominal prices of short- and long-term government bonds issued in period $t$, while $b_{S,t}$ and $b_{L,t}$ denote outstanding quantities of these bonds. A short bond issued in period $t-1$ pays $P_{t-1}$ in nominal terms in period $t$, while a long term bond issued in period $t-1$ pays decaying coupon payments of $P_{t-1}, \kappa P_{t-1}, \kappa^2 P_{t-1}, \ldots$ in periods $t$, $t+1$, $t+2, \ldots$, respectively. Note that in period $t$, the *ex coupon* nominal price of a long term bond issued in period $t-1$ is given by $\frac{\kappa}{\Pi_t} Q_{L,t}$, where $\Pi_t = P_t / P_{t-1}$ is the inflation factor, which allows us to write the long term bonds

[1] Since renter households are hand-to-mouth, $\beta_R$ does not play a role in the dynamics. We consider $\beta_R = \beta_I$ in the welfare calculations.

in the households' budget constraint above in recursive fashion.[2]

$L_t$ is the amount of new lending extended to impatient households, while $D_{t-1}$ denotes the stock of mortgage debt carried from the previous period. On the latter, the patient households receive a $\kappa_d$ fraction of principal payments and an average interest payment of $R^d_{t-1}$ from borrowers. The laws of motion for the stock of debt and the average interest charged on the debt are respectively given by

$$\frac{D_t}{P_t} = (1-\kappa_d)\frac{D_{t-1}}{P_t} + \frac{L_t}{P_t}, \tag{6}$$

and

$$\frac{D_t}{P_t}R^d_t = (1-\kappa_d)\frac{D_{t-1}}{P_t}R^d_{t-1} + \frac{L_t}{P_t}R^l_t. \tag{7}$$

where $R^l_t$ is current nominal fixed rate on new mortgage loans. Note that $R^d_t$ is a choice variable for the patient households (joint with their choice of new extended loans, $L_t$), while they take as given the economy-wide current fixed mortgage rate, $R^l_t$.

Following Chen et al. (2012), agents pay a transaction cost $\Gamma_t$ on their long-term debt holdings and this cost is given by

$$\Gamma_t = \Gamma_1\left(\frac{Q_{L,t}b_{L,t}}{Q_{S,t}b_{S,t}}\right)^{\Gamma_2}\exp(\widetilde{\varepsilon}_{\Gamma,t}) - 1, \tag{8}$$

where $\Gamma_1$ and $\Gamma_2$ denote level and elasticity parameters, and $\widetilde{\varepsilon}_{\Gamma,t}$ is an exogenous AR(1) process. Note that the stock of mortgage debt is subject to the transaction cost $\Gamma_t$ as long-term government bonds are; this will ensure that changes in the long rate will affect the interest burden and therefore the aggregate demand of borrower households.

The patient households' objective is to maximize utility subject to their budget constraint and appropriate No-Ponzi conditions. The optimality conditions for labor supply, owner-occupied housing, rental housing, and capital are respectively given by

$$n^{\vartheta}_{P,t} = \lambda_{P,t}w_{P,t}, \tag{9}$$

$$q_{h,t} = \xi_h\frac{c_{P,t}}{h_{P,t}} + E_t\left[\beta_P\frac{\lambda_{P,t+1}}{\lambda_{P,t}}(1-\delta_h)q_{h,t+1}\right], \tag{10}$$

$$q_{h,t} = r_{h,t} + E_t\left[\left(\beta_P\frac{\lambda_{P,t+1}}{\lambda_{P,t}}\right)(1-\delta_h)q_{h,t+1}\right], \tag{11}$$

$$q_{k,t} = E_t\left[\left(\beta_P\frac{\lambda_{P,t+1}}{\lambda_{P,t}}\right)\left[(1-\delta_k)q_{k,t+1} + r_{k,t+1}\right]\right], \tag{12}$$

where $\lambda_{P,t} = 1/c_{P,t}$ denotes the Lagrange multiplier on the period budget constraint. Similarly,

[2]Note that $\kappa = 0$ reduces the long-term bond to a short-term bond.

optimal short- and long-term government bond holdings imply

$$q_{S,t} = E_t\left[\left(\beta_P\frac{\lambda_{P,t+1}}{\lambda_{P,t}}\right)\frac{1}{\Pi_{t+1}}\right], \tag{13}$$

$$(1+\Gamma_t)\,q_{L,t} = E_t\left[\left(\beta_P\frac{\lambda_{P,t+1}}{\lambda_{P,t}}\right)\frac{1+\kappa q_{L,t+1}}{\Pi_{t+1}}\right], \tag{14}$$

with the related nominal yields on short and long bonds given by

$$1+R_t = \frac{1}{q_{S,t}}, \tag{15}$$

$$1+R_{L,t} = \frac{1}{q_{L,t}}+\kappa. \tag{16}$$

Finally, the optimality conditions for the flow and stock of mortgage loans and the average interest on them are given, respectively, by

$$1+\Gamma_t = \Omega_{dP,t}+\Omega_{rP,t}R_t^l, \tag{17}$$

$$\Omega_{dP,t}+\Omega_{rP,t}R_t^d = E_t\left[\left(\beta_P\frac{\lambda_{P,t+1}}{\lambda_{P,t}}\right)\frac{R_t^d+\kappa_d-(1-\kappa_d)\,\Gamma_{t+1}+(1-\kappa_d)\left[\Omega_{dP,t+1}+\Omega_{rP,t+1}R_t^d\right]}{\Pi_{t+1}}\right], \tag{18}$$

$$\Omega_{rP,t} = E_t\left[\left(\beta_P\frac{\lambda_{P,t+1}}{\lambda_{P,t}}\right)\frac{1+(1-\kappa_d)\,\Omega_{rP,t+1}}{\Pi_{t+1}}\right], \tag{19}$$

where $\Omega_{dP,t}$ and $\Omega_{rP,t}$ are the Lagrange multipliers on the laws of motion for the mortgage debt stock and the average interest on debt given in (6) and (7), respectively. Note that with full principal repayment each period (i.e., $\kappa_d = 1$), we have $R_t^d = R_t^l$ for all $t$, and the above expressions would collapse to the more familiar

$$1+\Gamma_t = E_t\left[\left(\beta_P\frac{\lambda_{P,t+1}}{\lambda_{P,t}}\right)\frac{1+R_t^l}{\Pi_{t+1}}\right], \tag{20}$$

with $\Gamma_t$ capturing the spread on mortgages relative to the short rate. In our general case, this holds only at the steady state with $R^d = R^l = R_L$ and

$$\frac{1+R^d}{1+R} = 1+\Gamma. \tag{21}$$

#### 2.1.2 Impatient households (borrowers)

The impatient households borrow from patient households to finance their investment purchases in their owner-occupied housing; otherwise, they do not accumulate any other assets. Their period budget constraint is given by

$$c_{I,t} + q_{h,t} i_{hI,t} + \left( R^d_{t-1} + \kappa_d \right) \frac{D_{t-1}}{P_t} \leq w_{I,t} n_{I,t} + \frac{L_t}{P_t}, \tag{22}$$

where $w_{I,t}$ denotes their wage rate and $i_{hI,t}$ is their residential investment purchases. The related law of motion for their housing stock is given by

$$h_{I,t} = (1 - \delta_h)\, h_{I,t-1} + i_{hI,t}. \tag{23}$$

Impatient households face a borrowing constraint on their new loans each period as

$$\frac{L_t}{P_t} \leq \phi q_{h,t} i_{hI,t}, \tag{24}$$

where $\phi$ is the regulatory LTV ratio on mortgages.

The impatient households' optimality conditions for labor supply and owner-occupied housing are respectively given by

$$n^{\vartheta}_{I,t} = \lambda_{I,t} w_{I,t}, \tag{25}$$

$$(1 - \phi\mu_t)\, q_{h,t} = \xi_h \frac{c_{I,t}}{h_{I,t}} + E_t \left[ \left( \beta_I \frac{\lambda_{I,t+1}}{\lambda_{I,t}} \right) (1 - \delta_h)\, (1 - \phi\mu_{t+1})\, q_{h,t+1} \right], \tag{26}$$

where $\lambda_{I,t}$ and $\mu_t$ denote the Lagrange multipliers on the period budget constraint and the borrowing constraint, respectively. Similarly, the optimality conditions for the flow and stock of mortgage loans and the average interest on them are given respectively by

$$1 - \mu_t = \Omega_{dI,t} + \Omega_{rI,t} R^l_t, \tag{27}$$

$$\Omega_{dI,t} + \Omega_{rI,t} R^d_t = E_t \left[ \left( \beta_I \frac{\lambda_{I,t+1}}{\lambda_{I,t}} \right) \frac{R^d_t + \kappa_d + (1 - \kappa_d) \left[ \Omega_{dI,t+1} + \Omega_{rI,t+1} R^d_t \right]}{\Pi_{t+1}} \right], \tag{28}$$

$$\Omega_{rI,t} = E_t \left[ \left( \beta_I \frac{\lambda_{I,t+1}}{\lambda_{I,t}} \right) \frac{1 + (1 - \kappa_d)\, \Omega_{rI,t+1}}{\Pi_{t+1}} \right], \tag{29}$$

where $\Omega_{dI,t}$ and $\Omega_{rI,t}$ are the Lagrange multipliers on the laws of motion for the mortgage debt stock and the average interest on debt given in (6) and (7), respectively. Note again that with full principal repayment each period (i.e., $\kappa_d = 1$), the above expressions would collapse to the more familiar

$$1 - \mu_t = E_t \left[ \left( \beta_I \frac{\lambda_{I,t+1}}{\lambda_{I,t}} \right) \frac{1 + R^l_t}{\Pi_{t+1}} \right]. \tag{30}$$

### 2.1.3 Renter households (hand-to-mouth)

Renter households are hand-to-mouth, and consume all their wage income on consumption goods and rental housing as

$$c_{R,t} + r_{h,t} h_{R,t} = w_{R,t} n_{R,t}, \tag{31}$$

where $w_{R,t}$ is their wage rate. Their optimality conditions imply the following static expressions:

$$n_{R,t}^{\vartheta} = \lambda_{R,t} w_{R,t}, \tag{32}$$

$$r_{h,t} = \xi_h \frac{c_{R,t}}{h_{R,t}}, \tag{33}$$

where $\lambda_{R,t} = 1/c_{R,t}$ denotes the Lagrange multiplier on the period budget constraint.

## 2.2 Production

The production side of the model is standard. There is a unit measure of imtermediate goods producers, which produce a slightly differentiated product and face price adjustment costs similar to Rotemberg (1982). Perfectly competitive final goods producers then aggregate these differentiated goods into a final good, which can be used for consumption, investment in capital and housing, and government expenditure. We also introduce adjustment costs in investment goods production through "investment goods producers" to ensure that the relative price of investment goods can deviate from consumption goods.

### 2.2.1 Final goods producers

Final goods producers are perfectly competitive, and aggregate the differentiated intermediate goods $y_t(j)$ for $j \in [0,1]$ into a final good $y_t$ using a standard Dixit-Stiglitz aggregator:

$$y_t = \left( \int_0^1 y_t(j)^{\frac{\eta_{y,t}-1}{\eta_{y,t}}} dj \right)^{\frac{\eta_{y,t}}{\eta_{y,t}-1}}, \tag{34}$$

where $\eta_{y,t}$ is the elasticity of substitution between the differentiated goods that follows a stationary $AR(1)$ process. The resulting demand curve facing each intermediate goods firm is thus given by

$$y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\eta_{y,t}} y_t. \tag{35}$$

The final goods can then be used for consumption, investment in capital and housing, government expenditure, and resources spent on price adjustment costs by intermediate goods producers. Price adjustment costs are described in more detail in the next subsection. Note that aggregate consumption and residential investment are the sum of the three types of households described previously as

$$c_t = c_{P,t} + c_{I,t} + c_{R,t}, \tag{36}$$

$$i_{h,t} = i_{hP,t} + i_{hI,t} + i_{hR,t}. \tag{37}$$

### 2.2.2 Intermediate goods producers

There is a unit measure of monopolistically competitive intermediate goods producers indexed by $j$. Their technology is described by the following production function:

$$y_t(j) = z_t k_{t-1}(j)^{\alpha} \left[ n_{P,t}(j)^{\theta_P} n_{I,t}(j)^{\theta_I} n_{R,t}(j)^{\theta_R} \right]^{1-\alpha} - f, \tag{38}$$

where $f$ denotes the fixed cost, $\alpha$ is the share of capital in overall production, and $\theta_P$, $\theta_I$, and $\theta_R$ denote the share of patient, impatient, and renter households in the labor input, respectively, with $\theta_P + \theta_I + \theta_R = 1$. The aggregate productivity shock, $z_t$, follows an AR(1) process.

Firm $j$'s profits at period $t$ is given by

$$\begin{aligned} \frac{\Xi_t(j)}{P_t} = \frac{P_t(j)}{P_t} y_t(j) - r_{k,t} k_{P,t-1}(j) - w_{P,t} n_{P,t}(j) \\ - w_{I,t} n_{I,t}(j) - w_{R,t} n_{R,t}(j) - \frac{\kappa_p}{2} \left( \frac{P_t(j)}{\pi P_{t-1}(j)} - 1 \right)^2 y_t, \end{aligned} \tag{39}$$

where price stickiness is introduced through quadratic adjustment costs, as in Rotemberg (1982), with $\kappa_p$ as the level parameter.

A firm's objective is to choose the quantity of inputs, output and its own output price each period to maximize the present value of profits (using the patient households' stochastic discount factor) subject to the demand function they are facing with respect to their output from the goods aggregators. The firm's optimality conditions with respect to its inputs are given by

$$\Omega_t \alpha \frac{y_t + f}{k_{t-1}} = r_{k,t}, \tag{40}$$

$$\Omega_t (1-\alpha) \theta_P \frac{y_t + f}{n_{P,t}} = w_{P,t}, \tag{41}$$

$$\Omega_t (1-\alpha) \theta_I \frac{y_t + f}{n_{I,t}} = w_{I,t}, \tag{42}$$

$$\Omega_t (1-\alpha) \theta_R \frac{y_t + f}{n_{R,t}} = w_{R,t}, \tag{43}$$

where $\Omega_t$ is the Lagrange multiplier on the demand function of final goods producers given in (35) and captures marginal cost of firms. The optimality with respect to pricing implies the following New Keynesian Phillips :

$$\left(\frac{\Pi_t}{\Pi}-1\right)\frac{\Pi_t}{\Pi}=E_t\left[\left(\beta_P\frac{\lambda_{P,t+1}}{\lambda_{P,t}}\right)\left(\frac{\Pi_{t+1}}{\Pi}-1\right)\frac{\Pi_{t+1}}{\Pi}\frac{y_{t+1}}{y_t}\right]-\frac{\eta_{y,t}-1}{\kappa_p}\left(1-\frac{\eta_{y,t}}{\eta_{y,t}-1}\Omega_t\right). \tag{44}$$

### 2.2.3 Capital and Housing producers

Investment goods producers are perfectly competitive, and they purchase $i_{k,t}$ and $i_{h,t}$ units of new investment goods from final goods firms at a relative price of 1, and turn these into effective units of installed capital and housing that can be purchased by end-users at relative prices of $q_{k,t}$ and $q_{h,t}$, respectively. The change in relative prices are due to adjustment costs in investment similar to Christiano et al. (2005) and Smets and Wouters (2007), which can potentially differ between capital and housing. The investment-goods producers' objective is thus to maximize

$$E_t\sum_{\tau=t}^{\infty}\beta_P^{\tau-t}\frac{\lambda_{P,\tau}}{\lambda_{P,t}}\left\{\left[1-\frac{\kappa_{ik}}{2}\left(\frac{i_{k,\tau}}{i_{k,\tau-1}}-1\right)^2\right]q_{k,\tau}i_{k,\tau}+\left[1-\frac{\kappa_{ih}}{2}\left(\frac{i_{h,\tau}}{i_{h,\tau-1}}-1\right)^2\right]q_{h,\tau}i_{h,\tau}-(i_{k,\tau}+i_{h,\tau})\right\}, \tag{45}$$

where $\kappa_{ik}$ and $\kappa_{ih}$ are the investment adjustment cost parameters, and future profits are discounted using the patient households' stochastic discount factor. The first-order conditions for capital and residential investment yield the Tobin's marginal $q$ expressions, which are summarized as follows:

$$\begin{aligned} i_{k,t}:\ \left(\frac{i_{k,t}}{i_{k,t-1}}-1\right)\frac{i_{k,t}}{i_{k,t-1}} &= E_t\left[\left(\beta_P\frac{\lambda_{P,t+1}}{\lambda_{P,t}}\right)\left(\frac{i_{k,t+1}}{i_{k,t}}-1\right)\left(\frac{i_{k,t+1}}{i_{k,t}}\right)^2\frac{q_{k,t+1}}{q_{k,t}}\right] \\ &+\frac{1}{\kappa_{ik}}\left(1-\frac{1}{q_{k,t}}\right)-\frac{1}{2}\left(\frac{i_{k,t}}{i_{k,t-1}}-1\right)^2 \end{aligned} \tag{46}$$

$$\begin{aligned} i_{h,t}:\ \left(\frac{i_{h,t}}{i_{h,t-1}}-1\right)\frac{i_{h,t}}{i_{h,t-1}} &= E_t\left[\left(\beta_P\frac{\lambda_{P,t+1}}{\lambda_{P,t}}\right)\left(\frac{i_{h,t+1}}{i_{h,t}}-1\right)\left(\frac{i_{h,t+1}}{i_{h,t}}\right)^2\frac{q_{h,t+1}}{q_{h,t}}\right] \\ &+\frac{1}{\kappa_{ih}}\left(1-\frac{1}{q_{h,t}}\right)-\frac{1}{2}\left(\frac{i_{h,t}}{i_{h,t-1}}-1\right)^2 \end{aligned} \tag{47}$$

## 2.3 Monetary and fiscal policy

The consolidated government's budget constraint is given by

$$g_t+\frac{P_{t-1}}{P_t}\left(b_{S,t-1}+b_{L,t-1}\right)=tax_t+\frac{Q_{S,t}}{P_t}b_{S,t}+\frac{Q_{L,t}}{P_t}\left(b_{L,t}-\frac{\kappa}{\Pi_t}b_{L,t-1}\right), \tag{48}$$

where the bond quantities above refer to privately-held bonds in circulation, and therefore exclude potential purchases by the central bank. The lump-sum taxes collected from patient households (relative to steady state output $y$) are assumed to respond positively to aggregate output and to the government's debt level so that the government cannot run a Ponzi scheme:

$$\frac{tax_t}{y} = \tilde{\tau} \left(\frac{y_t}{y}\right)^{\tau_y} \left(\frac{q_{S,t-1}b_{S,t-1} + q_{L,t-1}b_{L,t-1}}{q_S b_S + q_L b_L}\right)^{\tau_b}, \tag{49}$$

where $\tilde{\tau}$ is a level parameter, and $\tau_y$ and $\tau_b$ determine the elasticity of taxes to income and government debt, respectively. Government expenditure, $g_t$, follows an exogenous AR(1) process given by

$$\log g_t = (1 - \rho_g) \log g + \rho_g \log g_{t-1} + \varepsilon_{g,t}. \tag{50}$$

The central bank targets the short-term interest rate and the yield curve slope. Hitting the latter target requires the central bank to adjust the outstanding quantities of long versus short bonds, since the transactions cost term, $\Gamma_t$, which essentially determines the spread, depends on their relative holdings of long government bonds by private agents. The target for the short-term nominal interest rate is determined using a standard Taylor rule:

$$1 + R_t = (1 + R_{t-1})^{\rho} \left[(1 + R) \left(\frac{\Pi_t}{\Pi}\right)^{a_\pi} \left(\frac{y_t}{y}\right)^{a_y}\right]^{1-\rho} \widetilde{\varepsilon}_{R,t}, \tag{51}$$

where $\rho$ determines the extent of interest rate smoothing, $R$ is the steady-state value of the short rate, and $a_\pi$ and $a_y$ are the long-run response coefficients for inflation and the output gap, respectively. $\widetilde{\varepsilon}_{R,t}$ denotes the monetary policy shock, which follows an AR(1) process.

Finally, we define variable $\gamma_{b,t} = Q_{L,t}b_{L,t}/Q_{S,t}b_{S,t}$ as the supply of long-term bonds relative to short-term bonds which follows a log-stationary AR(1) process:

$$\hat{\gamma}_{b,t} = \rho_{\gamma_b} \hat{\gamma}_{b,t-1} + \varepsilon_{\gamma_b,t} \tag{52}$$

where $\varepsilon_{\gamma_b,t}$ is an i.i.d. process. Similar to Chen et al. (2012), we treat large scale asset purchase programs as shocks to relative supply of long-term bonds since the way we have defined $\gamma_{b,t}$ it implies that changes to it (expected or unexpected) change the composition of the outstanding government liabilities.

The model's equilibrium is defined as prices and allocations such that households maximize the discounted present value of their utility, all firms maximize the discounted present value of profits subject to their constraints, and all markets clear. Combining the budget constraints of all households with the budget constraint of the government, we arrive at the resource constraint of the economy:

$$c_t + i_t + g_t = y_t - \frac{\kappa_p}{2}\left(\frac{\pi_t}{\pi_{t-1}^{\varsigma_p}\pi^{1-\varsigma_p}} - 1\right)^2 y_t - \frac{\kappa_u}{1+\varpi}\left(u_t^{1+\varpi} - 1\right) k_{t-1} - \Gamma_t \left(q_{L,t} b_{L,t} + d_t\right) \tag{53}$$

## 3 Dissecting the monetary transmission mechanism

In this section, we look at the monetary policy transmission channels. The short-term rate affects the consumption of patient households directly as they are the only holders of short-term debt, while its expected future path affects the long-term rate, $R_{L,t}$, and thereby the average mortgage rate, $R_t^d$, they charge on impatient households' stock of debt. Combining the first-order conditions for short-term bonds, $b_{S,t}$, long-term bonds, $b_{L,t}$, and log-linearizing around the zero-inflation steady state, we receive:

$$\widehat{R}_{L,t} = \frac{1}{1+\kappa q_L} \sum_{s=0}^{\infty} \left(\frac{\kappa q_L}{1+\kappa q_L}\right)^s \left[\widehat{R}_{t+s} + \frac{\widehat{\Gamma}_{t+s}}{1+\Gamma}\right] \tag{54}$$

where variables with a hat denote log deviations from the steady state. Equation (54) shows that the long-term rate depends on the current and expected future paths of the short-term policy rate, $\hat{R}_t$, and the relative supply of long-term bonds, $\hat{\Gamma}_t$. The latter is summarized by:

$$\hat{\Gamma}_t = \Gamma_2 \hat{\gamma}_{b,t} + \tilde{\varepsilon}_{\Gamma,1} \tag{55}$$

where $\hat{\gamma}_{b,t} = \widehat{q_L b}_{L,t} - \widehat{q_S b}_{S,t}$.[3] A drop, for instance, in the relative supply of long-term bonds and a rebalancing towards short-term bonds results in a fall in the long-term rate. As we show in the next section, it is this ratio that the central bank can adjust in order to affect the yield curve in its effort to shield the economy against various shocks. The induced changes in the long-term rate affect the present value of the wealth of patient households and thereby their consumption since they are the only holders of government bonds.

Impatient households instead borrow from the patient. Therefore, the interest rate on their mortgages is indirectly affected by monetary policy decisions. Combining the first-order conditions of the patient households with respect to short-term bonds, long-term bonds, the issuance of new mortgages, ($l_t$) and the average mortage interest rate, we arrive at the following expression after log-linearization:

[3] When log-linearizing, we have considered $Q_{L,t}b_{L,t}$ and $Q_{S,t}b_{S,t}$ as one variable, respectively. This is in order to neutralize this variable from price effects, when designing the optimal monetary policy. In this case, a drop in the relative supply of long-term bonds decided by the central bank would coincide with a fall in $\hat{\gamma}_{b,t}$ and thereby a drop in the long-term rate.

$$\hat{R}_t^l = \frac{\Gamma}{1+\Gamma}\hat{R}_t + \frac{1}{1+\Gamma}\hat{R}_{L,t} - \frac{\kappa q_L}{1+\Gamma}\Delta\hat{R}_{L,t+1} \tag{56}$$

where we have imposed that mortgages are paid in full within one period (i.e. $\kappa_d = 1$), for tractability and in order to facilitate the intuition. Equation (56) reveals how the central bank can affect the mortgage rate via conventional and unconventional monetary policy. As regards conventional monetary policy, the equation shows that the short-term policy rate, $\hat{R}_t$, has a positive impact on the mortgage rate. Crucially though, the presence of transaction costs at the steady state ($\Gamma > 0$) makes the transmission of conventional monetary policy incomplete. Obviously, conventional monetary policy affects the mortgage rate indirectly as well, via its impact on the long-term rate, $\hat{R}_{L,t}$. The impact of unconventional monetary policy is captured by the effect of adjustments in relative supply of long-term bonds, $\hat{\gamma}_{b,t}$, on the long-term rate, $\hat{R}_{L,t}$, via equation (54). Again, the presence of transation costs at the steady state weakens the transmission of unconventional monetary policy via $\hat{R}_{L,t}$. Based on the model mechanisms, the intuition for these effects carries as follows. When the central bank decides, for instance, to lift the short-term rate and trigger a rise in the long-term rate, the patient households experience a drop in the present discounted value of their wealth. Since they finance the mortgages of borrowers, they pass partially the incidence of a tighter monetary policy on impatient households.

Finally, monetary policy impacts housing demand by renters and thereby the equilibrium rent, $\hat{r}_{h,t}$.[4] Combining, for instance, the first-order conditions of the patient households with respect to rental housing, $h_{R,t}$, and short-term bonds, $b_{S,t}$ with the first order condition of renters with respect to housing we receive after log-linearization:

$$\hat{h}_{R,t} = -\frac{1}{r_h}\hat{q}_{h,t} + \hat{c}_{R,t} + \frac{\beta_P(1-\delta_h)}{r_h}\left[-\left(\hat{R}_t - \pi_{t+1}\right) + \hat{q}_{h,t+1}\right] \tag{57}$$

The equation above defines the equilibrium quantity for rental housing. A rise, say, of the policy rate triggers a decline in real wages which in turn suppresses the demand for housing by renters, as shown by combining the first order conditions (32) and (33), respectively. At the same time, as argued above, the higher policy rate leads to a rise in the long-term rate and the mortgage rate. The rise in the latter results in a fall in housing investment by the impatient households. Consequently, the real price of housing declines, and patient households are thus less inclined to invest in housing, rebalancing their portfolios towards short- and long-term bonds. This results in a drop in the supplied quantity of rental housing. This drop essentially accommodates the drop in demand for housing by the renters.

[4] Patient households do not have market power over housing rents. Instead housing rents are determined by demand and supply in the market for rental housing.

## 4 Optimal Monetary Policy

In this section, we analyze the optimal monetary policy from a utility-based welfare perspective. Given the richness of the model and in order to simplify, we derive analytical results abstracting from capital accumulation.[5] In our numerical exercises later though we return back to allow for it. We focus more on the optimal response of the central bank to inflationary shocks, and as such we abstract for zero lower bound considerations. We construct a social welfare measure, $V$, as a weighted average of the three types of agents' welfare, where we pick the weights such that the same constant consumption stream would result in equal welfare across the three types:

$$V = (1-\beta_P)\, V_P + (1-\beta_I)\, V_I + (1-\beta_R)\, V_R. \tag{58}$$

where, $V_\alpha$, for $\alpha \in \{P, I, R\}$, is the welfare of patient, impatient and renter households, respectively. The central bank thus seeks to maximize the above welfare subject to the economy's equilibrium conditions. We approximate the above welfare by taking a second order approximation of the utility of each type of households. Hence, the household-type welfare measures, $V_\alpha$, correspond to the second-order approximations of the utility functions. Our optimal monetary policy welfare measure is summarized in the following proposition.

**Proposition 1.** *The discounted sum of the household utilities is given by:*

$$\begin{aligned}
\sum_{t=0}^{\infty}\beta^t U_t = \sum_{t=0}^{\infty}\beta^t \Bigg\{ & \omega_{c_P}\hat{c}_{P,t} - \frac{1}{2}\tilde{\omega}_{c_P}\hat{c}_{P,t}^2 + \omega_{c_I}\hat{c}_{I,t} - \frac{1}{2}\tilde{\omega}_{c_I}\hat{c}_{I,t}^2 + \omega_{c_R}\hat{c}_{R,t} - \frac{1}{2}\tilde{\omega}_{c_R}\hat{c}_{R,t}^2 \\
& + \omega_{i_{hP}}\hat{i}_{hP,t} - \frac{1}{2}\tilde{\omega}_{i_{hP}}\hat{i}_{hP,t}^2 + \omega_{i_{hI}}\hat{i}_{hI,t} - \frac{1}{2}\tilde{\omega}_{i_{hP}}\hat{i}_{hI,t}^2 + \omega_{i_{hR}}\hat{i}_{hR,t} - \frac{1}{2}\tilde{\omega}_{i_{hR}}\hat{i}_{hR,t}^2 \\
& - \frac{1}{2}\omega_y\hat{y}_t^2 - \frac{1}{2}\omega_\pi\pi_t^2 - \omega_{\gamma_b}\left(\Gamma_2\hat{\gamma}_{b,t} + \frac{\Gamma_2(\Gamma_2-1)}{2\gamma_b}\hat{\gamma}_{b,t}^2\right) \\
& - \omega_{yc_P}\hat{y}_t\hat{c}_{P,t} - \omega_{yc_I}\hat{y}_t\hat{c}_{I,t} - \omega_{yc_R}\hat{y}_t\hat{c}_{R,t} + \omega_{c_Pc_I}\hat{c}_{P,t}\hat{c}_{I,t} + \omega_{c_Pc_R}\hat{c}_{P,t}\hat{c}_{R,t} + \omega_{c_Rc_I}\hat{c}_{R,t}\hat{c}_{I,t}\Bigg\} \\
& + t.i.p. + O\left(||\xi^3||\right)
\end{aligned}$$

*Proof.* In appendix A.2 ∎

Notice that in the welfare criterion derived above linear terms appear as well. As shown in Benigno and Woodford (2005), a properly defined welfare criterion involves linear terms next to quadratic ones when the steady state is distorted. That is, the central bank must take into account the effects of stabilization policy on the average level of output or, in terms of the current setup, its components. In fact, the steady state in the current setup is distorted not only due to monopolistic competition in the intermediate goods sector but also because impatient households are credit constrained. Importantly,

[5] The conclusions derived in this section do not change if we consider capital accumulation as well.

as shown in proposition 1, given heterogeneity in our setup, the welfare-relevant components of aggregate output instead of output per-se appear in the linear part of the welfare criterion. This reflects the fact that the policymaker is interested not only in minimizing the variation of output and its components but also in correcting, at least partly, heterogeneity in consumption levels in the economy arising from the various shocks. As we show in Appendix C, given the distorted steady state, steady state consumption levels of the different types are not equalized.[6] This implies that the policy-maker will not seek to eliminate consumption heterogeneity completely. What the policymaker is after is to minimize further widening of consumption heterogeneity arising outside the steady state due to the various shocks or due to the general equilibrium effects accompanying them.

To gain a better insight consider the case of an adverse supply shock. In this case, the monetary authority will increase the short-term rate which, via (54), implies a rise in the long-term rate. This suppresses the present discounted value of patient households' wealth and triggers them to pass part of the burden onto the impatient households lifting thereby the average mortgage rate, $R_t^d$. The latter squeezes the consumption of impatient households. Moreover, their investment in housing declines as well given that the supply shock leads to a fall in real house prices, $q_{h,t}$, making them more credit constrained through (24), suppressing thereby their consumption further. At the same time, patient households mitigate the effects of higher short- and long-term rates on their consumption not only by raising the average mortgage rate they charge but also by lifting the rents, $r_{h,t}$, putting downward pressures on the consumption of renters on top of those stemming from the drop in real wages. Apart from volatility thus, it is also the direction of these effects that now matters for the policy-maker, a result that would not hold had the steady state not been distorted. Therefore, heterogeneity affects the optimal trade-offs the policymaker is facing. This is formalized in the proposition below.

**Proposition 2.** *In the absence of heterogeneity, with only patient households residing in the economy and no mortgage debt, the central bank faces the traditional inflation-output trade-off following a supply shock. In the presence of heterogeneity though, after a supply shock, the central bank can stabilize inflation by trading off all or some of the components of aggregate output.*

*Proof.* In appendix A.3 ■

As we show in the appendix, with optimal monetary under commitment when patient households only reside in the economy (i.e. $\theta_P = 1$), the optimal trade-off receives the following form:

$$\omega_y \left(\hat{y}_t - \hat{y}_{t-1}\right) = -\frac{\left(\eta_y - 1\right)\omega_\pi \vartheta}{\kappa_p}\pi_t \tag{59}$$

[6] The steady-state consumption of impatient households depends on their steady-state stock of housing. Moreover, given that they are always credit constrained, their steady-state consumption depends on the LTV ratio, $\phi$, as well. The steady-state consumption of renters instead depends entirely on their steady-state labor income.

where the terms $\omega_y$ and $\omega_\pi$ are nonlinear functions of the model structural parameters. Under heterogeneity instead, the optimal targeting criterion becomes:

$$\left(\frac{\omega_y c_P}{y}+\omega_{yc_P}\right)\Delta\hat{c}_{P,t}+\left(\frac{\omega_y c_I}{y}+\omega_{yc_I}\right)\Delta\hat{c}_{I,t}+\left(\frac{\omega_y c_R}{y}+\omega_{yc_R}\right)\Delta\hat{c}_{R,t}+\frac{\omega_y i_h}{y}\Delta\hat{i}_{h,t}=-\frac{(\eta_y-1)\,\omega_\pi\vartheta}{\kappa_p}\pi_t \tag{60}$$

where the coefficient in the consumption of each group is positive and $\Delta\hat{c}_{\alpha,t}=\hat{c}_{\alpha,t}-\hat{c}_{\alpha,t-1}$ for $\alpha\in\{P,I,R\}$ and $\Delta\hat{i}_{h,t}=\hat{i}_{h,t}-\hat{i}_{h,t-1}$. Under heterogeneity, the central bank is faced with additional trade-offs captured by the covariance between output and the consumption of each household type in the welfare criterion discussed in Proposition 1 above. These covariance terms vanish when considering the model without heterogeneity (i.e. $\theta_P=1$). In this case thus $\omega_{yc_P}=\omega_{yc_I}=\omega_{yc_R}=0$ in the welfare criterion. From expression (60), in the face of higher inflation, the central bank, ceteris paribus, has to lower the consumption of one or more population groups and/or housing investment or to lower simultaneously all of them. In either of the above cases, this translates to lower output, $y_t$. Contrary to the textbook result where there is only one representative household in the economy, here the central bank is not constrained to trade all components of output off when faced with higher inflation. Even though the outcome is the same (i.e. lower output), not every group or every sector in the economy bears the burden of stabilization to the same extent necessarily. This implies that inflation stabilization may come at the cost of consumption inequality.

### 4.1 Heterogeneity and the slope of the yield curve

In this section, we explore the interaction between heterogeneity and the implied optimal decision of the central bank about the short- and long-term rate. From Proposition 1, we have shown that the relative long-term bond supply, $\hat{\gamma}_{b,t}$, enters the welfare criterion and is hence one of the stabilization objectives of the central bank.[7] Setting this objective alters the supply of long-term bonds relative to short-term bonds having an immediate impact on their price and thereby on long-term rates. Following thus demand or supply shocks, the central bank decides optimally not only upon the trade-off between inflation and output, affecting thereby the short-term rate through the Euler equation of patient households for short-term bonds, but also upon the long-term rate through its decision for the relative supply for long-term bonds.

The key question in the current setup is to what extent does the optimal decision about the

[7] We abstract from central bank balance sheet considerations. An alternative would be to assume a central bank balance sheet where the central bank issues reserves (as in Sims and Wu, 2019) to finance its asset purchases, where a representative financial intermediary is born in each period and exits the industry in the subsequent period (as opposed to Gertler and Karadi, 2011, 2013). Another alternative would be to assume that the central bank issues short-term bonds to finance its asset purchases (as in Kabaca et al., 2023) and faces an efficiency cost associated with asset purchases (as in Karadi and Nakov, 2021). In both cases, the central bank would have to transfer its profit/losses to the government in the form of remittances. The conclusions of this section would not change had we considered this option.

relative supply of long-term bonds qualitatively depend on heterogeneity conditional on demand or supply shocks. From Proposition 2, it becomes clear that the implied optimal decision about the short-term rate is not dependent on the heterogeneity. That is, in the absence of heterogeneity, the central bank has to lower inflation by triggering a recession in the presence of an adverse supply shock, for instance. Similarly, in the presence of heterogeneity, the central bank has to lower inflation trading-off some or all of the components of aggregate output, affecting thus the latter negatively. Clearly, in both cases, it can be shown that this is achieved by raising the short-term rate. What heterogeneity changes in this case is simply the magnitude of the necessary increase in the short-term rate. When it comes to the effect on the long-term rate though, heterogeneity can have important qualitative implications for the optimal relative supply of long-term bonds. The first question that we ask is whether it is only inflation that affects the optimal decision for the relative supply of long-term bonds or whether output (and thereby its components) also plays a role. We show that under heterogeneity the optimal relative supply of long-term bonds depends not only on inflation but also on output and the consumption of each type. When heterogeneity is instead turned off, the optimal relative supply is solely dependent on inflation. We formalize this result in Proposition 3 below.

**Proposition 3.** *The presence of heterogeneity gives rise to a positive weight on output stabilization in the targeting criterion for the relative long-term bond supply. In the absence of heterogeneity, the weight on output stabilization in the targeting rule for long-term bonds is zero.*

*Proof.* In appendix A.4 ■

Solving for the optimal relative supply of long-term bonds, $\hat{\gamma}_{b,t}$, we receive:

$$\begin{aligned}\omega_{\gamma_b}\left(1-\Gamma_2\right)\hat{\gamma}_{b,t} &= \omega_{\gamma_b}\gamma_b - \frac{\omega_{c_P}-\omega_{c_I}+\omega_{c_R}}{R^l\Omega_{rP}} \\ &+ \frac{\tilde{\omega}_{c_P}+\omega_{c_Pc_I}-\omega_{c_Pc_R}}{R^l\Omega_{rP}}\hat{c}_{P,t} + \frac{\omega_{c_Pc_I}+\omega_{c_Rc_I}-\tilde{\omega}_{c_I}}{R^l\Omega_{rP}}\hat{c}_{I,t} + \frac{\tilde{\omega}_{c_R}+\omega_{c_Rc_I}-\omega_{c_Pc_R}}{R^l\Omega_{rP}}\hat{c}_{R,t} \\ &- \frac{\left(\eta_y-1\right)\omega_\pi\left(\theta_R+\theta_I-\theta_P\right)}{\kappa_p R^l\Omega_{rP}}\pi_t + \frac{\omega_{yc_P}+\omega_{yc_R}-\omega_{yc_I}}{R^l\Omega_{rP}}\hat{y}_t \end{aligned} \tag{61}$$

where using the definitions of weights in appendix A.2 it is easy to show that $\omega_{yc_I}+\omega_{yc_R}-\omega_{yc_P}>0$.[8] Note also that $\theta_R+\theta_I-\theta_P>0$. Given that $0<\Gamma_2<1$, expression (61) shows that when deciding upon the optimal relative supply of long-term bonds, the central bank takes into account not only inflation, but also aggregate output as well as the consumption of each group. That is, heterogeneity matters for the optimal setting of long-term bond supply and thereby for the long-term rate. In the appendix, we show that in the absence of heterogeneity (i.e. $\theta_P=1$ and $\theta_I=\theta_R=0$), the optimal decision about the relative long-term bond supply collapses to:

[8] Similarly, using the derivations in Appendix A.2, it is also easy to show that $\tilde{\omega}_{c_P}+\omega_{c_Pc_I}-\omega_{c_Pc_R}>0$, $\omega_{c_Pc_I}+\omega_{c_Rc_I}-\tilde{\omega}_{c_I}>0$ and that $\tilde{\omega}_{c_R}+\omega_{c_Rc_I}-\omega_{c_Pc_R}>0$, so that the coefficient on each group's consumption is positive.

$$\omega_{\gamma_b}\left(1-\Gamma_2\right)\hat{\gamma}_{b,t}=\omega_{\gamma_b}\gamma_b+\frac{\left(\eta_y-1\right)\omega_\pi}{\kappa_p R^l\Omega_{rP}}\pi_t \tag{62}$$

In the absence of heterogeneity thus, the central bank sets the optimal relative supply of long-term bonds according to developments in inflation only. Notice also that the sign on the weight on inflation in the optimal relative long-term bond supply rule changes when heterogeneity is turned off. In (61) there is a negative relation between the inflation rate the optimal relative long-term bond supply, given that $\theta_R+\theta_I-\theta_P>0$ and $\eta_y>1$.[9] In the absence of heterogeneity in (62) instead the sign on inflation is reversed implying a positive relation with the optimal relative long-term bond supply. This implies that heterogeneity matters for the optimal response of the central bank following supply shocks. The optimal targeting criterion under heterogeneity, (61), prescribes that the central bank has to lower the relative supply of long-term bonds after an adverse supply shock. In fact, since in this case inflation goes up and output as well as the consumption of each group shrink, the negative weight on inflation in (61), implies that the developments in inflation and output, although of opposite directions, have exactly the same (negative) effect on the optimal relative long-term bond supply. In this case thus, the central bank triggers a drop in the long-term rate, flattening thereby the yield curve. In the absence of heterogeneity instead, the central bank has to increase the relative supply of long-term bonds (positive weight on inflation in (62)), triggering thereby an increase in the long-term rate.

**Corollary.** *Following an adverse supply shock in the absence of heterogeneity, the central bank raises both the short-term rate and the relative supply of long-term bonds (higher long-term rate). In the presence of heterogeneity instead, the central bank raises the short-term rate, but lowers the relative supply of long-term bonds.*

The intuition behind this result carries as follows. Following an adverse supply shock the central bank has to lift the short term rate.[10] This will in turn trigger a rise in both the long-term rate and the mortgage rate, putting an additional burden on impatient households that borrow to invest in housing. To dampen thus these effects, the central bank has to lower the relative supply of long-term bonds counteracting the upward pressure of the policy rate on the long-term and the mortgage rate (see also (56)). As a consequence, the downward pressure on investment in housing is mitigated and the economy can experience a faster recovery, as we show later in our quantitative exercise. This sequence of effects is absent under homogeneity, where the main interest of the central bank is to lower inflation.

[9] Parameter $\eta_y$ is the steady state value of the elasticity of substitution between the differentiated goods which we set at 6 in line with Galí (2008).

[10] In fact, plugging the inflation output trade-off, implied by (59) under homogeneity and by (60) under heterogeneity, in the Euler equation for short-term bonds of patient households and solving for the short-term rate, $\hat{R}_t$, it is easy to show that a rise in inflation translates into an increase in the short-term rate.

| **Table 1. Calibrated parameters** | | |
|---|---|---|
| | Symbol | Value |
| Inflation target (gross, qtr.) | $\pi$ | 1.005 |
| Time discount factor | $\beta_P$, $\beta_I = \beta_R$ | 0.9925, 0.9875 |
| Inverse of the Frisch elasticity | $\vartheta$ | 1 |
| Level for housing in utility | $\xi$ | 1 |
| LTV ratio on new regular mortgages | $\phi$ | 0.8 |
| Amortization rate on HH loans | $\kappa_d$ | 0.0175 |
| Share of capital in production | $\alpha$ | 0.25 |
| Share of Patient agents and Renters in production function | $\theta_P$, $\theta_R$ | 0.30, 0.22 |
| Share of Impatient agents in production function | $\theta_I$ | 0.48 |
| Depreciation rates | $\delta_h$, $\delta_k$ | 0.015, 0.03 |
| Gross markup in price | $\theta_p$ | 1.1 |
| Fixed costs in production | $f$ | 0. 0.11 |
| Utilization cost level | $\kappa_u$ | 0.03 |
| Tax level | $\Xi$ | 0.22 |

The next question that we ask is what heterogeneity implies for the optimal response following a positive demand shock. When it comes to the short-term policy rate, it is easy to show that the cental bank needs to lift it.[11] When it comes to the relative supply of long-term bonds the conclusion is rather ambiguous. Contrary to the supply shock where inflation rises and output drops, observing again the optimal criterion (61) it is not obvious how the central bank has to move the optimal relative supply of long-term bonds following a demand shock, as output (and the consumption of each type) and inflation move in the same direction. We show that the share of impatient households in the production function, $\theta_I$, is crucial for the optimal decision of the central bank. We summarize the condition determining how the central bank should set the optimal relative supply of long-term bonds after a demand shock with heterogeneity in the following proposition.

**Proposition 4.** *Following a positive demand shock under heterogeneity, the central bank must lower the relative supply of long-term bonds if and only if:*

$$\theta_I > \left( \frac{1}{1 + \varsigma - \frac{2(1-\beta_I)}{c_I \zeta}} \right) (\theta_R(1-\varsigma) + \theta_P (1+\varsigma)) , \quad \textit{where} \quad \varsigma = \left( \frac{\eta_y - 1}{\kappa_p} \right)^2 .$$

*Otherwise, the central bank must increase the relative supply of long-term bonds.*

*Proof.* In appendix A.5 ■

[11]Again, this is shown by plugging the targeting criterion (60) in the euler equation of the patient household and then solve for the policy rate, $\hat{R}_t$.

## 5 Calibration

Table 1 lists calibrated parameter values, and Table 2 reports the main ratios at the steady state of our model. The trend inflation factor, $\pi$, is set to 1.005, corresponding to 2% annual inflation. The time-discount factor of savers, $\beta_P$, is set to 0.9925 to match an annualized 3% real risk-free interest rate, while that of borrower and renters, $\beta_I = \beta_R$, is set to 0.9875, implying a 200 bps spread on the risk-free rate if borrowers were allowed to engage in non-mortgage borrowing. The level parameter for housing in the utility function, $\xi$, is set to 1 to ensure that the value of housing relative to annual GDP is 1.07, consistent with FOF data.

In the data, residential and non-residential investments are about 4.5% and 13% of output, respectively, while housing-to-GDP and capital-to-GDP ratios are 1.07 and 1.85 on an annualized basis.[12] Based on these, we calibrate the quarterly depreciation rates for housing and capital stocks, $\delta_h$ and $\delta_k$, to 1.5% and 3%, respectively. The share of capital in domestic production, $\alpha$, is calibrated to 0.25 using the capital-output ratio and the model-implied after-tax rental rate of capital. The demand elasticity for differentiated intermediate goods, $\eta$, is set to 6, implying a net markup in prices of 20%. The fixed cost of production, $f$, is set equal to 0.1 times the steady-state level of output to ensure that pure economic profits are zero at the steady state, thus eliminating the incentive for entry and exit in the long run of the stochastic economy. We calibrate the coefficients, $\Gamma_1$ and $\Gamma_2$, in the transaction cost function $\Gamma_t$ such that the steady state spread between the long-term rate and the short-term rate equals to 0.75 approximately. We consider this spread reasonable when looking at the spread between the 10-year treasury yield and the policy rate-sensitive 2-year yield in the US historically. The implied values for $\Gamma_1$ and $\Gamma_2$ thus are 1.0025 and 0.0025, respectively, with a steady state ratio of long-term to short-term government debt, $\gamma_b$, equal to 2.

The LTV ratio on new mortgages, $\phi$, is set to 0.75. Based on FOF data, the ratio of mortgage debt owed by all households relative to their real estate holdings, $d/h$, is around 0.37. Given that the LTV ratio is 0.75 for the average borrower, we can infer that borrower households own about 56% of the total housing stock. We therefore calibrate the wage share of patient households, $\theta_P$, to 0.30, to hit this target. Steady-state government expenditure is calibrated to ensure that its share in output, $g/y$, is 20%. The level parameter for taxes, $\Xi$, is set to 0.2. We are interested in demand and supply shocks captured by the preference, $v_t$, and the time varying elasticity of subsitution across varieties, $\eta_{y,t}$, respectively. We set the persistence in both shocks to 0.5.

## 6 Results

In this section, we analyze the effects of a demand and a supply shock. The demand shock is captured by the preference shock, $v_t$, that enters the utility function of all household types symmetrically.

[12] The capital stock reflects the tangible asset holdings of non-financial corporations, non-corporate businesses, and households minus the real estate and consumer durable holdings of households, using FOF data.

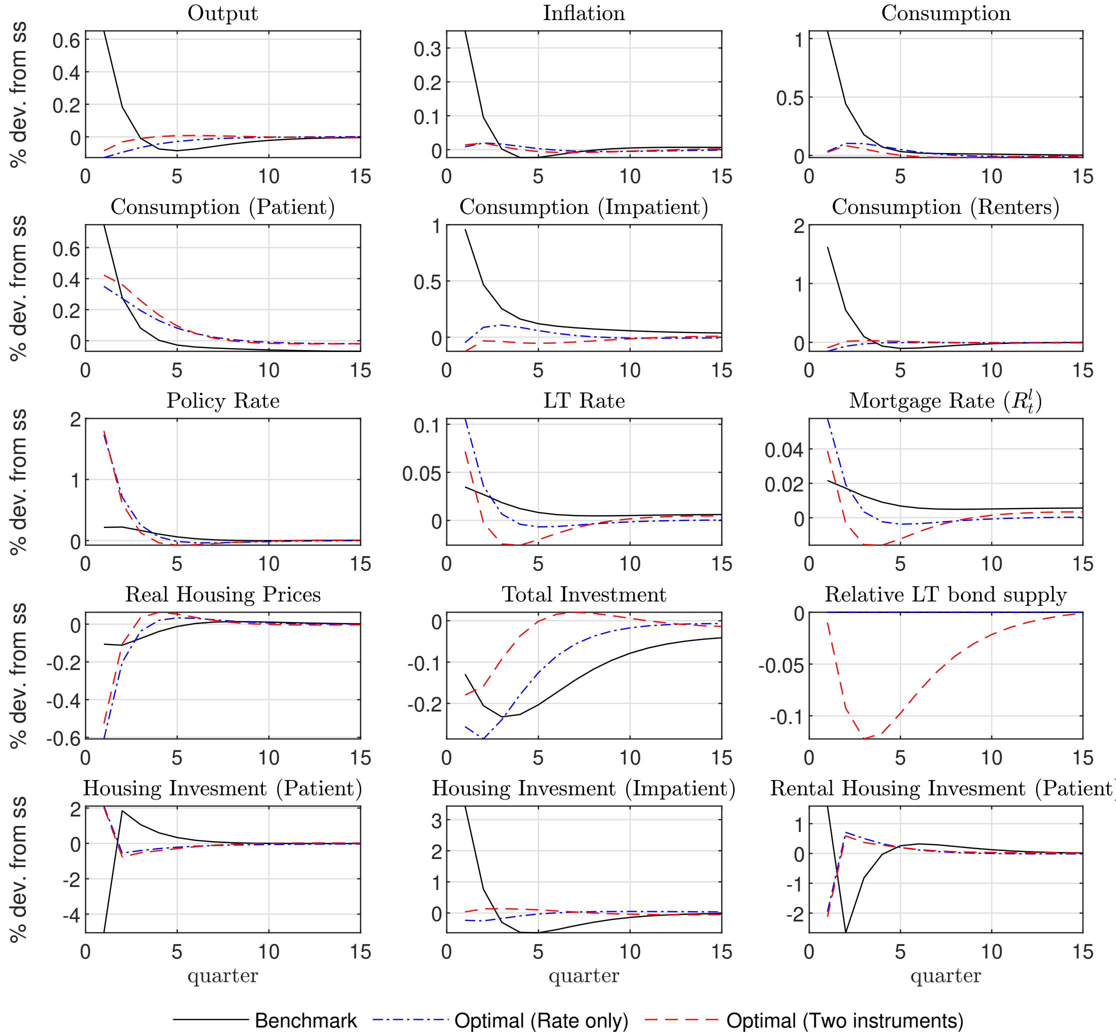
**Table 2. Model steady-state target ratios**

| | Symbol | Model |
|---|---|---|
| Total consumption / GDP | $c/y$ | 0.625 |
| share of patient households | $c_P/c$ | 0.2325 |
| share of impatient households | $c_I/c$ | 0.5351 |
| share of renter households | $c_R/c$ | 0.23 |
| Residential investment / GDP | $i_h/y$ | 0.2173 |
| Non-residential investment / GDP | $i_k/y$ | 0.1997 |
| Government expenditure / GDP | $g/y$ | 0.20 |
| Tax revenue / GDP | $tax/y$ | 0.22 |
| Capital stock / GDP (qtr) | $k/y$ | 6.7 |
| Housing stock / GDP (qtr) | $h/y$ | 3.62 |
| share of patient households | $h_P/h$ | 0.48 |
| share of impatient households | $h_I/h$ | 0.52 |
| Mortgage debt / total housing value | $d/h$ | 0.37 |
| average LTV on all outstanding loans | $d/h_I$ | 0.75 |
| LTV on new regular loans | $\phi$ | 0.75 |

The supply shock is captured by a markup shock, namely a shock to the elasticity of substitution across varieties $\eta_{y,t}$. We look at three cases. That is, the benchmark case where monetary policy is conducted only through an interest rate rule setting the short-term rate, as in (51) and the relative supply of long-term bonds follows a stationary process, as in (52), the case where the central bank sets only the short-term rate optimally (one instrument optimal monetary policy), while leaving the relative supply of long-term bonds to follow a stationary process, as in (52) and, finally, the fully optimal policy case where the central bank sets both the short-term rate and the relative supply of long-term bonds optimally (optimal monetary policy with two instruments). The analytical results of the previous section provide an indication about the trade-offs the central faces under the type of heterogeneity considered in the paper as well as on the implications of the latter on the optimal decision about the relative long-term bond supply. However, they do not provide a clear picture about what the net effect on the long-term rate and thereby on the mortgage rate will be. This section serves this purpose. We shock the economy and look at the overall impact on interest rates, and then on the macroeconomy, which we try to rationalize based on our analytical results.

**Demand shock.** We start with the case of a one standard deviation positive demand (preference) shock. We display the impulse responses in Figure 1. Looking at the responses in the benchmark case first (black solid lines), where the central bank sets the short-term rate according to the Taylor rule in (51) and the relative supply of long-term bonds follows the exogenous process in (52), the positive demand shock yields the usual effects. The consumption of each household type rises boosting demand and thereby leading to higher inflation. Following the positive demand shock, the central bank raises the short-term rate in response to the rise in inflation. The hike in the short-term rate results in a rise in the long-term rate via (54) and the mortgage rate via (56). The resulting increase

FIGURE 1. Impulse responses following a positive demand shock.




Notes: Impulse response functions following a positive demand shock, modeled as a positive preference shock. The solid black lines display the responses from the benchmark model where the central bank follows a Taylor rule and the relative supply of long-term bonds follows an $AR(1)$. The blue dashed-dotted are the responses where the central bank sets only the short-term (policy) rate optimally and the red-dashed lines are the responses where the central bank sets optimal both the short-term (policy) rate and the relative supply of long-term bonds.

in the real interest rates suppresses investment in capital. The expansion in economic activity drives wages upwards, increasing the demand for consumption goods and housing by renters. The latter effect in turn provides incentives to patient households to invest more in rental housing. Similarly, impatient households increase their demand for housing due to the lower real housing prices. Although lower real housing prices make them more credit constrained, observing their first order condition with respect to housing, equation (26), their demand for housing depends not only on current real housing prices, $q_{h,t}$, but also on future ones. Taking into account that future real housing prices will start rising to revert back to their steady-state level they decide to invest in housing today. This happens even though they will be less credit-constrained in the future due to higher housing prices.[13] This effect in conjuction with the rise in their wages offsets the rise in borrowing costs driving up their investment in housing. Patient households lower their investment for own housing on impact. This is because they find it more profitable to invest in rental housing given the stronger demand by renters and to provide loans to impatient households. Specifically, given fixed-rate mortgages, patient households seek to benefit from the rise in borrowing costs extending thereby the provision of loans. It is only until impatient households' housing investment and rental housing investment undershoot that patient households increase their investment in their own housing.[14]

Contrary to Iacoviello (2005), the supply of housing is not fixed. The expansionary demand shock results in a rise in the demand for housing by impatient households and renters. However, the demand for housing by patient households declines sharply at least on impact, outweighing the rise in demand for housing by the other two types of households (see Figure A.1 in Appendix A). The net effect is a decline in total demand for housing which drives real housing prices downwards. The decline in prices is mitigated though by the decline in the supply of housing. The drop in total housing investment, and thereby in total housing supply, instead puts upward pressure on real housing prices. In the end, the demand effect dominates leading to a decline in real housing prices. Had the supply of housing been fixed, the decline in real housing prices would have been larger.

Let us now focus on the case where the central bank sets optimally the nominal interest rate (blue dashed-dotted responses in Figure 1) whereas the relative long-term bond supply follows an exogenous $AR(1)$. This case reveals that the central bank has to optimally raise the policy rate considerably above the benchmark scenario under a Taylor rule. This yields, via (54), a sharper increase in the long-term term rate on impact followed by a persistent undershooting in the medium-run. The mortgage rate displays a similar pattern. Interestingly, the sharper hikes of the policy rate, the long-term rate, and hence the mortgage rate more than offset the positive effects of the shock on output, leading to an, albeit mild, recession. The consumption response of impatient households is now substantially dampened compared to the benchmark case owing to the higher borrowing costs on impact and to the decline in their wages on impact due to the induced recession.[15] In fact, the higher rate implies higher accumulated debt repayment costs that suppress their consumption.

The decline in real housing prices is now deeper in the first quarters following the positive demand shock. The response of impatient households' housing investment now reverses sign owing to lower wages (due to the induced recession) and to the higher mortgage rate on impact. At the same time, the induced recession suppresses real wages (see top panel in Figure A.2 in Appendix A) and thereby

[13] Given that the credit constraint binds always, impatient households will always borrow at their limit. Hence, being less credit-constrained implies that they will face a higher debt repayment burden which will suppress their consumption. As a result, they find it optimal to invest more today at the lower real housing prices than in the future.

[14] These patterns also reveal a substitutability for patient households between investment in own housing and the provision of loans to impatient or investing in rental housing.

[15] The real wage of impatient households drops on impact but starts rising soon after and is subject to a persistent overshooting in the quarters that follow the initial impact of the shock. The results are available upon request.

the demand for housing by renters (see Figure A.1 in Appendix A), which explains the decline in rental housing investment by patient households. The net effect is a decline in housing demand which dominates the decline in housing supply.[16] As a result real housing prices now decline more than in the benchmark case. This in turn puts additional pressure on impatient households who are now more credit-constrained (via (24)) contributing further to the drop in their demand for housing.

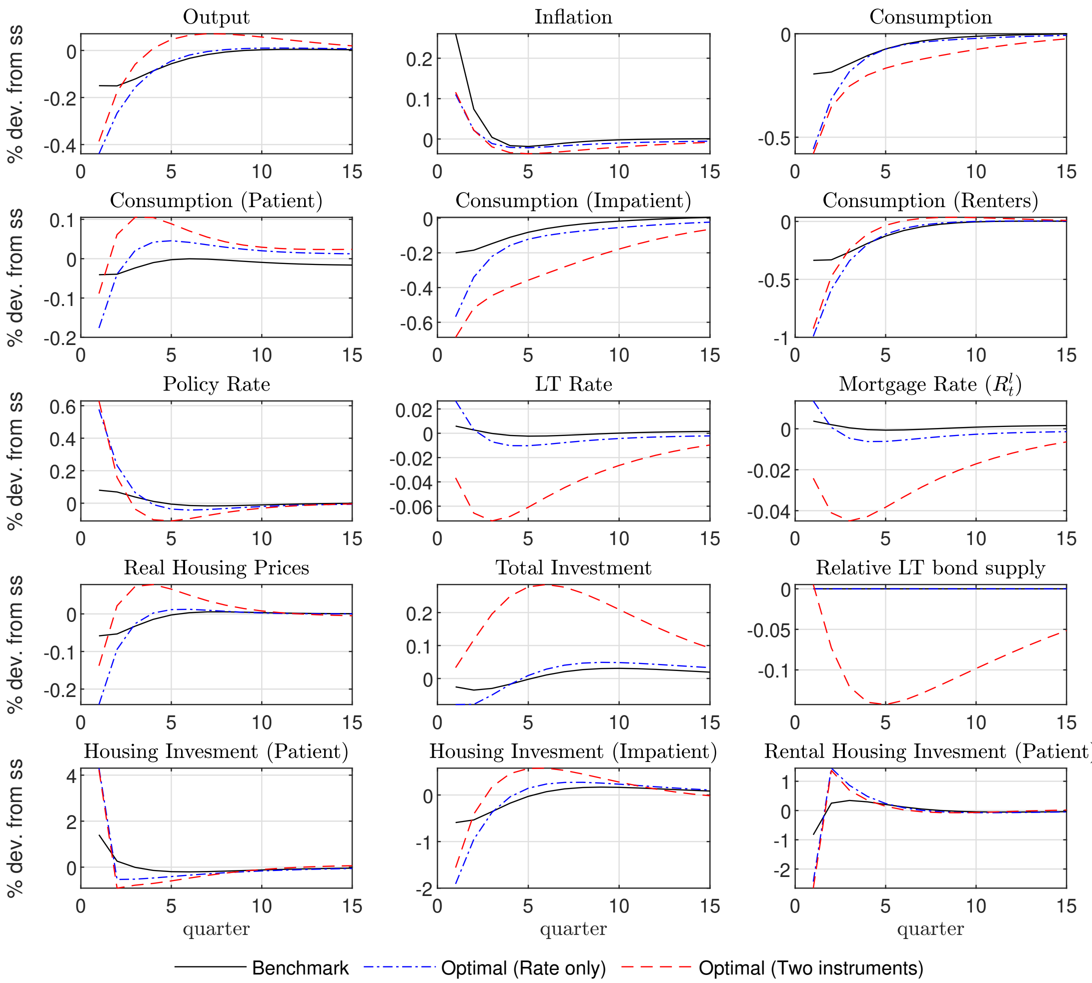
Compared to the benchmark, the consumption of patient households now rises less due to the sharper increase of the short- and the long-term rates that dampen the positive effects of the preference shock through their impact on the present discounted value of wealth. The rise in housing rents is also milder (see bottom panel in Figure A.2 in Appendix A) implying a milder rise in their income from rental housing which adds to the dampened response of their consumption.

Overall, the central bank is successful in mitigating the inflationary pressures due to the demand shock, with inflation being substantially dampened contrary to the benchmark, at the expense of triggering a mild recession, outweighing completely the positive effects of the preference shock on output. In this case, the crowding out of total investment dominates. At the same time, the decline in output results in lower real wages contributing to the muted response of impatient households' consumption and explaining the decline in the consumption of renters, as discussed above (see top panel in Figure A.2 in Appendix A). It is important to note that the stabilization of the economy, leads to a widening of the asymmetries across households and to higher consumption inequality, with a loose use of the term, just by looking at the responses of private consumption of the different household types. The impatient households and the renters, whose consumption now declines, seem to bear the burden of the stabilization.

Turning now to the case where the central bank chooses optimally both the policy rate and the relative supply of long-term bonds (red-dashed lines in Figure 1), the central bank manages to control inflation equally well to when it sets only the policy rate optimally (blue dashed-dotted lines). However, the induced recession is marginally milder. As in the other scenarios, both the policy rate and the long-term rate rise. However, compared the case where only the short-term rate is set optimally, the upward pressure on the long-term rate is now mitigated by the central bank through lowering the relative supply of long-term bonds. This partially undoes the strong pressures on the long-term rate stemming from the sharp rise in the short-term rate. This, via (56), leads to a milder increase in the mortgage rate on impact compared to when only the policy rate is set optimally. In the quarters that follow, the mortgage rate undershoots the steady state. These dynamics allow housing investment by impatient households to rise, though mildly, instead of falling. The rise in the demand for housing by the impatient dampens substantially the decline in total demand for housing which also explains why real housing prices decline less on impact and overshoot earlier. Therefore, by controlling the relative supply of long-term bonds, the central bank makes impatient households less credit constrained as well. Hence, total investment declines less while it overshoots its steady

[16] In fact, the negative housing demand effect on real housing prices in this case is stronger than in the benchmark case, since both impatient households and renters drop their demand for housing. In the benchmark case instead their demand increases. See Figure A.1 in Appendix A.

Figure 2. Impulse responses following a supply shock.




Notes: Impulse response functions following a supply shock. The solid black lines display the responses from the benchmark model where the central bank follows a Taylor rule and the relative supply of long-term bonds follows an $AR(1)$. The blue dashed-dotted are the responses where the central bank sets only the short-term (policy) rate optimally and the red-dashed lines are the responses where the central bank sets optimal both the short-term (policy) rate and the relative supply of long-term bonds.

state value 5 quarters after the shock. This sequence of effects alleviate the induced decline in output.

**Supply shock.** Let us now look at the case of a stagflationary supply shock. This is modeled as a one standard deviation shock to the time varying elasticity of substitution across varieties, $\eta_{y,t}$, in the optimal pricing equation (44) of the intermediate goods producers. We display the impulse responses of the three cases that we consider in Figure 2. In the benchmark case (black solid lines) where the central bank sets the policy rate according to the Taylor rule in (51) and the relative supply of long-term bonds follows the exogenous process in (52), a monetary tightening is triggered in order to control inflation, adding to the recession already induced by the supply shock. Similar to Iacoviello (2005), the supply shock leads to a decline in real house prices on impact. The higher policy rate leads to a rise in mortgage rates on new mortgages, which combined with lower real house prices that make impatient households more credit-constrained, results in a decline in their housing investment.

Patient households benefit from lower real house prices and thus increase their housing investment adding to their own stock of housing. However, the adverse supply shock leads to a decline in real wages (see top panel in Figure A.4 in Appendix A). This is quite costly, particularly for the renters whose only source of income comes from labor. As a result, they lower their demand for housing which is accommodated by lower investment in rental housing by the patient households. The above of events entail a decline in the consumption of both the impatient households and the renters. The consumption of patient households declines owing to lower expected future income from the provision of loans to the impatient households, lower wages as well as a drop in the present discounted value of their wealth due to overall higher interest rates.

When the central bank sets optimally only the policy rate (blue dashed-dotted lines), it lifts it more aggressively than in the benchmark case. Subsequently, the policy rate slightly undershoots its steady state value and this is because the central bank takes into account not only the jump in inflation but also the fact that the consumption of impatient and renters and their housing demand depends on the long-term rate.[17] It, therefore, finds it optimal to keep the policy rate optimally lower than otherwise for a number of periods. Given that via (54) the long-term rate depends on the sequence of the policy (or short-term) rate, it undershoots after its initial increase and stays persistently below the steady state. Through (56), this results to lower mortgage rates in the medium-run. This allows for a quick recovery of impatient households' housing investment which overshoots its benchmark path approximately a year after the initial impact.

Overall, this policy is very successful at mitigating the impact of the supply shock on inflation, at the expense of a deeper recession in the first three quarters, compared to the benchmark case. This is due to the following facts. First, the induced deeper recession results in lower wages dampening further the consumption of all households. Subsequently, the higher interest rates on impact dampen investment in both capital and housing more than in the benchmark case. Once total investment and consumption start to recover, output starts to return back to its steady state relatively fast. Allowing the policy rate, and thereby the long-term and mortgage rates, to undershoot in the medium-run is the main reason why the economy manages to recover fast despite the initial aggressive hike.

When the central bank sets optimally both the policy rate and the relative supply of long-term assets (red-dashed lines) it lifts the policy rate to the same extent as when it sets only the policy rate optimally, but allows it to decline faster and to undershoot more. As regards, the optimal relative long-term bond supply, it shrinks, as Proposition 3 and Corollary 1 suggest, bringing about a drop in the long-term rate and subsequently in the mortgage rate. Contrary to the demand shock, the

[17] Recall from section 3 that the long-term rate affects the mortgage rate impatient households pay via (56) and the demand for housing of renters via (57).

decline in the optimal relative supply of long-term bonds now dominates the rise in the policy rate and leads to a decline in the long-term rate and mortgage rate. The drop in the latter contributes to a faster recovery, with an earlier overshooting, in housing investment by impatient households who benefit not only from the lower borrowing costs but also from the quick overshooting in real house prices becoming thus less credit-constrained. The resulting rise of total investment mitigates the downward pressures on output giving rise to a faster recovery relative to the other two cases.

Note that the consumption of impatient households now declines more compared to the other two cases. This is because they derive higher utility from housing due to the lower mortgage rates, cutting more on their consumption of goods. Specifically, lower mortgage rates together with the expected overshooting in real housing prices force them to reduce their consumption today more relative to the other two cases, in order to raise their investment in housing, since they know that they will be less credit-constrained. As regards the consumption of renters it now declines more relative to the benchmark since the induced larger contraction of output suppresses the real wages more (see top panel in Figure A.4 in Appendix A). Compared to the case where only the policy rate is set optimally, their consumption recovers slightly faster due to the faster recovery of real wages that is led by the steeper recovery of economic activity.

Overall, when the central bank sets optimally both the policy rate and the relative supply of long-term bonds, it is successful in stabilizing inflation while also triggering a quick recovery of economic activity with a persistent overshooting. It is important to note though that this policy widens consumption inequality across households. In fact, contrary to what happens to impatient households and renters, the consumption of patient households falls less than when only the policy rate is set optimally subsequently also rising above the steady state more. This is fueled by the rise in the present discounted value of their wealth caused by the persistent undershooting of the policy rate and the decline in the long-term rate. Moreover, the overshooting in housing investment by the impatient in the medium run together with the higher real housing prices, which make the impatient less credit-constrained, generate additional income for the patient. These facts outweigh the negative impact of the supply shock on their consumption.

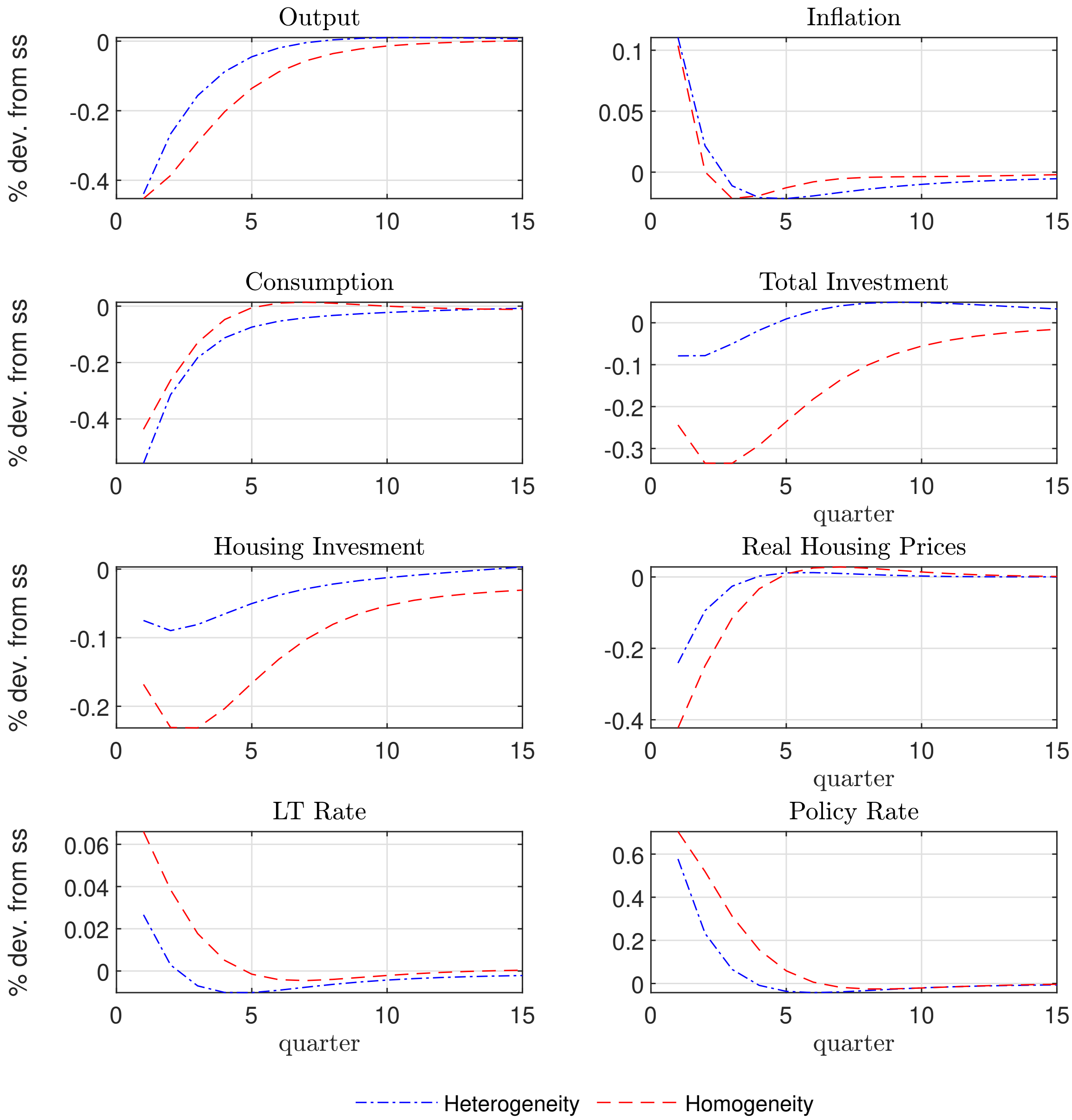
### 6.1 Optimal monetary policy in the presence and in the absence of heterogeneity

In this section, we contrast our results to those from the same model but with patient households only, namely a model abstracting from household heterogeneity.[18] Specifically, we twist our model by assuming that the household sector of the economy is comprised of patient households only who hold short- and long-term government bonds, invest in their own housing, and supply labor to intermediate goods firms. Since this is the only type of household now in the simpler economy, this means that the model does not feature the provision of loans or investment in rental housing. The rest of the model is described in section 2 above. In what follows, we focus on the supply shock to save space.

We start with the case where optimal monetary policy is conducted by setting the policy rate

[18] In Appendix B, we also discuss the version of the model without renters and look at the implications for optimal policy of the existence of this group of households.

Figure 3. Homogeneity vs Heterogeneity: Optimal Policy under One Instrument




Notes: Impulse response functions following a supply shock when optimal policy is conducted by setting the short-term (policy) rate only. The blue dashed-dotted lines are the responses corresponding to the original model taken from figure 2 while the red dashed are the responses in the absence of heterogeneity with patient households only.

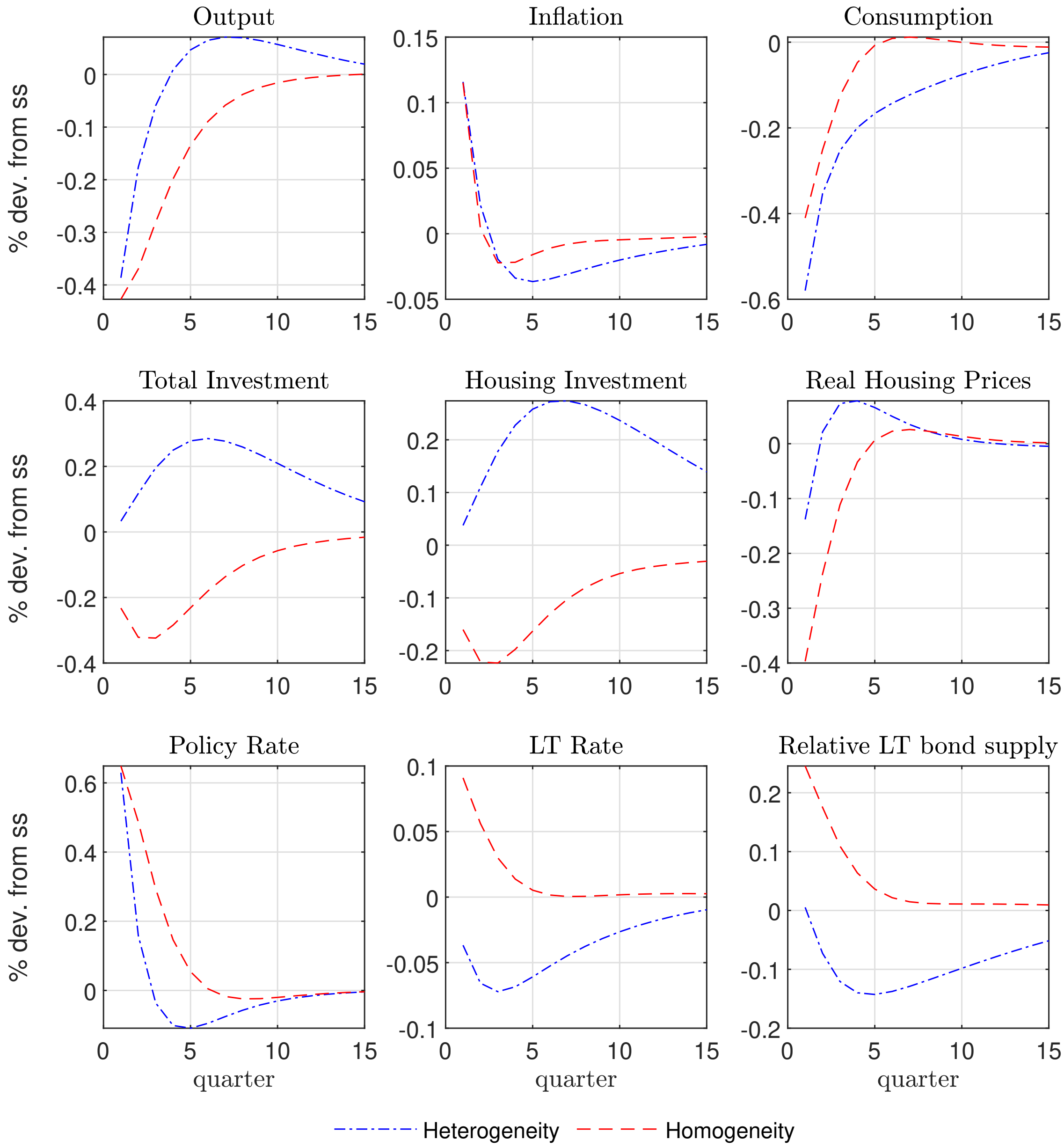
only. The responses are displayed in Figure 3. Under homogeneity (red-dashed lines), with patient households only residing in the economy, the central bank achieves a better stabilization of inflation at the expense of a slightly more persistent contraction. Optimal monetary policy is more aggressive under homogeneity, which is reflected in the higher policy rate hike. Under heterogeneity instead, the central bank tolerates a more persistent deflation in order to mitigate the recessionary impact of the supply shock. These observations are in line with what is formalized in Proposition 2. That is, under heterogeneity, the central bank faces a more complex trade-off when stabilizing inflation because it is optimal to take into account the induced changes in the consumption of each type as well as the implications for housing investment. Under homogeneity, the fact that patient households hold a rich set of assets, from government bonds to investments in physical capital and housing, makes them less vulnerable to the negative effects of the supply shock and to those stemming from the stronger monetary policy tightening. For instance, although the higher interest rates suppress the present discounted value of their wealth, they benefit from the lower real house prices. This mitigates the negative impact on their consumption and gives more leeway to the central bank to stabilize inflation.

Under heterogeneity instead, the central bank accounts for the fact that a substantial fraction of the households is indebted without any assets in their portfolios other than housing (impatient) or whose only source of income comes from labor (renters). For that reason, as argued in the previous section, the consumption of impatient households is very sensitive to fluctuations in the short-term rate and thereby in the mortgage rate. Moreover, it has been also shown, how sensitive the consumption of renters is to the degree of contraction in the economy and thereby to the fall in real wages. In its effort to curb inflation, the central bank needs to take into account these channels. This explains why it lifts its policy rate optimally less than what it does under homogeneity.

We now turn to the case where the central bank sets optimally both the policy rate and the relative supply of long-term bonds. We compare the two cases in Figure 4. A few key differences stand out. First, and most importantly, the differences in the responses of the optimal relative long-term bond supply. Under homogeneity, the central bank lifts the relative supply of long-term bonds as opposed to what it does under heterogeneity. This is what Proposition 3 and Corollary 1 prescribe. Specifically, under homogeneity, we have shown that the optimal relative long-term bond supply (see equation (62)) depends positively on inflation. Therefore, following a stagflationary supply shock, the central bank has to raise the relative supply of long-term bonds, triggering thus a rise in the long-term rate. Under heterogeneity instead, we have shown (see equation (61)) that the sign on inflation in the optimal rule for the relative long-term bond supply becomes negative, and in this case, the central bank has to lower the relative supply of long-term bonds, as discussed in the previous section. This now triggers a drop in the long-term rate.

Additionally, we showed in Proposition 3 that under homogeneity the central bank does not place a weight on output fluctuations when deciding about the relative supply of long-term bonds. This is not the case under heterogeneity (see (61)) where the central bank apart from inflation accounts also for output and household-specific consumption fluctuations when setting the relative long-term bond supply optimally. Hence, it accounts for the fact that impatient households and renters are largely affected by the induced movements in the long-term rate via the channels discussed in section 3 (see also equations (56) and (57)). That said, it finds optimal to decrease the relative supply of long-term bonds following the supply shock in order to offset the negative effects of the shock and the higher short-term rate on impatient households and on renters.

Figure 4. Homogeneity vs Heterogeneity: Optimal Policy under Two Instruments




Notes: Impulse response functions following a supply shock when optimal policy is conducted by setting the short-term (policy) rate as well as the relative supply of long-term bonds. The blue-circled are the responses corresponding to the original model taken from figure 2 while the red dashed are the responses in the absence of heterogeneity with patient households only.

The second key observation is the difference in the responses of real house prices. They decline more under homogeneity making housing for patient households cheaper that in turn mitigates the decline in their consumption. Under heterogeneity instead, the rise in housing demand due to the persistent decline in mortgage rate leads to a substantially milder decline in real house prices on impact followed by a quick overshooting. This makes own-housing more costly for patient households on the one hand, but makes impatient households less credit constrained on the other hand. As regards total consumption, it falls more under heterogeneity mainly driven by the fact that impatient households derive now more utility from housing cutting thus on their consumption of goods, as also explained in the previous section. The rise in total investment partially offsets the larger drop in total consumption allowing for a faster recovery of economic activity. Looking at the policy rate path, it rises in both cases, but declines faster and undershoots more under heterogeneity. It follows thus that heterogeneity matters quite substantially for the design of optimal monetary policy, especially when it comes to the decision about the relative supply of long-term bonds and thereby their effects on the long-term rates and on the macroeconomy.

# 7 Conclusion

This paper investigates the optimal mix of conventional and unconventional monetary policy in an economy with household heterogeneity and mortgage debt. Using a DSGE model with savers, borrowers, and renters, we show that the presence of heterogeneity fundamentally alters the transmission and design of optimal monetary policy. In particular, following adverse supply shocks, the optimal policy involves raising the short-term rate to stabilize inflation while simultaneously lowering the long-term rate to mitigate the financial burden on indebted households and renters. This dual policy approach supports a faster recovery in investment and output but comes at the cost of increased consumption inequality.

Our analysis highlights the importance of considering the distributional consequences of monetary policy, especially in economies with high levels of household debt. The results suggest that unconventional tools such as yield curve control or quantitative easing should be deployed in a targeted manner that accounts for household balance sheet vulnerabilities. Moreover, we demonstrate that the optimal response of long-term bond supply depends critically on the nature of the shock and the degree of heterogeneity in the economy.

Future research could extend this framework by incorporating financial intermediaries, endogenous default risk, or central bank balance sheet constraints. Additionally, exploring the interaction between fiscal and monetary policy in heterogeneous-agent settings remains a promising avenue for further investigation.

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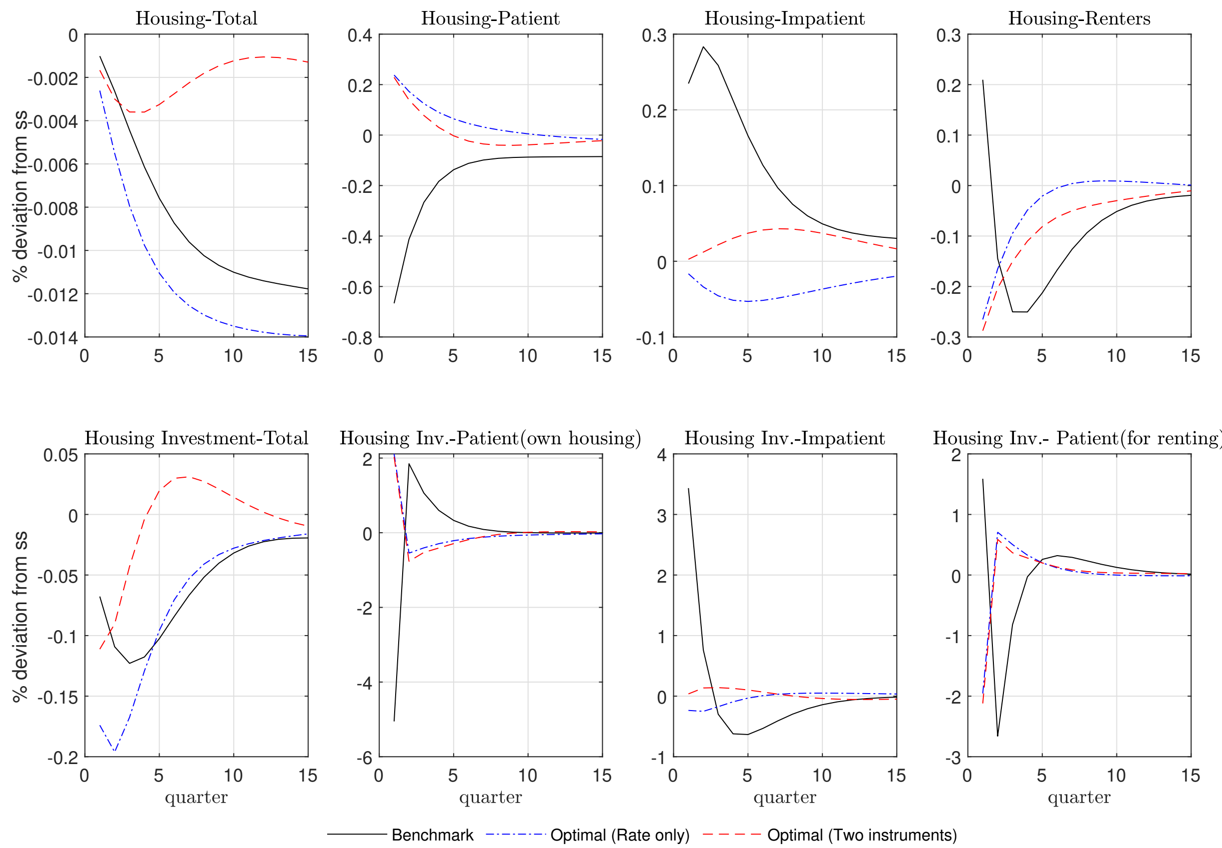
# Online Appendix

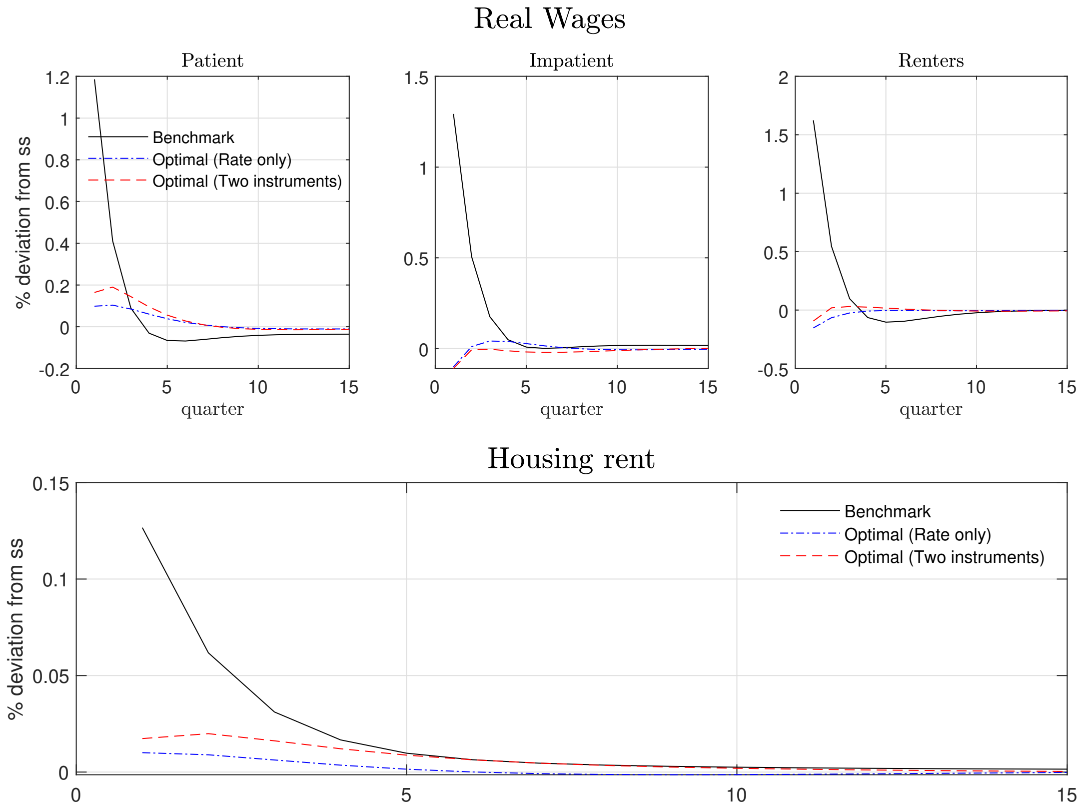
## A Additional Figures

Figure A.1. Housing Demand and Housing Investment following a positive preference shock



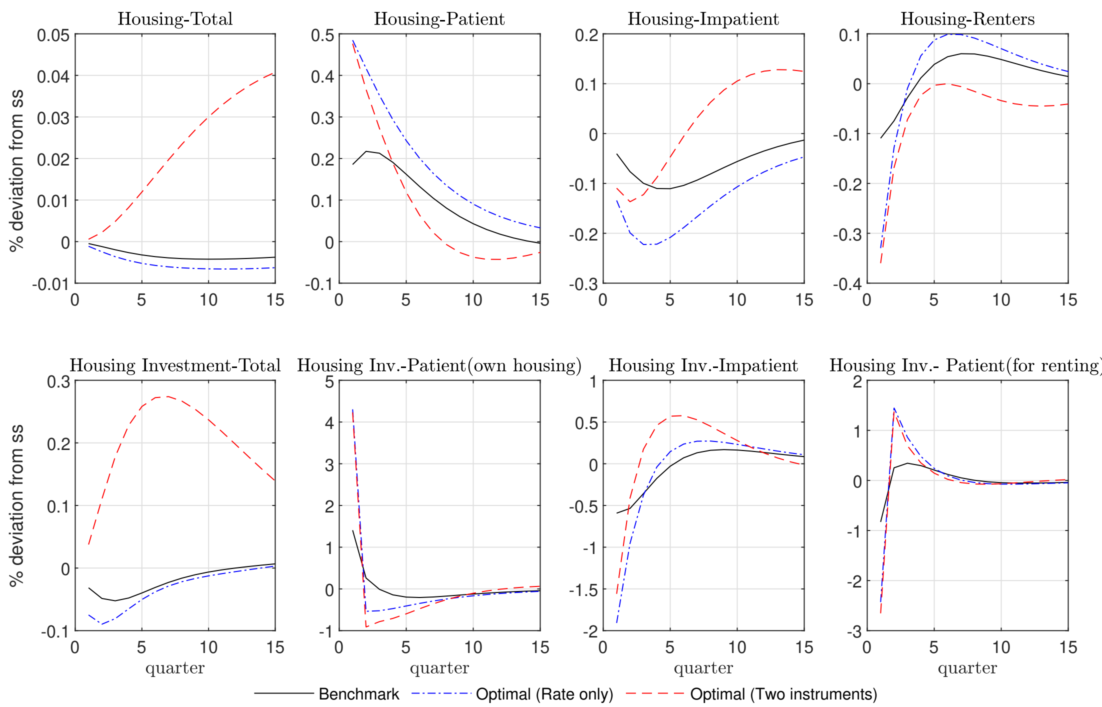

Notes: Impulse response functions following a positive demand shock, modeled as a positive preference shock. The solid black lines display the responses from the benchmark model where the central bank follows a Taylor rule and the relative supply of long-term bonds follows an $AR(1)$. The blue dashed-dotted are the responses where the central bank sets only the short-term (policy) rate optimally and the red-dashed lines are the responses where the central bank sets optimal both the short-term (policy) rate and the relative supply of long-term bonds.

FIGURE A.2. Real Wages and Housing Rent following a positive preference shock



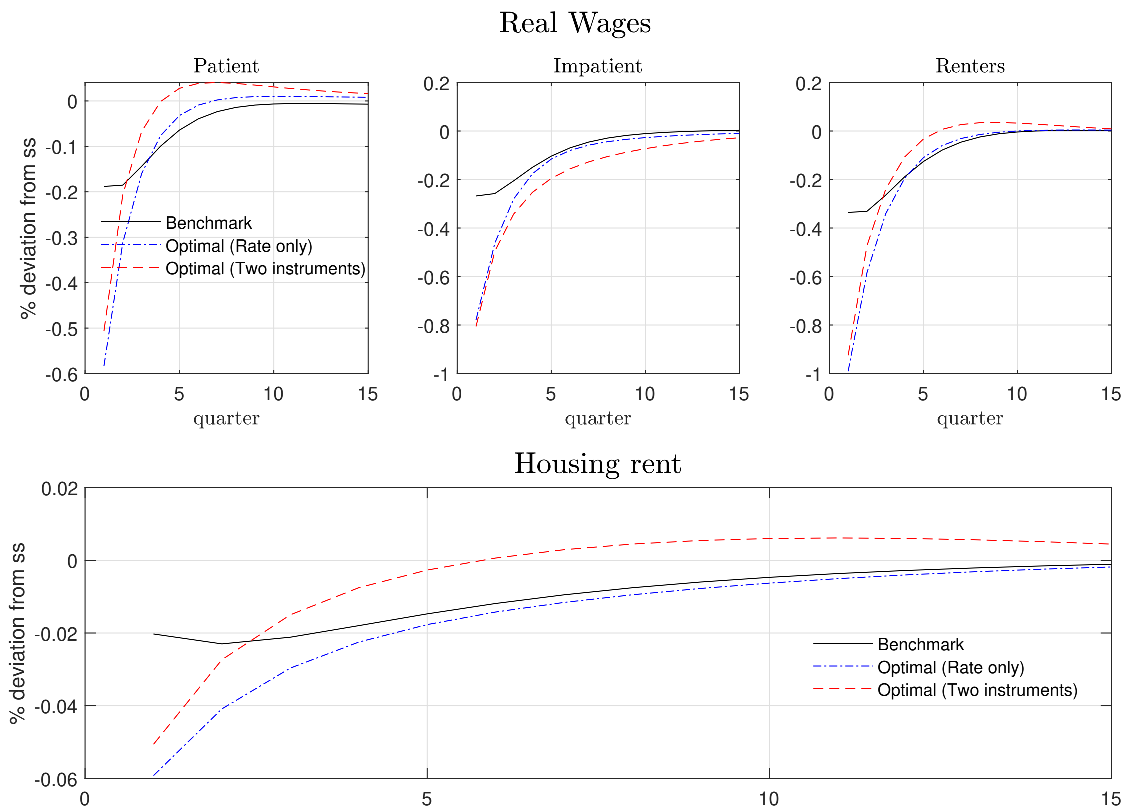

Notes: Impulse response functions following a positive demand shock, modeled as a positive preference shock. The solid black lines display the responses from the benchmark model where the central bank follows a Taylor rule and the relative supply of long-term bonds follows an $AR(1)$. The blue dashed-dotted lines are the responses where the central bank sets only the short-term (policy) rate optimally and the red-dashed lines are the responses where the central bank sets optimal both the short-term (policy) rate and the relative supply of long-term bonds.

Figure A.3. Housing Demand and Housing Investment following a supply shock




Notes: Impulse response functions following an adverse supply shock, modeled as a positive cost-push shock. The solid black lines display the responses from the benchmark model where the central bank follows a Taylor rule and the relative supply of long-term bonds follows an $AR(1)$. The blue dashed-dotted are the responses where the central bank sets only the short-term (policy) rate optimally and the red-dashed lines are the responses where the central bank sets optimal both the short-term (policy) rate and the relative supply of long-term bonds.

Figure A.4. Real Wages and Housing Rent following a supply shock




Notes: Impulse response functions following an adverse supply shock, modeled as a positive cost-push shock. The solid black lines display the responses from the benchmark model where the central bank follows a Taylor rule and the relative supply of long-term bonds follows an $AR(1)$. The blue dashed-dotted lines are the responses where the central bank sets only the short-term (policy) rate optimally and the red-dashed lines are the responses where the central bank sets optimal both the short-term (policy) rate and the relative supply of long-term bonds.

## A.1 Derivation of the resource constraint

Solving for the stock of mortgage debt in aggregate budget constraint of impatient households and substituting in the patient households' aggregate budget constraint, we receive:

$$\begin{aligned} & c_{P,t} + c_{I,t} + q_{h,t}\left(i_{hP,t} + i_{hI,t} + i_{hR,t}\right) + q_{k,t} i_{k,t} + q_{S,t} b_{S,t} + q_{L,t} b_{L,t} + \Gamma_t l_t \\ & = w_{P,t} nP,t + w_{I,t} nI,t + r_{h,t} h_{R,t} + r_{k,t} k_{t-1} + \frac{b_{S,t-1}}{\pi_t} + \frac{1+\kappa q_{L,t}}{\pi_t} b_{L,t-1} - (1-\kappa_d)\Gamma_t \frac{d_{t-1}}{\pi_t} + \frac{\Pi_t}{P_t} - tax_t \end{aligned} \tag{A.1}$$

while solving for the rental costs in the renters' aggregate budget constraint and substituting in the expression above, we receive:

$$\begin{aligned} & c_{P,t} + c_{I,t} + c_{R,t} + q_{h,t}\left(i_{hP,t} + i_{hI,t} + i_{hR,t}\right) + q_{k,t} i_{k,t} + q_{S,t} b_{S,t} + q_{L,t} b_{L,t} \\ & = w_{P,t} nP,t + w_{I,t} nI,t + w_{R,t} nR,t + r_{k,t} k_{t-1} + \frac{b_{S,t-1}}{\pi_t} + \frac{1+\kappa q_{L,t}}{\pi_t} b_{L,t-1} - \Gamma_t d_t + \frac{\Pi_t}{P_t} - tax_t \end{aligned} \tag{A.2}$$

Note that real aggregate profits read as follows:

$$\frac{\Pi_t}{P_t} = \frac{1}{P_t}\int_0^1 P_t(i) y_t(i) di - w_{P,t} n_{P,t} - w_{I,t} n_{I,t} - w_{R,t} n_{R,t} - r_{k,t} k_{t-1} - \frac{\kappa_u}{1+\varpi}\left[u_t^{1+\varpi} - 1\right] k_{t-1} - \frac{\kappa_p}{2}\left(\frac{\pi_t}{\pi_{t-1}^{\varsigma_p}\pi^{1-\varsigma_p}} - 1\right)^2 y_t \tag{A.3}$$

Substituting (A.3) in (A.2), and abstracting from capital adjustment costs (i.e. $\kappa_u = 0$), we receive:

$$\begin{aligned} & c_{P,t} + c_{I,t} + c_{R,t} + q_{h,t}\left(i_{hP,t} + i_{hI,t} + i_{hR,t}\right) + q_{k,t} i_{k,t} + q_{S,t} b_{S,t} + q_{L,t} b_{L,t} \\ & = y_t + \frac{b_{S,t-1}}{\pi_t} + \frac{1+\kappa q_{L,t}}{\pi_t} b_{L,t-1} - \Gamma_t d_t - \frac{\kappa_p}{2}\left(\frac{\pi_t}{\pi_{t-1}^{\varsigma_p}\pi^{1-\varsigma_p}} - 1\right)^2 y_t - tax_t \end{aligned} \tag{A.4}$$

The real government budget constraint is summarized by:

$$g_t + \frac{b_{S,t-1}}{\pi_t} + \frac{1+\kappa q_{L,t}}{\pi_t} b_{L,t-1} = tax_t + q_{S,t} b_{S,t} + q_{L,t} b_{L,t} \tag{A.5}$$

Substituting out for newly issued and accumulated debt in (A.4) using the government budget constraint, we receive the resource constraint of the economy:

$$c_t + i_t + g_t = y_t - \frac{\kappa_p}{2}\left(\frac{\pi_t}{\pi_{t-1}^{\varsigma_p}\pi^{1-\varsigma_p}} - 1\right)^2 y_t - \Gamma_t\left(q_{L,t}b_{L,t} + d_t\right) \tag{A.6}$$

where we assume that $\Gamma_t$ has the following functional form:

$$\Gamma_t = \Gamma_1\left(\frac{q_{L,t}b_{L,t}}{q_{S,t}b_{S,t}}\right)^{\Gamma_2} - 1 \tag{A.7}$$

## A.2 Proof of Proposition 1: Derivation of the welfare criterion

The derivation of the welfare criterion consists of taking the second order approximation to the utility of patient, impatient and renter households, respectively. For simplicity, we abstract from physical capital assuming labor as the only input in the production function.

**Patient households**

The second order approximation to the utility function of patient households reads as follows:

$$\begin{aligned} W_{P,t} = U_P + U_{c_P}x_P\left(\widehat{c}_{P,t} + \frac{1}{2}(1 + \frac{U_{c_Pc_P}c_P}{U_{c_P}})\widehat{c}^2_{P,t}\right) + U_{h_P}h_P\left(\hat{h}_{P,t} + \frac{1}{2}\left(1 + \frac{U_{h_Ph_P}h_P}{U_{h_P}}\right)\hat{h}^2_{P,t}\right) \\ - U_{n_P}n_P\left(\hat{n}_{P,t} + \frac{1}{2}(1 + \frac{U_{n_Pn_P}n_P}{U_{n_P}})\hat{n}^2_{P,t}\right) \end{aligned} \tag{A.8}$$

where $U_{c_P}, U_{h_P}, U_{n_P}$ is the marginal utility of consumption, the marginal utility of housing and the marginal disutility of labor, respectively. Computing the marginal utility of consumption and housing, the marginal disutility of and the FOC w.r.t. labor at the steady state, we receive:

$$W_{P,t} = U_P + U_{c_P}\left[\frac{c_P}{1-\zeta}\hat{c}_{P,t} + \frac{U_{h_P}}{U_{c_P}}h_P\hat{h}_{P,t} - \frac{U_{n_P}}{U_{c_P}}n_P\left(\hat{n}_{P,t} + \frac{1}{2}\left(1+\vartheta\right)\hat{n}^2_{P,t}\right)\right] \tag{A.9}$$

**Impatient households**

The second order approximation to the utility function of impatient households reads as follows:

$$\begin{aligned} W_{I,t} = U_I + U_{c_I}x_I\left(\widehat{c}_{I,t} + \frac{1}{2}(1 + \frac{U_{c_Ic_I}c_I}{U_{c_I}})\widehat{c}^2_{I,t}\right) + U_{h_I}h_I\left(\hat{h}_{I,t} + \frac{1}{2}\left(1 + \frac{U_{h_Ih_I}h_I}{U_{h_I}}\right)\hat{h}^2_{I,t}\right) \\ - U_{n_I}n_I\left(\hat{n}_{I,t} + \frac{1}{2}(1 + \frac{U_{n_In_I}n_I}{U_{n_I}})\hat{n}^2_{I,t}\right) \end{aligned} \tag{A.10}$$

where $U_{c_I}, U_{h_I}, U_{n_I}$ is the marginal utility of consumption, the marginal utility of housing and the marginal disutility of labor, respectively.Computing the marginal utility of consumption and housing, the marginal disutility of labor and the FOC w.r.t. labor at the steady state, we receive:

$$W_{I,t} = U_I + U_{c_I}\left[c_I\hat{c}_{I,t} + \frac{U_{h_I}}{U_{c_I}}h_I\hat{h}_{I,t} - \frac{U_{n_I}}{U_{c_I}}n_I\left(\hat{n}_{I,t} + \frac{1}{2}(1+\vartheta)\,\hat{n}_{I,t}^2\right)\right] \tag{A.11}$$

**Renter households**

The second order approximation to the utility function of renter households reads as follows:

$$\begin{aligned} W_{R,t} = U_R + U_{c_R}c_R\left(\widehat{c}_{R,t} + \frac{1}{2}(1+\frac{U_{c_Rc_R}c_R}{U_{c_R}})\widehat{c}_{R,t}^2\right) + U_{h_R}h_R\left(\hat{h}_{R,t} + \frac{1}{2}\left(1+\frac{U_{h_Rh_R}h_R}{U_{h_R}}\right)\hat{h}_{R,t}^2\right) \\ - U_{n_R}n_R\left(\hat{n}_{R,t} + \frac{1}{2}(1+\frac{U_{n_Rn_R}n_R}{U_{n_R}})\hat{n}_{R,t}^2\right) \end{aligned} \tag{A.12}$$

where $U_{c_R}, U_{h_R}, U_{n_R}$ is the marginal utility of consumption, the marginal utility of housing and the marginal disutility of labor, respectively. Computing the marginal utility of consumption and housing, the marginal disutility of and the FOC w.r.t. labor at the steady state, we receive:

$$W_{R,t} = U_R + U_{c_R}\left[c_R\hat{c}_{R,t} + \frac{U_{h_R}}{U_{c_R}}h_R\hat{h}_{R,t} - \frac{U_{n_R}}{U_{c_R}}n_R\left(\hat{n}_{R,t} + \frac{1}{2}(1+\vartheta)\,\hat{n}_{R,t}^2\right)\right] \tag{A.13}$$

The objective function of the policy maker is a weighted average of the three welfare measures:

$$W_t = (1-\beta_P)\,W_{P,t} + (1-\beta_I)\,W_{I,t} + (1-\beta_R)\,W_{R,t} \tag{A.14}$$

Note that the terms $\frac{U_{n_j}}{U_{c_j}}n_j$, for $j = P, I, R$, in the welfare of each group is equivalent to:

$$\frac{U_{n_j}}{U_{c_j}}n_j = w_jn_j = \theta_jy \tag{A.15}$$

Using each group's corresponding second order approximation of the utility, we can write the the objective function as:

$$\begin{aligned} W_t = (1-\beta_P)\,\hat{c}_{P,t} + (1-\beta_I)\,\hat{c}_{I,t} + (1-\beta_R)\,\hat{c}_{R,t} + (1-\beta_P)\,\xi_P\hat{h}_{P,t} + (1-\beta_I)\,\xi_I\hat{h}_{I,t} + (1-\beta_R)\,\xi_R\hat{h}_{R,t} \\ - \frac{1-\beta_P}{c_P}\theta_Py\left(\hat{n}_{P,t} + \frac{1+\vartheta}{2}\hat{n}_{P,t}^2\right) - \frac{1-\beta_I}{c_I}\theta_Iy\left(\hat{n}_{I,t} + \frac{1+\vartheta}{2}\hat{n}_{I,t}^2\right) - \frac{1-\beta_R}{c_R}\theta_Ry\left(\hat{n}_{R,t} + \frac{1+\vartheta}{2}\hat{n}_{R,t}^2\right) \end{aligned} \tag{A.16}$$

Combining the FOC of the firm's maximization problem w.r.t. to labor inputs, that define input demands, with the FOCs of each group w.r.t. labor supply and taking a first order approximation of the resulting expression (and setting production costs $f$ to zero), we receive:

$$\hat{n}_{j,t} = \frac{1}{1+\vartheta}\left[\hat{\Omega}_t + \hat{y}_t - \hat{c}_{j,t}\right] \tag{A.17}$$

for $j = P, I, R$. Plugging the above expression in (A.16) and gathering terms:

$$\begin{aligned}
W_t = {} & (1-\beta_P)\left(1+\frac{\theta_P y}{c_P(1+\vartheta)}\right)\hat{c}_{P,t} + (1-\beta_I)\left(1+\frac{\theta_I y}{c_I(1+\vartheta)}\right)\hat{c}_{I,t} + (1-\beta_R)\left(1+\frac{\theta_R y}{c_R(1+\vartheta)}\right)\hat{c}_{R,t} \\
& + (1-\beta_P)\,\xi_P\hat{h}_{P,t} + (1-\beta_I)\,\xi_I\hat{h}_{I,t} + (1-\beta_R)\,\xi_R\hat{h}_{R,t} \\
& - \left(\frac{(1-\beta_P)\,\theta_P}{c_P} + \frac{(1-\beta_I)\,\theta_I}{c_I} + \frac{(1-\beta_R)\,\theta_R}{c_R}\right)\frac{y}{1+\vartheta}\left(\hat{\Omega}_t + \frac{1}{2}\hat{\Omega}_t^2 + \hat{y}_t + \frac{1}{2}\hat{y}_t^2 + \hat{\Omega}_t\hat{y}_t\right) \\
& - \left(\frac{(1-\beta_P)\,\theta_P y}{2c_P(1+\vartheta)}\right)\hat{c}_{P,t}^2 - \left(\frac{(1-\beta_I)\,\theta_I y}{2c_I(1+\vartheta)}\right)\hat{c}_{I,t}^2 - \left(\frac{(1-\beta_R)\,\theta_R y}{2c_R(1+\vartheta)}\right)\hat{c}_{R,t}^2 \\
& + \left(\hat{\Omega}_t + \hat{y}_t\right)\left[\frac{(1-\beta_P)\,\theta_P y}{c_P(1+\vartheta)}\hat{c}_{P,t} + \frac{(1-\beta_I)\,\theta_I y}{c_I(1+\vartheta)}\hat{c}_{I,t} + \frac{(1-\beta_R)\,\theta_R y}{c_R(1+\vartheta)}\hat{c}_{R,t}\right]
\end{aligned} \tag{A.18}$$

Let us now work with the marginal cost term, $\hat{\Omega}_t$. Log-linearization leads to:

$$\hat{\Omega}_t = \theta_P\hat{w}_{P,t} + \theta_I\hat{w}_{I,t} + \theta_R\hat{w}_{R,t} - \hat{z}_t \tag{A.19}$$

Using the log-linearized FOCs of households w.r.t. to labor, we may rewrite the expression above as follows:

$$\hat{\Omega}_t = \vartheta\theta_P\hat{n}_{P,t} + \vartheta\theta_I\hat{n}_{I,t} + \vartheta\theta_R\hat{n}_{R,t} + \theta_P\hat{c}_{P,t} + \theta_I\hat{c}_{I,t} + \theta_R\hat{c}_{R,t} - \hat{z}_t \tag{A.20}$$

and using the output equation we can simplify to:

$$\hat{\Omega}_t = \vartheta\hat{y}_t + \theta_P\hat{c}_{P,t} + \theta_I\hat{c}_{I,t} + \theta_R\hat{c}_{R,t} \tag{A.21}$$

where *t.i.p.* denotes terms independent of policy (e.g. productivity shocks in our case) Plugging (A.21) in (A.18) and gathering terms, we receive:

$$
\begin{aligned}
W_t = & \left(\varpi_P - \zeta \frac{y\theta_P}{1+\vartheta}\right) \hat{c}_{P,t} + \left(\varpi_I - \zeta \frac{y\theta_I}{1+\vartheta}\right) \hat{c}_{I,t} + \left(\varpi_R - \zeta \frac{y\theta_R}{1+\vartheta}\right) \hat{c}_{R,t} \\
& + (1-\beta_P)\, \xi_P \hat{h}_{P,t} + (1-\beta_I)\, \xi_I \hat{h}_{I,t} + (1-\beta_R)\, \xi_R \hat{h}_{R,t} \\
& - \frac{1}{2} \widetilde{\varpi}_P \hat{c}_{P,t}^2 - \frac{1}{2} \widetilde{\varpi}_I \hat{c}_{I,t}^2 - \frac{1}{2} \widetilde{\varpi}_R \hat{c}_{R,t}^2 \\
& + \vartheta \hat{y}_t \left[ \left( \frac{(1-\beta_P)\, \theta_P y}{c_P} - \zeta y \theta_P \right) \hat{c}_{P,t} + \left( \frac{(1-\beta_I)\, \theta_I y}{c_I} - \zeta y \theta_I \right) \hat{c}_{I,t} + \left( \frac{(1-\beta_R)\, \theta_R y}{c_R} - \zeta y \theta_R \right) \hat{c}_{R,t} \right] \\
& + \left( \theta_P(\varpi_I - 1 + \beta_I) + \theta_I(\varpi_P - 1 + \beta_P) - \frac{\zeta y \theta_P \theta_I}{(1+\vartheta)} \right) \hat{c}_{P,t} \hat{c}_{I,t} \\
& + \left( \theta_P(\varpi_R - 1 + \beta_R) + \theta_R(\varpi_P - 1 + \beta_P) - \frac{\zeta y \theta_P \theta_R}{(1+\vartheta)} \right) \hat{c}_{P,t} \hat{c}_{R,t} \\
& + \left( \theta_I(\varpi_R - 1 + \beta_R) + \theta_R(\varpi_I - 1 + \beta_I) - \frac{\zeta y \theta_R \theta_I}{(1+\vartheta)} \right) \hat{c}_{R,t} \hat{c}_{I,t} \\
& - \zeta y \left( \hat{y}_t + \frac{1}{2} \left( \vartheta + \frac{1}{1+\vartheta} \right) \hat{y}_t^2 \right) + t.i.p. + O\left(||\xi^3||\right) \qquad \text{(A.22)}
\end{aligned}
$$

where:

$$
\varpi_j = (1-\beta_j) \left( 1 + \frac{\theta_j y}{c_j\,(1+\vartheta)} \right) \qquad for \quad j = P, I, R
$$

$$
\zeta = \frac{(1-\beta_P)\, \theta_P}{c_P} + \frac{(1-\beta_I)\, \theta_I}{c_I} + \frac{(1-\beta_R)\, \theta_R}{c_R}
$$

$$
\widetilde{\varpi}_j = \left[ \frac{\theta_j^2 \zeta y}{1+\vartheta} + \frac{(1-\beta_j)\, \theta_j y}{c_j\,(1+\vartheta)} - \frac{2\,(1-\beta_j)\, \theta_j^2 y}{c_j\,(1+\vartheta)} \right] \qquad \text{for} \quad j = P, I, R
$$

Taking a second order approximation of the resource constraint (A.6) under zero indexation:

$$
\begin{aligned}
\hat{y}_t = & \frac{c_P}{y} \left( \hat{c}_{P,t} + \frac{1}{2} \hat{c}_{P,t}^2 \right) + \frac{c_I}{y} \left( \hat{c}_{I,t} + \frac{1}{2} \hat{c}_{I,t}^2 \right) + \frac{c_R}{y} \left( \hat{c}_{R,t} + \frac{1}{2} \hat{c}_{R,t}^2 \right) \\
& + \frac{i_P}{y} \left( \hat{i}_{P,t} + \frac{1}{2} \hat{i}_{P,t}^2 \right) + \frac{i_I}{y} \left( \hat{i}_{I,t} + \frac{1}{2} \hat{i}_{I,t}^2 \right) + \frac{i_R}{y} \left( \hat{i}_{R,t} + \frac{1}{2} \hat{i}_{R,t}^2 \right) \\
& + \frac{\kappa_p}{2} \pi_t^2 + \frac{(q_L b_L + d)}{y} \left( \hat{\Gamma}_t + \frac{1}{2} \hat{\Gamma}_t^2 \right) - \frac{1}{2} \hat{y}_t^2 + O\left(||\xi^3||\right) \qquad \text{(A.23)}
\end{aligned}
$$

Substituting the expression above in (A.22) and gathering terms, we get:

$$
\begin{aligned}
W_t = &\left(\varpi_P - \zeta \frac{y\theta_P}{1+\vartheta} - \zeta c_P\right)\hat{c}_{P,t} + \left(\varpi_I - \zeta \frac{y\theta_I}{1+\vartheta} - \zeta c_I\right)\hat{c}_{I,t} + \left(\varpi_R - \zeta \frac{y\theta_R}{1+\vartheta} - \zeta c_R\right)\hat{c}_{R,t} \\
&+ (1-\beta_P)\,\xi_P \hat{h}_{P,t} + (1-\beta_I)\,\xi_I \hat{h}_{I,t} + (1-\beta_R)\,\xi_R \hat{h}_{R,t} \\
&- \left(\widetilde{\varpi}_P + \frac{1}{2}\zeta c_P\right)\hat{c}_{P,t}^2 - \left(\widetilde{\varpi}_I + \frac{1}{2}\zeta c_I\right)\hat{c}_{I,t}^2 - \left(\widetilde{\varpi}_R + \frac{1}{2}\zeta c_R\right)\hat{c}_{R,t}^2 \\
&- \zeta i_P\left(\hat{i}_{P,t} + \frac{1}{2}\hat{i}_{P,t}^2\right) - \zeta i_I\left(\hat{i}_{I,t} + \frac{1}{2}\hat{i}_{I,t}^2\right) - \zeta i_R\left(\hat{i}_{R,t} + \frac{1}{2}\hat{i}_{R,t}^2\right) \\
&+ \vartheta \hat{y}_t\left[\left(\frac{(1-\beta_P)\,\theta_P y}{c_P} - \zeta y\theta_P\right)\hat{c}_{P,t} + \left(\frac{(1-\beta_I)\,\theta_I y}{c_I} - \zeta y\theta_I\right)\hat{c}_{I,t} + \left(\frac{(1-\beta_R)\,\theta_R y}{c_R} - \zeta y\theta_R\right)\hat{c}_{R,t}\right] \\
&+ \left(\theta_P(\varpi_I - 1 + \beta_I) + \theta_I(\varpi_P - 1 + \beta_P) - \frac{\zeta y\theta_P\theta_I}{(1+\vartheta)}\right)\hat{c}_{P,t}\hat{c}_{I,t} \\
&+ \left(\theta_P(\varpi_R - 1 + \beta_R) + \theta_R(\varpi_P - 1 + \beta_P) - \frac{\zeta y\theta_P\theta_R}{(1+\vartheta)}\right)\hat{c}_{P,t}\hat{c}_{R,t} \\
&+ \left(\theta_I(\varpi_R - 1 + \beta_R) + \theta_R(\varpi_I - 1 + \beta_I) - \frac{\zeta y\theta_R\theta_I}{(1+\vartheta)}\right)\hat{c}_{R,t}\hat{c}_{I,t} \\
&- \frac{\zeta y\vartheta^2}{2\,(1+\vartheta)}\hat{y}_t^2 - \frac{\zeta y\kappa_p}{2}\pi_t^2 - \zeta\,(q_L b_L + d)\left(\hat{\Gamma}_t + \hat{\Gamma}_t^2\right) + t.i.p. + O\left(||\xi^3||\right) \qquad \text{(A.24)}
\end{aligned}
$$

Since we have abstracted from capital, $\hat{i}_{h,t}$ comprises total investment, $\hat{i}_t$. The second order approximation of the laws of motion of owner occupied, rental housing and the law of motion of housing of impatient households reads as follows:

$$
\hat{h}_{j,t} = (1-\delta_h)\,\hat{h}_{j,t-1} + \frac{i_{hj}}{h_j}\hat{i}_{hj,t} - \frac{1}{2}(1-\delta_h)^2\,\hat{h}_{j,t-1}^2 - \frac{1}{2}\frac{i_{hj}^2}{h_j^2}\hat{i}_{hj,t}^2 - \frac{1-\delta_h}{h_j^2}(i_{hj} + 1 - \delta_h)\left(\hat{i}_{hj,t}\hat{h}_{j,t-1}\right) + \frac{1}{2}\hat{h}_{j,t}^2 \qquad \text{(A.25)}
$$

for $j = P, I, R$. Iterating (A.25) backwards, we receive the following expression:

$$
\hat{h}_{j,t} = (1-\delta_h)^{t+1}\left(\hat{h}_{P,-1} - \frac{1}{2}\hat{h}_{P,-1}^2\right) + \frac{i_h j}{h_P}\sum_{s=0}^{t}(1-\delta_h)^{t-s}\,\hat{i}_{hj,s} - \frac{1}{2}\frac{i_h j^2}{h_P^2}\sum_{s=0}^{t}(1-\delta_h)^{t-s}\,\hat{i}_{hj,s}^2 + O\left(||\xi^3||\right) \qquad \text{(A.26)}
$$

where $O\left(||\xi^3||\right)$ captures terms of order higher than two. Note that the first term in parenthesis on the RHS is independent of the policy that one chooses to apply in periods $t \geq 0$. Thus, if one takes the discounted value of these terms over all periods $t \geq 0$, one obtains:

$$
\sum_{t=0}^{\infty}\beta^t\hat{h}_{j,t} = \left(\frac{1}{1-(1-\delta_h)\,\beta}\right)\frac{i_h j}{h_P}\sum_{t=0}^{\infty}\beta^t\hat{i}_{hj,s} - \left(\frac{1}{1-(1-\delta_h)\,\beta}\right)\frac{i_h j^2}{2h_P^2}\sum_{t=0}^{\infty}\beta^t\hat{i}_{hj,s}^2 + O\left(||\xi^3||\right) \qquad \text{(A.27)}
$$

We can now substitute (A.27) in the discounted sum of household's utility. Following Woodford

(2001), the discounted sum of utility of the representative household can be approximated by:

$$\sum_{t=0}^{\infty}\beta^t U_t = \sum_{t=0}^{\infty}\beta^t W_t + t.ip. + O\left(||\xi^3||\right) \tag{A.28}$$

or

$$\begin{aligned}
\sum_{t=0}^{\infty}\beta^t U_t = \sum_{t=0}^{\infty}\beta^t \Bigg\{ & \left(\varpi_P - \zeta\frac{y\theta_P}{1+\vartheta} - \zeta c_P\right)\hat{c}_{P,t} + \left(\varpi_I - \zeta\frac{y\theta_I}{1+\vartheta} - \zeta c_I\right)\hat{c}_{I,t} + \left(\varpi_R - \zeta\frac{y\theta_R}{1+\vartheta} - \zeta c_R\right)\hat{c}_{R,t} \\
& + \left(\frac{1}{1-(1-\delta_h)\beta}\right)\Bigg[(1-\beta_P)\,\xi_P\frac{i_{hP}}{h_P}\left(\hat{i}_{hP,t} - \frac{i_{hP}}{2h_P}\hat{i}^2_{hP,t}\right) + (1-\beta_I)\,\xi_I\frac{i_{hI}}{h_I}\left(\hat{i}_{hI,t} - \frac{i_{hI}}{2h_P}\hat{i}^2_{hI,t}\right) \\
& + (1-\beta_R)\,\xi_R\frac{i_{hR}}{h_R}\left(\hat{i}_{hR,t} - \frac{i_{hR}}{2h_R}\hat{i}^2_{hR,t}\right)\Bigg] \\
& - \zeta i_{hP}\left(\hat{i}_{hP,t} + \frac{1}{2}\hat{i}^2_{hP,t}\right) - \zeta i_{hI}\left(\hat{i}_{hI,t} + \frac{1}{2}\hat{i}^2_{hI,t}\right) - \zeta i_{hr}\left(\hat{i}_{hR,t} + \frac{1}{2}\hat{i}^2_{hR,t}\right) \\
& - \frac{1}{2}\left(\widetilde{\varpi}_P + \zeta c_P\right)\hat{c}^2_{P,t} - \frac{1}{2}\left(\widetilde{\varpi}_I + \zeta c_I\right)\hat{c}^2_{I,t} - \frac{1}{2}\left(\widetilde{\varpi}_R + \zeta c_R\right)\hat{c}^2_{R,t} \\
& + \vartheta\hat{y}_t\left[\left(\frac{(1-\beta_P)\,\theta_P y}{c_P} - \zeta y\theta_P\right)\hat{c}_{P,t} + \left(\frac{(1-\beta_I)\,\theta_I y}{c_I} - \zeta y\theta_I\right)\hat{c}_{I,t} + \left(\frac{(1-\beta_R)\,\theta_R y}{c_R} - \zeta y\theta_R\right)\hat{c}_{R,t}\right] \\
& + \left(\theta_P(\varpi_I - 1 + \beta_I) + \theta_I(\varpi_P - 1 + \beta_P) - \frac{\zeta y\theta_P\theta_I}{(1+\vartheta)}\right)\hat{c}_{P,t}\hat{c}_{I,t} \\
& + \left(\theta_P(\varpi_R - 1 + \beta_R) + \theta_R(\varpi_P - 1 + \beta_P) - \frac{\zeta y\theta_P\theta_R}{(1+\vartheta)}\right)\hat{c}_{P,t}\hat{c}_{R,t} \\
& + \left(\theta_I(\varpi_R - 1 + \beta_R) + \theta_R(\varpi_I - 1 + \beta_I) - \frac{\zeta y\theta_R\theta_I}{(1+\vartheta)}\right)\hat{c}_{R,t}\hat{c}_{I,t} \\
& - \frac{\zeta y\vartheta^2}{2\,(1+\vartheta)}\hat{y}^2_t - \frac{\zeta y\kappa_p}{2}\pi^2_t - \zeta\left(q_L b_L + d\right)\left(\hat{\Gamma}_t + \frac{1}{2}\hat{\Gamma}^2_t\right)\Bigg\} + t.i.p. + O\left(||\xi^3||\right)
\end{aligned} \tag{A.29}$$

Simplifying and gathering terms leads to:

$$\begin{aligned}
\sum_{t=0}^{\infty}\beta^t U_t = \sum_{t=0}^{\infty}\beta^t \Bigg\{ & \omega_{c_P}\hat{c}_{P,t} - \frac{1}{2}\tilde{\omega}_{c_P}\hat{c}^2_{P,t} + \omega_{c_I}\hat{c}_{I,t} - \frac{1}{2}\tilde{\omega}_{c_I}\hat{c}^2_{I,t} + \omega_{c_R}\hat{c}_{R,t} - \frac{1}{2}\tilde{\omega}_{c_R}\hat{c}^2_{R,t} \\
& + \omega_{i_{hP}}\hat{i}_{hP,t} - \frac{1}{2}\tilde{\omega}_{i_{hP}}\hat{i}^2_{hP,t} + \omega_{i_{hI}}\hat{i}_{hI,t} - \frac{1}{2}\tilde{\omega}_{i_{hI}}\hat{i}^2_{hI,t} + \omega_{i_{hR}}\hat{i}_{hR,t} - \frac{1}{2}\tilde{\omega}_{i_{hR}}\hat{i}^2_{hR,t} \\
& - \frac{1}{2}\omega_y\hat{y}^2_t - \frac{1}{2}\omega_\pi\pi^2_t - \omega_{\gamma_b}\left(\Gamma_2\hat{\gamma}_{b,t} + \frac{\Gamma_2\left(\Gamma_2 - 1\right)}{2\gamma_b}\hat{\gamma}^2_{b,t}\right) \\
& - \omega_{yc_P}\hat{y}_t\hat{c}_{P,t} - \omega_{yc_I}\hat{y}_t\hat{c}_{I,t} - \omega_{yc_R}\hat{y}_t\hat{c}_{R,t} + \omega_{c_Pc_I}\hat{c}_{P,t}\hat{c}_{I,t} + \omega_{c_Pc_R}\hat{c}_{P,t}\hat{c}_{R,t} + \omega_{c_Rc_I}\hat{c}_{R,t}\hat{c}_{I,t}\Bigg\} \\
& + t.i.p. + O\left(||\xi^3||\right)
\end{aligned} \tag{A.30}$$

where:

$$\omega_{c_P} = \varpi_P - \zeta \frac{y\theta_P}{1+\vartheta} - \zeta c_P, \quad \omega_{c_I} = \varpi_I - \zeta \frac{y\theta_I}{1+\vartheta} - \zeta c_I, \quad \omega_{c_R} = \varpi_R - \zeta \frac{y\theta_R}{1+\vartheta} - \zeta c_R,$$
$$\tilde{\omega}_{c_P} = \widetilde{\varpi}_P + \zeta c_P - (1-\beta_P), \quad \tilde{\omega}_{c_I} = \widetilde{\varpi}_I + \zeta c_I - (1-\beta_I), \quad \tilde{\omega}_{c_R} = \widetilde{\varpi}_R + \zeta c_R - (1-\beta_R),$$
$$\omega_{i_{hP}} = \left(\frac{1}{1-(1-\delta_h)\beta_P}\right)(1-\beta_P)\,\xi_P \frac{i_{hP}}{h_P} - \zeta i_{hP}, \quad \omega_{i_{hI}} = \left(\frac{1}{1-(1-\delta_h)\beta_I}\right)(1-\beta_I)\,\xi_I \frac{i_{hI}}{h_I} - \zeta i_{hI}$$
$$\omega_{i_{hR}} = \left(\frac{1}{1-(1-\delta_h)\beta_R}\right)(1-\beta_R)\,\xi_R \frac{i_{hR}}{h_R} - \zeta i_{hR}$$
$$\tilde{\omega}_{i_{hP}} = \left(\frac{1}{1-(1-\delta_h)\beta_P}\right)(1-\beta_P)\,\xi_P \frac{i_{hP}^2}{h_P^2} + \zeta i_{hP}, \quad \tilde{\omega}_{i_{hI}} = \left(\frac{1}{1-(1-\delta_h)\beta_I}\right)(1-\beta_I)\,\xi_I \frac{i_{hI}^2}{h_I^2} + \zeta i_{hI}$$
$$\tilde{\omega}_{i_{hR}} = \left(\frac{1}{1-(1-\delta_h)\beta_R}\right)(1-\beta_R)\,\xi_R \frac{i_{hR}^2}{h_R^2} + \zeta i_{hR}$$
$$\omega_y = \frac{\zeta y \vartheta^2}{(1+\vartheta)}, \quad \omega_\pi = \zeta y \kappa_p, \quad \omega_{\gamma_b} = \frac{\zeta\,(q_L b_L + d)}{\gamma_b}$$
$$\omega_{yc_P} = \vartheta\left(\zeta y\theta_P - \frac{(1-\beta_P)\,\theta_P y}{c_P}\right), \quad \omega_{yc_I} = \vartheta\left(\zeta y\theta_I - \frac{(1-\beta_I)\,\theta_I y}{c_I}\right), \quad \omega_{yc_R} = \vartheta\left(\zeta y\theta_R - \frac{(1-\beta_R)\,\theta_R y}{c_R}\right)$$
$$\omega_{c_P c_I} = \theta_P(\varpi_I - 1 + \beta_I) + \theta_I(\varpi_P - 1 + \beta_P) - \frac{\zeta y \theta_P \theta_I}{(1+\vartheta)}, \quad \omega_{c_P c_R} = \theta_P(\varpi_R - 1 + \beta_R) + \theta_R(\varpi_P - 1 + \beta_P) - \frac{\zeta y \theta_P \theta_R}{(1+\vartheta)}$$
$$\omega_{c_R c_I} = \theta_I(\varpi_R - 1 + \beta_R) + \theta_R(\varpi_I - 1 + \beta_I) - \frac{\zeta y \theta_R \theta_I}{(1+\vartheta)} \tag{A.31}$$

Note that in expression (A.30) we have used the second-order approximation of the transaction cost function, $\Gamma_t$. The latter reads as follows:

$$\hat{\Gamma}_t + \frac{1}{2}\hat{\Gamma}_t^2 = \frac{\Gamma_2}{\gamma_b}\hat{\gamma}_{b,t} + \frac{\Gamma_2\,(\Gamma_2 - 1)}{2\gamma_b^2}\hat{\gamma}_{b,t}^2 \tag{A.32}$$

where, $\hat{\gamma}_{b,t}$, is the linearized version of $\gamma_{b,t}$. At this stage, it is convenient to linearize the equilibrium conditions that serve as constraints to the maximization problem of the central bank under commitment. Note that below, we consider the version of the model without capital. Taking a first-order Taylor approximation of the optimal pricing equation, we receive the Phillips curve:

$$\pi_t = \beta_P E_t \pi_{t+1} + \frac{\eta_y - 1}{\kappa_p}\left(\hat{\Omega}_t + \hat{\Theta}_{p,t}\right) \tag{A.33}$$

where $\hat{\Theta}_{p,t}$:

$$\hat{\Theta}_{p,t} = -\frac{1}{\eta_y - 1}\hat{\eta}_{y,t} \tag{A.34}$$

Log-linearizing the equilibrium conditions from the optimization problem of patient households, we receive:

(1) $h_{P,t}(i)$ :

$$\hat{q}_{h,t} + \hat{\lambda}_{P,t} = -\frac{\xi_h}{q_h \lambda_P}\hat{h}_{P,t} + (1-\delta_h)\,\beta_P E_t\left(\hat{\lambda}_{P,t+1} + E_t \hat{q}_{h,t+1}\right) \tag{A.35}$$

where $\hat{\lambda}_{P,t}$:

$$\hat{\lambda}_{P,t} = -\hat{c}_{P,t} \tag{A.36}$$

(2) $h_{R,t}(i)$ :

$$\hat{q}_{h,t} + \left(1 - \frac{r_h}{q_h}\right)\hat{\lambda}_{P,t} = \frac{r_h}{q_h}\hat{r}_{h,t} + (1-\delta_h)\beta_P E_t\left(\hat{\lambda}_{P,t+1} + E_t\hat{q}_{h,t+1}\right) \tag{A.37}$$

(3) $k_{P,t}(i)$ :

$$\hat{q}_{k,t} + \hat{\lambda}_{P,t} = E_t\left[\left(1 - \delta_k + \frac{r_k}{q_k}\right)\hat{\lambda}_{P,t+1} + \frac{r_k}{q_k}\hat{r}_{P,t+1} + (1-\delta_k)\hat{q}_{h,t+1}\right] \tag{A.38}$$

(4) $n_{P,t}(i)$ :

$$\vartheta\hat{n}_{P,t} = \hat{\lambda}_{P,t} + \hat{w}_{P,t} \tag{A.39}$$

(5) $b_{S,t}(i)$ :

$$\hat{\lambda}_{P,t} = E_t\hat{\lambda}_{P,t+1} - E_t\pi_{t+1} - \hat{q}_{S,t} \tag{A.40}$$

(6) $b_{L,t}(i)$ :

$$\hat{\Gamma}_t + \hat{q}_{L,t} = E_t\hat{\lambda}_{P,t+1} - \hat{\lambda}_{P,t} - E_t\pi_{t+1} + \frac{\kappa q_L}{1+\kappa q_L}E_t\hat{q}_{L,t+1} \tag{A.41}$$

(7) $l_t(i)$ :

$$\hat{\Gamma}_t = \Omega_{dP}\hat{\Omega}_{dP,t} + R^l\Omega_{rP}\left(\hat{\Omega}_{rP,t} + \hat{R}^l_t\right) \tag{A.42}$$

(8) $d_t$ :

$$\frac{1}{\Omega_{dP} + \Omega_{rP}R^d}\left(\Omega_{dP}\hat{\Omega}_{dP,t} + \Omega_{rP}R^d\hat{\Omega}_{rP,t} + \Omega_{rP}R^d\hat{R}^d_t\right) = E_t\hat{\lambda}_{P,t+1} - \hat{\lambda}_{P,t} - E_t\pi_{t+1}$$
$$+ \frac{1-\kappa_d}{R^d + \kappa_d - (1-\kappa_d)\Gamma + (1-\kappa_d)(\Omega_{dP} + \Omega_{rP}R^d)}\left[\left(\frac{1}{1-\kappa_d} + \Omega_{rP}\right)R^d\hat{R}^d_t - E_t\hat{\Gamma}_{t+1} + \Omega_{dP}E_t\hat{\Omega}_{dP,t+1} + \Omega_{rP}R^dE_t\hat{\Omega}_{rP,t+1}\right] \tag{A.43}$$

(9) $R^d_t$ :

$$\hat{\Omega}_{rP,t} = E_t\hat{\lambda}_{P,t+1} - \hat{\lambda}_{P,t} - E_t\pi_{t+1} + \frac{(1-\kappa_d)\Omega_{rP}}{1+(1-\kappa_d)\Omega_{rP}}E_t\hat{\Omega}_{rP,t+1} \tag{A.44}$$

Log-linearizing the FOCs of the impatient households' maximization, we receive:

(10) $h_{I,t}(i)$ :

$$-\frac{\phi\mu}{1-\phi\mu}\hat{\mu}_t + \hat{q}_{h,t} =$$
$$\frac{\xi_h}{(1-\phi\mu)q_h\lambda_I h_I}\left(\hat{\lambda}_{I,t} - \hat{h}_{I,t}\right) + \beta_I(1-\delta_h)\left(E_t\hat{\lambda}_{I,t+1} - \hat{\lambda}_{I,t} + E_t\hat{q}_{h,t+1} - \frac{\phi\mu}{1-\phi\mu}E_t\hat{\mu}_{t+1}\right) \tag{A.45}$$

where $\hat{\lambda}_{I,t}$:

$$\hat{\lambda}_{I,t} = -\hat{c}_{I,t} \tag{A.46}$$

(11) $n_{I,t}(i)$ :

$$\vartheta\hat{n}_{I,t} = \hat{\lambda}_{I,t} + \hat{w}_{I,t} \tag{A.47}$$

(12) $l_t(i)$ :

$$-\mu\hat{\mu}_t = \Omega_{dI}\hat{\Omega}_{dI,t} + R^l\Omega_{rI}\left(\hat{\Omega}_{rI,t} + \hat{R}_t^l\right) \tag{A.48}$$

(13) $d_t$ :

$$\frac{1}{\Omega_{dI} + \Omega_{rI}R^d}\left(\Omega_{dI}\hat{\Omega}_{dI,t} + \Omega_{rI}R^d\hat{\Omega}_{rI,t} + \Omega_{rI}R^d\hat{R}_t^d\right) =$$
$$E_t\hat{\lambda}_{I,t+1} - \hat{\lambda}_{I,t} - E_t\pi_{t+1} + \frac{1-\kappa_d}{R^d + \kappa_d + (1-\kappa_d)(\Omega_{dI} + \Omega_{rI}R^d)}\left[\left(\frac{1}{1-\kappa_d} + \Omega_{rI}\right)R^d\hat{R}_t^d + \Omega_{dI}E_t\hat{\Omega}_{dI,t+1} + \Omega_{rI}R^dE_t\hat{\Omega}_{rI,t+1}\right] \tag{A.49}$$

(14) $R_t^d$ :

$$\hat{\Omega}_{rI,t} = E_t\hat{\lambda}_{I,t+1} - \hat{\lambda}_{I,t} - E_t\pi_{t+1} + \frac{(1-\kappa_d)\Omega_{rI}}{1 + (1-\kappa_d)\Omega_{rI}}E_t\hat{\Omega}_{rI,t+1} \tag{A.50}$$

(15) Borrowing constraint

$$\hat{l}_t = \phi q_h i_{hI}\left(\hat{q}_{h,t} + \hat{i}_{hI,t}\right) \tag{A.51}$$

Log-linearizing the FOCs of renter households' maximization, we receive:

(16) $h_{R,t}^d$ :

$$\hat{r}_{h,t} = -\hat{\lambda}_{R,t} - \hat{h}_{R,t} \tag{A.52}$$

where $\hat{\lambda}_{R,t}$:

$$\hat{\lambda}_{R,t} = -\hat{c}_{R,t} \tag{A.53}$$

(17) $n_{R,t}(i)$ :

$$\vartheta\hat{n}_{R,t} = \hat{\lambda}_{R,t} + \hat{w}_{R,t} \tag{A.54}$$

Log-linearizing the resource constraint (excluding capital), we get:

$$\hat{y}_t = \frac{c_P}{y}\hat{c}_{P,t} + \frac{c_I}{y}\hat{c}_{I,t} + \frac{c_R}{y}\hat{c}_{R,t} + \frac{i_h}{y}\hat{i}_{h,t} \tag{A.55}$$

where in the expression above the terms associated with Rotemberg price adjustment cost have dropped out, since we assume a zero steady-state inflation, $\Pi = 1$. As mentioned in the main body of the text, for the sake of simplicity and exposition, in the optimal monetary policy analysis we abstract from capital accumulation.

## A.3 Proof of Proposition 2

Let us work with the case of no heterogeneity first, namely $\theta_P = 1$ and $\theta_I = \theta_R = 0$. Assuming these values in the coefficients of the welfare criterion summarized in (A.31), it is easy to show that the welfare criterion collapses to:

$$\sum_{t=0}^{\infty}\beta^t U_t = \sum_{t=0}^{\infty}\beta^t \Bigg\{ \omega_{c_P}\hat{c}_{P,t} - \frac{1}{2}\tilde{\omega}_{c_P}\hat{c}_{P,t}^2 + \omega_{i_{hP}}\hat{i}_{hP,t} - \frac{1}{2}\tilde{\omega}_{i_{hP}}\hat{i}_{hP,t}^2$$

$$- \frac{1}{2}\omega_y\hat{y}_t^2 - \frac{1}{2}\omega_\pi\pi_t^2 - \omega_{\gamma_b}\left(\Gamma_2\hat{\gamma}_{b,t} + \frac{\Gamma_2(\Gamma_2-1)}{2\gamma_b}\hat{\gamma}_{b,t}^2\right)\Bigg\} + t.i.p. + O\left(||\xi^3||\right) \tag{A.56}$$

where now the weights collapse to:

$$\omega_{c_P} = \frac{(1-\beta_P)y}{c_P(1+\vartheta)},\ \tilde{\omega}_{c_P} = 1-\beta_P$$

$$\omega_{i_{hR}} = \left(\frac{1}{1-(1-\delta_h)\beta_R}\right)(1-\beta_R)\,\xi_R\frac{i_{hR}}{h_R} - \frac{1-\beta_P}{c_P}i_{hR},\ \tilde{\omega}_{i_{hP}} = \left(\frac{1}{1-(1-\delta_h)\beta_P}\right)(1-\beta_P)\,\xi_P\frac{i_{hP}^2}{h_P^2}$$

$$\omega_y = \frac{1-\beta_P}{c_P}y\vartheta^2,\ \omega_\pi = \frac{1-\beta_P}{c_P}y\kappa_p$$

Setting $\theta_P = 1$ and $\theta_I = \theta_R = 0$, it is also easy to show that the weights on the covariance terms in (A.30) are zero. This implies that the central bank is not faced with additional trade-offs once heterogeneity is turned off. Notice also the linear terms in the welfare criterion (A.56). In this case, they arise because of the absence of a subsidy that would render the steady state efficient after removing the distortion due to monopolistic competition.

Updating and adjusting the equilibrium conditions (A.33) - (A.54) to account for $\theta_P = 1$ and $\theta_I = \theta_R = 0$ and taking the first order conditions of the central bank's maximization problem with respect to $\pi_t$ and $\hat{y}_t$ under commitment, we receive:

$$-\omega_\pi\pi_t - \lambda_{\pi,t} + \lambda_{\pi,t-1} = 0 \tag{A.57}$$

$$-\omega_y\hat{y}_t + \frac{(\eta_y-1)\vartheta}{\kappa_p}\lambda_{\pi,t} = 0 \tag{A.58}$$

where $\lambda_{\pi,t}$ is the Lagrange multiplier on the Phillips curve in the central bank's maximization problem. Substituting out for $\lambda_{\pi,t}$ and $\lambda_{\pi,t-1}$ in (A.57) using (A.58), we arrive at the standard targeting rule under commitment:

$$\omega_y\left(\hat{y}_t - \hat{y}_{t-1}\right) = -\frac{(\eta_y-1)\,\omega_\pi\vartheta}{\kappa_p}\pi_t$$

which is the standard trade-off under optimal monetary policy with commitment. Returning now to the case of heterogeneity and maximizing (A.30) subject to (A.33) - (A.54) with respect to $\pi, t$ and $y_t$, under commitment, we receive:

$$-\omega_\pi\pi_t - \lambda_{\pi,t} + \lambda_{\pi,t-1} = 0 \tag{A.59}$$

$$-\omega_y\hat{y}_t - \omega_{yc_P}\hat{c}_{P,t} - \omega_{yc_I}\hat{c}_{I,t} - \omega_{yc_R}\hat{c}_{R,t} + \frac{(\eta_y-1)\vartheta}{\kappa_p}\lambda_{\pi,t} = 0 \tag{A.60}$$

Substituting out for $\hat{y}_t$ using the log-linearized resource constraint (A.55) and gathering terms, we can write the FOC (A.60) as:

$$\left(\frac{\omega_y c_P}{y}+\omega_{yc_P}\right)\hat{c}_{P,t}+\left(\frac{\omega_y c_I}{y}+\omega_{yc_I}\right)\hat{c}_{I,t}+\left(\frac{\omega_y c_R}{y}+\omega_{yc_R}\right)\hat{c}_{R,t}+\frac{\omega_y i_h}{y}\hat{i}_{h,t}+\frac{(\eta_y-1)\vartheta}{\kappa_p}\lambda_{\pi,t}=0 \quad \text{(A.61)}$$

Solving for $\lambda_{\pi,t}$ in (A.61) and substituting in (A.59), we receive the optimal targeting criterion of the central bank under heterogeneity:

$$\left(\frac{\omega_y c_P}{y}+\omega_{yc_P}\right)(\hat{c}_{P,t}-\hat{c}_{P,t-1})+\left(\frac{\omega_y c_I}{y}+\omega_{yc_I}\right)(\hat{c}_{I,t}-\hat{c}_{I,t-1})+\left(\frac{\omega_y c_R}{y}+\omega_{yc_R}\right)(\hat{c}_{R,t}-\hat{c}_{R,t-1})+\frac{\omega_y i_h}{y}\left(\hat{i}_{h,t}-\hat{i}_{h,t-1}\right)$$
$$=-\frac{(\eta_y-1)\,\omega_\pi\vartheta}{\kappa_p}\pi_t$$

## A.4 Proof of Proposition 3

We derive the optimal relative supply for long-term bonds, $\hat{\gamma}_{b,t}$, by maximizing the welfare criterion (A.30) under commitment. To keep tractability and focus on the intuition of the result, we restrict ourselves to the case of full depreciation of housing ($\delta_h = 1$) and full mortgage debt repayment within one period ($\kappa_d = 1$). Doing so, we end up to the following optimal targeting criterion for the relative supply for long-term bonds:

$$\omega_{\gamma_b}(1-\Gamma_2)\,\hat{\gamma}_{b,t}=$$
$$\omega_{\gamma_b}\gamma_b-\frac{\omega_{c_P}-\omega_{c_I}+\omega_{c_R}}{R^l\Omega_{rP}}+\frac{\tilde{\omega}_{c_P}+\omega_{c_Pc_I}-\omega_{c_Pc_R}}{R^l\Omega_{rP}}\hat{c}_{P,t}+\frac{\omega_{c_Pc_I}+\omega_{c_Rc_I}-\tilde{\omega}_{c_I}}{R^l\Omega_{rP}}\hat{c}_{I,t}+\frac{\tilde{\omega}_{c_R}+\omega_{c_Rc_I}-\omega_{c_Pc_R}}{R^l\Omega_{rP}}\hat{c}_{R,t}$$
$$-\frac{(\eta_y-1)\,\omega_\pi\,(\theta_R+\theta_I-\theta_P)}{\kappa_p R^l\Omega_{rP}}\pi_t+\frac{\omega_{yc_P}+\omega_{yc_R}-\omega_{yc_I}}{R^l\Omega_{rP}}\hat{y}_t \quad \text{(A.62)}$$

where $\gamma_b$ is the steady state relative long-term to short-term bond ratio. In the targeting criterion above it becomes clear that in deciding about the optimal relative supply for long-term bonds, the central bank takes into account not only inflation, $\pi_t$, but also output, $\hat{y}_t$ as well as the consumption of each group. When instead we turn heterogeneity off (i.e. $\theta_P = 1$ and $\theta_I = \theta_R = 0$), the covariance terms drop out, meaning that $\omega_{c_Pc_I} = \omega_{c_Pc_R} = \omega_{c_Rc_I} = 0$ and $\omega_{yc_I} = \omega_{yc_R} = \omega_{yc_P} = 0$, which we receive after using the definition of these parameters in section A.2 above and the homogeneity assumption that $\theta_P = 1$ and $\theta_I = \theta_R = 0$. In this case, the optimal targeting criterion collapses thus to the following:

$$\omega_{\gamma_b}(1-\Gamma_2)\,\hat{\gamma}_{b,t}=\omega_{\gamma_b}\gamma_b+\frac{(\eta_y-1)\,\omega_\pi}{\kappa_p R^l\Omega_{rP}}\pi_t \quad \text{(A.63)}$$

Clearly, from the expression above, and given that $\eta_y > 1$, under homogeneity the optimal relative supply of long-term bonds depends solely on the inflation rate.

## A.5 Proof of Proposition 4

Plugging in the optimal targeting criterion for the relative long-term bond supply, (61), the log-linearized Phillips curve, (A.33), and gathering terms, we get that the composite coefficient on output, $\hat{y}_t$, in the resulting targeting criterion for the optimal relative supply of long-term bonds is:[19]

[19]Note that for simplicity we have set the inverse of the Firsch elasticity of labor supply, $\vartheta$, to 1, as we also do in our baseline calibration presented at table 1 in the main text.

$$\frac{\zeta y}{R^l \Omega_{rP}}\left[-(\varsigma+1)(\theta_I-\theta_P)+\theta_R(1-\varsigma)+\frac{2(1-\beta_I)\theta_I}{c_I\zeta}\right] \tag{A.64}$$

where $\varsigma = \left(\frac{\eta_y-1}{\kappa_p}\right)^2$. Note that since $\eta_y > 1$ and $\kappa_p > \eta_y$, as $\kappa_p$ is the Rotemberg price adjustment cost parameter (and hence high enough), it follows that $\varsigma \in (0,1)$. If the above composite coefficient is negative then the rise in output due to the positive demand shock will require a decline in the optimal relative supply of long-term bonds. If the composite coefficient is positive then, the optimal relative supply of long-term bonds must rise following a positive demand shock. We can now derive the share of impatient households that makes the composite coefficient on output negative. Considering thus the case where:

$$\left[-(\varsigma+1)(\theta_I-\theta_P)+\theta_R(1-\varsigma)+\frac{2(1-\beta_I)\theta_I}{c_I\zeta}\right] < 0$$

and solving for $\theta_I$, we receive:

$$\theta_I > \left(\frac{1}{1+\varsigma-\frac{2(1-\beta_I)}{c_I\zeta}}\right)\left(\theta_R(1-\varsigma)+\theta_P\left(1+\varsigma\right)\right)$$

which guarantees that the optimal relative supply of long-term bonds declines after a positive demand shock. Otherwise, if:

$$\theta_I \leq \left(\frac{1}{1+\varsigma-\frac{2(1-\beta_I)}{c_I\zeta}}\right)\left(\theta_R(1-\varsigma)+\theta_P\left(1+\varsigma\right)\right)$$

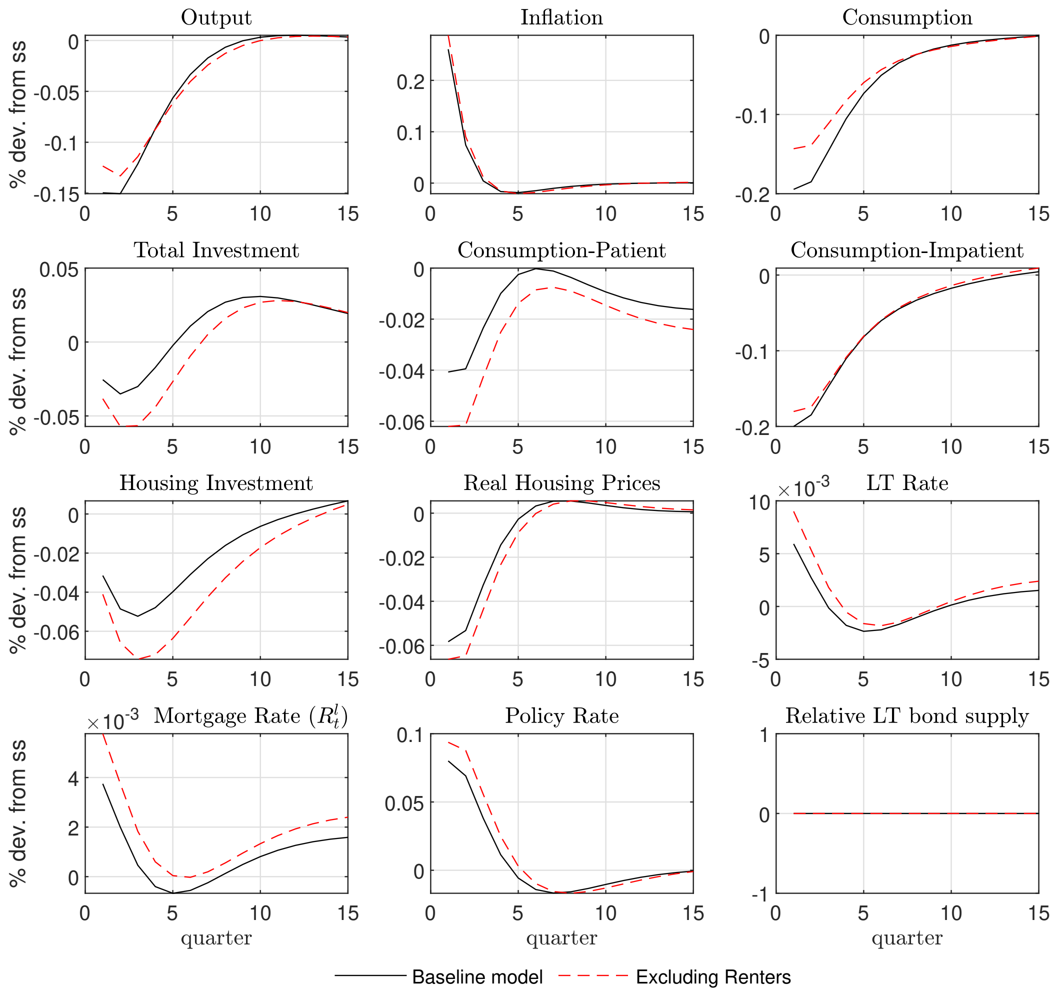
the optimal relative supply of long-term bonds must rise following a positive demand shock.

# B The model without renters

In this section, we look at the importance of renters for the dynamics of the economy. For this reason, we set the share of renters in the production function (38) to zero, $\theta_R = 0$. At the same time, we keep the ratio of impatient to patient households the same as in the benchmark calibration. That is, we set $\theta_P = 0.3846$ and $\theta_I = 0.6154$. We consider all three cases as in the main text, namely, the case (Benchmark) where the central bank sets the policy rate following Taylor rule (51) and relative long-term bond supply, $\hat{\gamma}_{b,t}$, is exogenous as in (52), the case where the central bank sets optimally the policy rate while the relative long-term bond supply is exogenous (i.e. one instrument policy), and the case where the central bank sets optimally both the policy rate and the relative supply of long-term bonds. To save space, we restrict our focus to the supply shock and compare the results to those from the baseline calibration with three types of households, as presented in Figure 2. The results are displayed in Figures A.5, A.6 and A.7.

Looking at the responses in the benchmark case (Figure A.5) a few observations stand out. Excluding renters (red-dashed lines) leads to a slightly milder recession while inflation jumps more on impact. The milder recession is, obviously, partly due to the weaker decline in total consumption since the renters are now excluded. Other than that, impatient households (borrowers) are now more credit-constrained because real housing prices decline more when renters are excluded. In addition to being more credit-constrained, they also face higher borrowing costs that dampen the demand for housing and housing investment more compared to the baseline model. When renters are present, the downward pressures on private consumption following

Figure A.5. Model without Renters vs Baseline model with three-types: Monetary Policy conducted via a Taylor Rule with exogenous relative Long-Term Bond supply



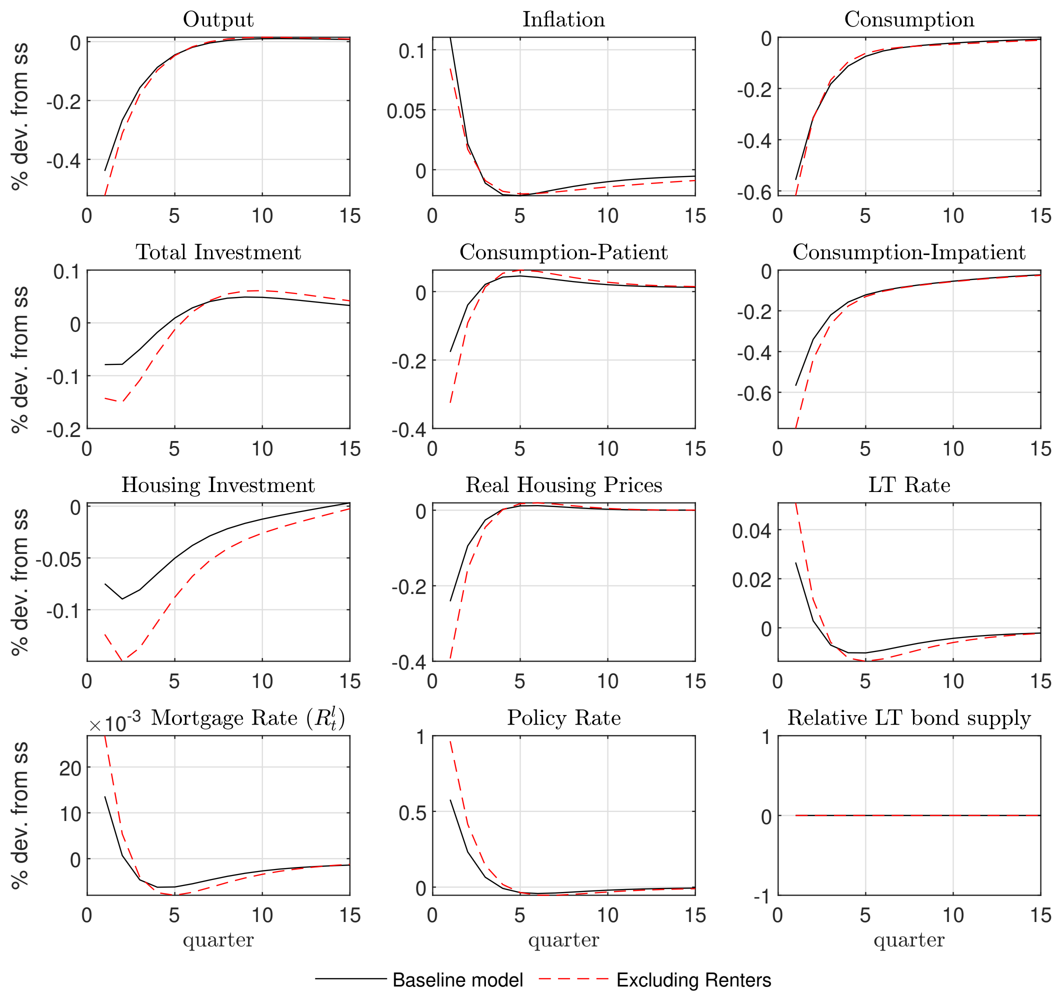

Notes: Impulse response functions following a supply shock. The solid black lines display the responses from the baseline model (black solid lines from Figure 2) with three agents, where the central bank follows a Taylor rule and the relative supply of long-term bonds, $\hat{\gamma}_{b,t}$, follows an $AR(1)$. The red-dashed lines are the corresponding impulse responses from the version of the model without renters.

the supply shock are stronger which results in a milder jump in inflation and thereby in a milder policy rate hike. Subsequently, given equation (54), this implies that in the model with three-types, the long-term rate and hence the mortgage rate rate increase less. Looking at the consumption of patient and impatient households, the latter type cuts more its consumption slightly less on impact compared to the baseline model, while patient households' consumption fluctuates at constantly lower levels in the absence of renters and this is due to the lower interest income from housing investment.

Turning now to Figure A.6 displaying the comparison under optimal policy with one instrument (i.e. only the policy rate set optimally), the key message is that in the absence of renters the central bank needs to raise the policy rate optimally more compared to the baseline model. This is because the absence of renters tends to make a given supply shock more inflationary and as such the central bank counteracts it by raising the policy rate more on impact. As a result, in the absence of renters, the long-term and mortgage rates rise higher than in the presence of renters. This explains why housing investment and real housing prices decline more than in the baseline model. Impatient households become thus more credit-constrained when renters are absent adding to the downward pressures in housing and thus total investment. The induced larger contraction of economic activity on impact reduces real wages more than in the baseline model, entailing a deeper dive in the consumption of impatient households in the first quarters after the shock. The inclusion of renters has thus visible implications for optimal monetary policy. Their presence implies a less contractionary monetary policy stance, so that the drop in housing investment is milder, while at the same time making impatient households less credit-constrained due to the milder drop in real housing prices.

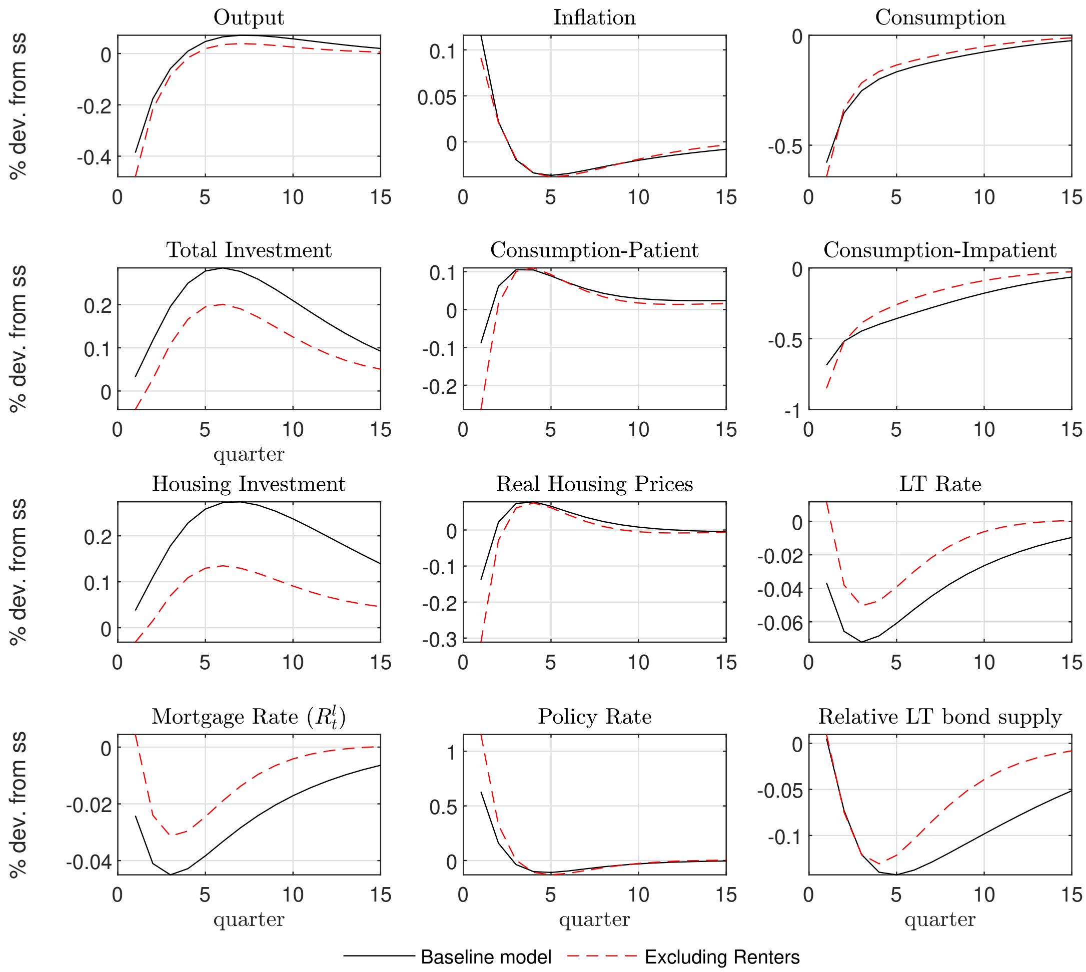
Finally, looking at the case of optimal policy with two instruments (i.e. policy rate and relative long-term bond supply), optimal monetary policy in the baseline model with three types implies a stronger flattening of the yield curve in the quarters following the shock. In fact, compared to the case where renters are absent, the central bank has to lift the policy rate substantially less (about 50bps less) and lower the long-term and the mortgage rate more by optimally decreasing the relative supply of long-term bonds more. Clearly, optimal policy in the presence of renters results to a widening of consumption inequality since the consumption of patient households drops substantially less on impact in the baseline model while the consumption of impatient stays persistently below its level in the absence of renters in the medium-run. The induced stronger flattening of the yield curve in the baseline model causes a boom in housing investment even on impact and to a quick rise in real housing prices. These imply higher wealth for patient households which explains why their consumption falls less. These effects are considerably weaker when renters are absent. The differences in the total consumption responses between the two models are marginal as the benefits from the improved response of patients' consumption are offset by the worsening of the response of impatients' consumption. When it comes to output, the amplified rise in housing and total investment in the baseline model due to the lower interest rates, compared to when renters are absent, seem to explain the milder contraction on impact and the subsequent amplified overshooting in the medium-term.

Figure A.6. Model without Renters vs Baseline model with three-types: Optimal Policy with One Instrument




Notes: Impulse response functions following a supply shock. The solid black lines display the responses from the baseline model (black solid lines from Figure 2) with three agents, where optimal monetary policy is conducted by setting only the policy rate, while the relative supply of long-term bonds, $\hat{\gamma}_{b,t}$, follows an $AR(1)$. The red-dashed lines are the corresponding impulse responses from the version of the model without renters.

FIGURE A.7. Model without Renters vs Baseline model with three-types: Optimal Policy under Two Instruments




Notes: Impulse response functions following a supply shock. The solid black lines display the responses from the baseline model (black solid lines from Figure 2) with three agents, where optimal monetary policy is conducted by setting optimally both the policy rate and the relative supply of long-term bonds, $\hat{\gamma}_{b,t}$. The red-dashed lines are the corresponding impulse responses from the version of the model without renters.

# C The steady state

## C.1 List of steady-state conditions

$$\frac{1}{c_P} = \lambda_P$$

$$1 = \xi_h \frac{c_P/y}{h_P/y} + \beta_P \left(1 - \delta_h\right)$$

$$1 = r_h + \beta_P \left(1 - \delta_h\right)$$

$$1 = \beta_P \left(1 - \delta_k + r_k\right)$$

$$n_P^{1+\vartheta} = \frac{w_P n_P/y}{c_P/y}$$

$$\frac{1}{c_I} = \lambda_I$$
$$1 - \phi\mu = \xi_h \frac{c_I/y}{h_I/y} + \beta_I \left(1 - \delta_h\right) \left(1 - \phi\mu\right)$$
$$n_I^{1+\vartheta} = \frac{w_I l_I/y}{c_I/y}$$

$$\frac{1}{c_R} = \lambda_R$$
$$r_h = \xi_h \frac{c_R/y}{h_R/y}$$
$$n_R^{1+\vartheta} = \frac{w_R l_R/y}{c_R/y}$$

$$q_S = \frac{\beta_P}{\pi}$$
$$\left(1 + \Gamma\right) q_L = \frac{\beta_P}{\pi} \left(1 + \kappa q_L\right)$$
$$1 + R = \frac{1}{q_S}$$
$$1 + R_L = \frac{1}{q_L} + \kappa$$
$$1 + \Gamma = \Gamma_1 \left(\frac{q_L b_L/y}{q_S b_S/y}\right)^{\Gamma_2}$$

$$\frac{i_k}{y} = \delta_k \frac{k}{y}$$
$$\frac{i_{hP}}{y} = \delta_h \frac{h_P}{y}$$
$$\frac{i_{hI}}{y} = \delta_h \frac{h_I}{y}$$
$$\frac{i_{hR}}{y} = \delta_h \frac{h_R}{y}$$

$$\left(1 - \frac{1 - \kappa_d}{\pi}\right) \frac{d}{y} = \frac{l}{y}$$
$$\frac{l}{y} = \phi \frac{i_{hI}}{y}$$

$$R^d = R^l$$
$$1 + \Gamma = \Omega_{dP} + \Omega_{rP} R^l$$
$$\Omega_{dP} + \Omega_{rP} R^d = \beta_P \frac{R^d + \kappa_d - (1-\kappa_d)\,\Gamma + (1-\kappa_d)\left(\Omega_{dP} + \Omega_{rP} R_t^d\right)}{\pi_{t+1}}$$
$$\Omega_{rP} = \beta_P \frac{1 + (1-\kappa_d)\,\Omega_{rP}}{\pi}$$

$$1 - \mu = \Omega_{dI} + \Omega_{rI} R^l$$
$$\Omega_{dI} + \Omega_{rI} R^d = \beta_I \frac{R^d + \kappa_d + (1-\kappa_d)\left(\Omega_{dI} + \Omega_{rI} R^d\right)}{\pi}$$
$$\Omega_{rI} = \beta_I \frac{1 + (1-\kappa_d)\,\Omega_{rI}}{\pi}$$

$$\frac{c_I}{y} + \frac{i_{hI}}{y} + \frac{R^d + \kappa_d}{\pi}\frac{d}{y} = \frac{w_I n_I}{y} + \frac{l}{y}$$
$$\frac{c_R}{y} + r_h \frac{h_R}{y} = \frac{w_R n_R}{y}$$

$$q_k = 1$$
$$q_h = 1$$

$$\alpha = r_k \frac{k}{y}$$
$$(1-\alpha)\,\theta_P = \frac{w_P n_P}{y}$$
$$(1-\alpha)\,\theta_I = \frac{w_I n_I}{y}$$
$$(1-\alpha)\,(1-\theta_P-\theta_I) = \frac{w_R n_R}{y}$$
$$u = 1 \text{ and } \kappa_u = \frac{\alpha}{k/y}$$
$$\Omega = \frac{1}{\Theta_p}$$
$$y = \frac{1}{\Theta_p} k^\alpha \left(n_P^{\theta_P} n_I^{\theta_I} n_R^{\theta_R}\right)^{1-\alpha}$$
$$\frac{c}{y} + \frac{i_k}{y} + \frac{i_h}{y} + \frac{g}{y} = 1$$

$$\frac{g}{y}+\left(\frac{1+R}{\pi}-1\right)\frac{q_S b_S}{y}+\left(\frac{1+R_L}{\pi}-1\right)\frac{q_L b_L}{y}=\frac{tax}{y}$$
$$\frac{q_L b_L}{q_S b_S}=\gamma_b$$
$$\frac{tax}{y}=\Xi$$

$$\frac{h}{y}=\frac{h_P}{y}+\frac{h_I}{y}+\frac{h_R}{y}$$
$$\frac{c}{y}=\frac{c_P}{y}+\frac{c_I}{y}+\frac{c_R}{y}$$
$$\frac{i}{y}=\frac{i_k}{y}+\frac{i_h}{y}$$
$$\frac{i_h}{y}=\frac{i_{hP}}{y}+\frac{i_{hI}}{y}+\frac{i_{hR}}{y}$$

## C.2 Solution algorithm to find the steady-state

$$q_k=q_h=1 \tag{A.65}$$

$$q_S=\frac{\beta_P}{\pi} \tag{A.66}$$

$$\begin{aligned}1+R &= \frac{1}{q_S}\\ R &= \frac{1}{q_S}-1\end{aligned} \tag{A.67}$$

$$\begin{aligned}\frac{q_L b_L}{q_S b_S} &= \gamma_b \text{ and } 1+\Gamma=\Gamma_1\left(\frac{q_L b_L}{q_S b_S}\right)^{\Gamma_2}\\ \Gamma &= \Gamma_1\gamma_b^{\Gamma_2}-1\end{aligned} \tag{A.68}$$

$$\begin{aligned}(1+\Gamma)\,q_L &= \frac{\beta_P}{\pi}\left(1+\kappa q_L\right)\\ q_L &= \frac{1}{(1+\Gamma)\frac{\pi}{\beta_P}-\kappa}\end{aligned} \tag{A.69}$$

$$\begin{aligned} 1 + R_L &= \frac{1}{q_L} + \kappa \\ R_L &= \frac{1}{q_L} + \kappa - 1 \\ \text{Note that } \frac{1 + R_L}{1 + R} &= 1 + \Gamma \end{aligned} \tag{A.70}$$

$$\begin{aligned} \Omega_{rP} &= \beta_P \frac{1 + (1 - \kappa_d)\,\Omega_{rP}}{\pi} \\ \Omega_{rP} &= \frac{1}{\frac{\pi}{\beta_P} - (1 - \kappa_d)} \end{aligned} \tag{A.71}$$

$$\begin{aligned} &R^d = R^l \text{ and } 1 + \Gamma = \Omega_{dP} + \Omega_{rP} R^l \\ &\text{and } \Omega_{dP} + \Omega_{rP} R^d = \beta_P \frac{R^d + \kappa_d - (1 - \kappa_d)\,\Gamma + (1 - \kappa_d)\left(\Omega_{dP} + \Omega_{rP} R_t^d\right)}{\pi_{t+1}} \\ &1 + \Gamma = \beta_P \frac{R^d + 1}{\pi} \\ &R^d = (1 + \Gamma)\,\frac{\pi}{\beta_P} - 1 \\ &\text{Note that } \frac{1 + R^d}{1 + R} = 1 + \Gamma = \frac{1 + R_L}{1 + R} \end{aligned} \tag{A.72}$$

$$R^l = R^d \tag{A.73}$$

$$\begin{aligned} 1 + \Gamma &= \Omega_{dP} + \Omega_{rP} R^l \\ \Omega_{dP} &= 1 + \Gamma - \Omega_{rP} R^l \end{aligned} \tag{A.74}$$

$$\begin{aligned} \Omega_{rI} &= \beta_I \frac{1 + (1 - \kappa_d)\,\Omega_{rI}}{\pi} \\ \Omega_{rI} &= \frac{1}{\frac{\pi}{\beta_I} - (1 - \kappa_d)} \end{aligned} \tag{A.75}$$

$$R^d = R^l \text{ and } 1-\mu = \Omega_{dI} + \Omega_{rI} R^l$$
$$\text{and } \Omega_{dI} + \Omega_{rI} R^d = \beta_I \frac{R^d + \kappa_d + (1-\kappa_d)\left(\Omega_{dI} + \Omega_{rI} R^d\right)}{\pi}$$
$$1-\mu = \beta_I \frac{1 + R^d - (1-\kappa_d)\mu}{\pi}$$
$$\mu = \frac{\frac{\pi}{\beta_I} - \left(1 + R^d\right)}{\frac{\pi}{\beta_I} - (1-\kappa_d)} \tag{A.76}$$

$$\begin{aligned} 1-\mu &= \Omega_{dI} + \Omega_{rI} R^l \\ \Omega_{dI} &= 1 - \mu - \Omega_{rI} R^l \end{aligned} \tag{A.77}$$

$$\Omega = \frac{1}{\Theta_p} \tag{A.78}$$

$$\begin{aligned} 1 &= r_h + \beta_P (1-\delta_h) \\ r_h &= 1 - \beta_P (1-\delta_h) \end{aligned} \tag{A.79}$$

$$\begin{aligned} 1 &= \beta_P (1 - \delta_k + r_k) \\ r_k &= \frac{1}{\beta_P} - 1 + \delta_k \end{aligned} \tag{A.80}$$

$$\begin{aligned} \alpha &= r_k \frac{k}{y} \\ \frac{k}{y} &= \frac{\alpha}{r_k} \end{aligned} \tag{A.81}$$

$$\frac{i_k}{y} = \delta_k \frac{k}{y} \tag{A.82}$$

$$\frac{w_P n_P}{y} = (1-\alpha)\,\theta_P \tag{A.83}$$
$$\frac{w_I n_I}{y} = (1-\alpha)\,\theta_I \tag{A.84}$$
$$\frac{w_R n_R}{y} = (1-\alpha)(1-\theta_P-\theta_I) \tag{A.85}$$

$$r_h = \xi_h \frac{(1-\zeta)\, c_R/y}{h_R/y} \text{ and } \frac{c_R}{y} + r_h \frac{h_R}{y} = \frac{w_R n_R}{y}$$
$$[1+\xi_h (1-\zeta)] \frac{c_R}{y} = \frac{w_R n_R}{y}$$
$$\frac{c_R}{y} = \frac{\frac{w_R n_R}{y}}{1+\xi_h (1-\zeta)} \tag{A.86}$$

$$r_h = \xi_h \frac{(1-\zeta)\, c_R/y}{h_R/y}$$
$$\frac{h_R}{y} = \xi_h \frac{(1-\zeta)\, c_R/y}{r_h} \tag{A.87}$$

$$\frac{i_{hR}}{y} = \delta_h \frac{h_R}{y} \tag{A.88}$$

$$n_R^{1+\vartheta} = \frac{w_R l_R/y}{(1-\zeta)\, c_R/y}$$
$$n_R = \left( \frac{w_R l_R/y}{(1-\zeta)\, c_R/y} \right)^{\frac{1}{1+\vartheta}} \tag{A.89}$$

$$1-\phi\mu = \xi_h \frac{(1-\zeta)\, c_I/y}{h_I/y} + \beta_I (1-\delta_h)(1-\phi\mu)$$
$$\frac{c_I}{y} = \frac{[1-\beta_I (1-\delta_h)](1-\phi\mu)}{\xi_h (1-\zeta)} \frac{h_I}{y}$$

$$\frac{l}{y} = \phi \frac{i_{hI}}{y} \text{ and } \frac{i_{hI}}{y} = \delta_h \frac{h_I}{y}$$
$$\frac{l}{y} = \delta_h \phi \frac{h_I}{y}$$

$$\frac{d}{y} = \frac{\pi}{\pi - 1 + \kappa_d} \frac{l}{y}$$
$$\frac{d}{y} = \frac{\pi \delta_h \phi}{\pi - 1 + \kappa_d} \frac{h_I}{y}$$

$$\frac{c_I}{y} + \frac{i_{hI}}{y} + \frac{R^d + \kappa_d}{\pi}\frac{d}{y} = \frac{w_I n_I}{y} + \frac{l}{y}$$

$$\frac{[1-\beta_I(1-\delta_h)](1-\phi\mu)}{\xi_h(1-\zeta)}\frac{h_I}{y} + \delta_h\frac{h_I}{y} + \frac{R^d+\kappa_d}{\pi-1+\kappa_d}\delta_h\phi\frac{h_I}{y} = \frac{w_I n_I}{y} + \delta_h\phi\frac{h_I}{y}$$

$$\frac{h_I}{y} = \frac{\frac{w_I n_I}{y}}{\frac{[1-\beta_I(1-\delta_h)](1-\phi\mu)}{\xi_h(1-\zeta)} + \delta_h + \left(\frac{R^d+\kappa_d}{\pi-1+\kappa_d} - 1\right)\delta_h\phi} \tag{A.90}$$

$$\frac{i_{hI}}{y} = \delta_h\frac{h_I}{y} \tag{A.91}$$

$$\frac{l}{y} = \phi\frac{i_{hI}}{y} \tag{A.92}$$

$$\frac{d}{y} = \frac{\pi}{\pi-1+\kappa_d}\frac{l}{y} \tag{A.93}$$

$$\frac{c_I}{y} = \frac{[1-\beta_I(1-\delta_h)](1-\phi\mu)}{\xi_h(1-\zeta)}\frac{h_I}{y} \tag{A.94}$$

$$n_I^{1+\vartheta} = \frac{w_I l_I/y}{(1-\zeta)\,c_I/y}$$

$$n_I = \left(\frac{w_I l_I/y}{(1-\zeta)\,c_I/y}\right)^{\frac{1}{1+\vartheta}} \tag{A.95}$$

$$1 = \xi_h\frac{(1-\zeta)\,c_P/y}{h_P/y} + \beta_P(1-\delta_h)$$

$$\frac{c_P}{y} = \frac{1-\beta_P(1-\delta_h)}{\xi_h(1-\zeta)}\frac{h_P}{y}$$

$$\frac{c}{y} + \frac{i_k}{y} + \frac{i_h}{y} + \frac{g}{y} = 1 \text{ and } \frac{c}{y} = \frac{c_P}{y} + \frac{c_I}{y} + \frac{c_R}{y} \text{ and } \frac{i_h}{y} = \frac{i_{hP}}{y} + \frac{i_{hI}}{y} + \frac{i_{hR}}{y}$$

$$\frac{1-\beta_P(1-\delta_h)}{\xi_h(1-\zeta)}\frac{h_P}{y} + \frac{c_I}{y} + \frac{c_R}{y} + \frac{i_k}{y} + \delta_h\frac{h_P}{y} + \frac{i_{hI}}{y} + \frac{i_{hR}}{y} + \frac{g}{y} = 1$$

$$\frac{h_P}{y} = \frac{1 - \frac{c_I}{y} - \frac{c_R}{y} - \frac{i_k}{y} - \frac{i_{hI}}{y} - \frac{i_{hR}}{y} - \frac{g}{y}}{\frac{1-\beta_P(1-\delta_h)}{\xi_h(1-\zeta)} + \delta_h} \tag{A.96}$$

$$\frac{i_{hP}}{y} = \delta_h\frac{h_P}{y} \tag{A.97}$$

$$\frac{c_P}{y} = \frac{1-\beta_P(1-\delta_h)}{\xi_h(1-\zeta)}\frac{h_P}{y} \tag{A.98}$$

$$
\begin{aligned}
n_P^{1+\vartheta} &= \frac{w_P n_P/y}{(1-\zeta)\, c_P/y} \\
n_P &= \left(\frac{w_P n_P/y}{(1-\zeta)\, c_P/y}\right)^{\frac{1}{1+\vartheta}}
\end{aligned} \tag{A.99}
$$

$$
\frac{i_h}{y} = \frac{i_{hP}}{y} + \frac{i_{hI}}{y} + \frac{i_{hR}}{y} \tag{A.100}
$$

$$
\frac{i}{y} = \frac{i_k}{y} + \frac{i_h}{y} \tag{A.101}
$$

$$
\frac{h}{y} = \frac{h_P}{y} + \frac{h_I}{y} + \frac{h_R}{y} \tag{A.102}
$$

$$
\frac{c}{y} = \frac{c_P}{y} + \frac{c_I}{y} + \frac{c_R}{y} \tag{A.103}
$$

$$
\frac{tax}{y} = \Xi \tag{A.104}
$$

$$
\begin{aligned}
&\frac{g}{y} + \left(\frac{1+R}{\pi} - 1\right)\frac{q_S b_S}{y} + \left(\frac{1+R_L}{\pi} - 1\right)\frac{q_L b_L}{y} = \frac{tax}{y} \text{ and } \frac{q_L b_L}{q_S b_S} = \gamma_b \\
&\frac{g}{y} + \left[\frac{1+R}{\pi} - 1 + \left(\frac{1+R_L}{\pi} - 1\right)\gamma_b\right]\frac{q_S b_S}{y} = \frac{tax}{y} \\
&\frac{q_S b_S}{y} = \frac{\frac{tax}{y} - \frac{g}{y}}{\frac{1+R}{\pi} - 1 + \left(\frac{1+R_L}{\pi} - 1\right)\gamma_b}
\end{aligned} \tag{A.105}
$$

$$
\frac{q_L b_L}{y} = \gamma_b \frac{q_S b_S}{y} \tag{A.106}
$$

$$
\begin{aligned}
y &= \frac{1}{\Theta_p} k^{\alpha} \left(n_P^{\theta_P} n_I^{\theta_I} n_R^{\theta_R}\right)^{1-\alpha} \\
y &= \left(\frac{1}{\Theta_p}\right)^{\frac{1}{1-\alpha}} \left(\frac{k}{y}\right)^{\frac{\alpha}{1-\alpha}} n_P^{\theta_P} n_I^{\theta_I} n_R^{\theta_R}
\end{aligned} \tag{A.107}
$$

Now all variables in levels can be determined. And note that

$$
\lambda_P = \frac{1}{c_P} \tag{A.108}
$$

$$
\lambda_I = \frac{1}{c_I} \tag{A.109}
$$

$$
\lambda_R = \frac{1}{c_R} \tag{A.110}
$$

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