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On the Limits of Hedging Inflation Risk in Investment Portfolios[a]

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Abstract

We explore to what extent real returns on investment portfolios can be hedged against inflation risk by using existing financial market instruments. We empirically find that inflation-linked bonds offer only limited protection against inflation risk, while nominal debt and stocks play at least comparable roles in this respect. These findings apply to both a static and a dynamic setting. To explain the empirical results, we develop a theoretical framework that incorporates real basis risk. The demonstrated limits of hedging inflation risk are of particular relevance for long-term investors, such as pension funds with participants concerned about the real value of their pension benefits.

Key words: unhedgeable inflation risk, incomplete markets, welfare loss, mean-variance frontiers, minimum-variance portfolio, nominal and index-linked bonds.
JEL Codes: C61, E21, G11, G23.

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1 Introduction

In the aftermath of the Covid-19 crisis, inflation has once again become a major economic concern. With higher average inflation, the uncertainty about future inflation also increases, which in turn raises macroeconomic risks and makes returns on investments more risky. Long-term investors generally aim to protect the real value of their assets. This is especially important for pension funds that try to provide a stream of benefits with stable purchasing power because most pensioners have only limited ability to absorb risk. Chen et al. (2020) have shown that the lack of adequate inflation hedging instruments can lead to welfare losses for retirees ranging from 1% to 8% in terms of certainty equivalent consumption, thus underscoring the importance of limiting inflation risk. However, effectively hedging inflation risk over a long horizon is hampered by market frictions, rollover risks and other complicating factors. Therefore, a key question is to what extent it is possible *in practice* to construct investment portfolios that generate stable real returns, and what would be the cost associated with such a strategy.

This paper explores to what extent real returns on investment portfolios composed of (indices of) nominal public debt, index-linked public debt and stocks can be hedged against inflation. The purpose of our analysis is expressly *not* to investigate optimal allocations of a consumption-based capital asset pricing model (CAPM) for real portfolio returns. Instead, we explore more narrowly to what extent it is possible *in practice* to hedge inflation risk with instruments that are widely traded on exchanges and therefore are easily available at low trading cost. To our knowledge, there is little to no empirical work that addresses this question. We address the issue by constructing minimum-variance portfolios that include and exclude index-linked bonds (ILBs).

In practice, minimum-variance portfolios feature only small fractions of index-linked bonds (ILBs), possibly because of their limited availability. Many countries, such as, for example, the Netherlands, do not even issue ILBs linked to their own price index.[1] Although Dutch pension funds can resort to foreign ILBs according to the pension supervisor, the Dutch central bank, these constituted only 1% of their total asset holdings in 2024.[2] Other assets, such as equities and commodities, have historically demonstrated some usefulness in hedging against inflation risk, but those hedges are far from perfect. In addition, financial innovation, derivatives and alternative investment strategies have emerged that do help protect portfolio returns against inflation risk. However, these possibilities come with costs, frictions and roll-over risks that reduce the scope of hedging inflation risks.

We demonstrate for four countries (the Netherlands, Germany, the United

[1] Common arguments against issuing ILBs are high issuance costs because of limited market liquidity, which may well be a circular argument, and the fear that their issuance would signal a reduced commitment to combating inflation. There is little evidence for the latter argument.

[2] See https://www.dnb.nl/statistieken/data-zoeken/ belegd vermogen voor risico pensioenfondsen.

Kingdom and the United States) that it is *not* possible to construct a portfolio with ILBs that is *even remotely* able to hedge away inflation risk. We demonstrate the limited effectiveness in hedging inflation risk for both the short- and the long-term. Our findings underscore the critical importance of deeper investigation of inflation hedging possibilities. This is in particular important for long-term investors, such as pension funds, who aim at protecting the purchasing power of their participants' benefits over their lifetime. Our findings should provide investment managers with leads to improve risk versus expected return trade-offs in real terms. From a policy perspective, our findings suggest the need for an enhanced supply of hedging instruments.

We also explore optimal (in real returns) long-horizon investment portfolios when risk-aversion is limited. The role of ILBs and stocks is substantially higher in these portfolios than in the minimum-variance portfolios. Finally, we set up a simple theoretical framework to clarify both the composition of the minimum-variance one-period portfolios and the long-horizon portfolios based on limited risk aversion. The framework has a reasonable capability in replicating both classes of portfolios, although it finds it difficult the replicate both on the basis of the same parameter combinations.

Our analysis relates to the literature that analyzes the methodologies and effectiveness of inflation risk hedging. Jeanblanc et al. (2012) present advanced techniques for mean-variance hedging, but do not provide empirical results. Kupfer (2019) presents methodologies for estimating inflation risk premia. Assessments of the effectiveness of hedging inflation risks vary in the literature, of which Arnold and Auer (2015) and Jarrow and Yildirim (2023) provide an overview. Specifically, they explore the relationship between inflation and asset classes like stocks, gold, fixed income and real estate. Our findings are consistent with those of Campbell et al. (2009), who also highlight a strong relationship between nominal bond returns and inflation, which contrast with those of Spierdijk and Umar (2015), who indicate that nominal bonds have only limited effectiveness in inflation hedging. He et al. (2024) find that 5-year nominal and inflation-protected US public debt securities generate similar returns, while 10-year nominal debt outperforms 10-year inflation-protected debt. Bekaert and Wang (2010) show that the inflation risk is difficult to hedge and that the inflation risk premium is large and volatile. Fleckenstein et al. (2014) indicate that inflation-linked bonds are mispriced in the market and that arbitrage opportunities persist. Mispricing limits capital flows into the ILBs, which causes mispricing to persist. This can be an additional source of hedging mismatch. Finally, Neville et al. (2021) explore different passive and active investment strategies for inflationary times. Inspired by Brière and Signori (2012), who show that economic regimes matter for the role of inflation-linked bonds, we also specifically explore inflation risk hedging during the period after the COVID-19 outbreak. Indeed, during this high-inflation period the fraction of ILBs in the optimal minimum-variance portfolio increases.

The remainder of this paper is structured as follows. Section 2 describes the methodology and data for our analysis. Section 3 analyses the mean-variance frontiers of portfolio returns, while Section 4 presents the results of an optimal investment strategy for a variable annuity. Section 5 rationalizes our empirical findings using a theoretical framework with time-varying real basis risk. Section 6 concludes, emphasizing some general lessons from our analysis. The Appendix provides technical details on the analytical solutions and some additional tables and figures with results.

2 Data and methodology

We explore to what extent real returns on investment portfolios can be hedged against inflation. In practice, the mismatch between the inflation rate that needs to be hedged and the protection provided by the instruments (that is, the "real basis risk") may be driven by several factors. First, not all durations of the relevant instruments may be available. Second, the measure of inflation to be hedged may differ from what is available in terms of inflation measures underlying the hedging instrument; for example, Dutch inflation can only be hedged using bonds linked to foreign inflation. This is the case when the domestic government does not issue ILBs. Third, there are rebalancing and transaction costs. Fourth, instruments used for hedging may have credit risks, like all financial instruments.

2.1 Instruments for inflation hedging

We confine ourselves to nominal bonds, ILBs and stocks as the market instruments to be used for inflation hedging. We could also consider inflation swaps, but this would not add much since their underlying assets are bonds, which we already include. We use the three criteria for the selection of specific instruments suggested by the Solvency II review of EIOPA[3]: market depth, measured by the average daily notional amount traded; market liquidity, measured by the average daily number of trades; and transparency.

2.2 Data

We use ILB price indices from S&P Global, because they are designed to track the performance of local-currency-denominated ILBs. These indices include broad and comprehensive developed market indices and indices specific to individual countries.[4] Each index is based on several instruments.[5]

[3]See https://www.eiopa.europa.eu/system/files/2020-12/eiopa-bos-20-750-background-analysis.pdf.

[4]See https://www.spglobal.com/spdji/en/index-family/fixed-income/inflation-linked/#overview.

[5]For example, the S&P Germany Sovereign Bond Index is composed of four instruments with an average maturity of 8.3 years (see https://www.spglobal.com/spdji/en/idsenhancedfactsheet/file.pdf?calcFrequency=M&force_download=true&hostIdentifier=48190c8c-42c4-46af-8

Table 1: Number of observations and the number of financial instruments

#instruments	**#observations**	$\sqrt{\#\textbf{instr}} * \sqrt{\#\textbf{obs}}$
23	1587	191.05
22	2294	224.65
21	2489	228.62
20	2508	223.96
19	4507	292.63
18	4768	292.96
17	**5082**	**293.93**
16	5227	289.19

Note: Trade-off between the number of observations and the number of financial instruments

Table 2: Average and standard deviation of CPI inflation rates in four countries: Germany, the UK, the Netherlands and the US.

inflation rates (in bps)	GE	UK	NL	US
monthly average	17.97	29.13	19.02	20.97
monthly standard deviation	36.96	40.80	45.72	29.58
quarterly average	54.12	89.62	57.42	63.80
quarterly standard deviation	82.02	94.93	80.57	75.50

The complete set of ILB-indices from S&P Global consists of a set of 10 nominal bond indices and 13 ILB indices. All indices are total return indices that reinvest coupon and dividend payments. These 23 indices have 1587 days in common. But Table 1 shows that there is a trade-off between the number of available instruments and the number of trading days in common. To strike a balance between the number of common trading days and the number of instruments, we select the set of indices for which the product of the numbers of instruments and observations is maximized. Table 1 shows that this is obtained with 17 bond indices: these all have have 5082 trading days in common, from 20 September 2004 to 18 April 2024. Table 3 lists these indices.[6] Using changes in the price levels of the indices, we construct nominal returns.

Figure 1 depicts the monthly Dutch, German, UK and US consumer price

d1a-0cd5db894797&indexId=91922574), while the S&P U.K. Gilt Bond Index is based on 32 instruments with an average maturity of 16.7 years (see https://www.spglobal.com/spdji/en/idsenhancedfactsheet/file.pdf?calcFrequency=M&force_download=true&hostIdentifier=48190c8c-42c4-46af-8d1a-0cd5db894797&indexId=91922572).

[6]The excluded bond indices are the S&P Global Developed Sovereign Inflation Linked Bond USD Index, S&P U.S. TIPS Index, S&P South Africa Sovereign Inflation-Linked Bond Index, S&P CLX Chile Sovereign Inflation-Linked Bond Index, S&P/B3 Brazil Sovereign Inflation Linked Bond Index and S&P Cyprus Sovereign Bond Index.

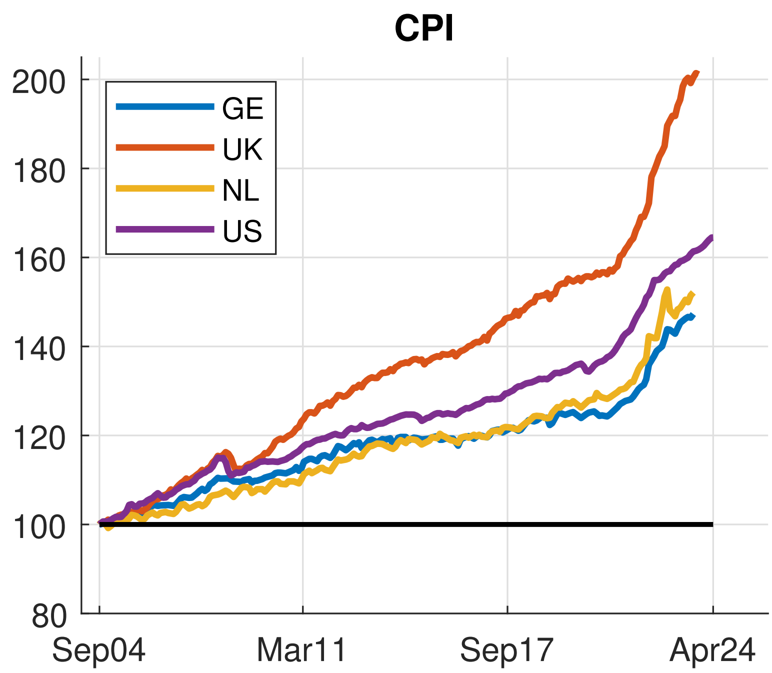


Figure 1: CPI of Germany, the UK, the Netherlands and the US.*Note: "GE" is Germany and "NL" is the Netherlands.*

indices (CPI). Except for the Netherlands, all of these countries issue ILBs. Table 2 reports the average and standard deviation of the corresponding inflation rates of the CPI per month and quarter. On an annual basis, the average inflation varies from 2.2% (= 12 * 17.97 bps in Germany) to 3.6% (= 4 * 89.62 bps in the U.K).

Table 3 reports the correlations (in %) between nominal bond index returns and the CPI inflation rates. All nominal bond index returns are negatively correlated with Dutch, German and UK inflation, but except for the US Aggregate Bond Index they are all positively correlated with US inflation. The prevalence of these negative correlations may well have a macroeconomic explanation. Higher inflation is associated with monetary tightening, which causes a fall in the price of the outstanding nominal debt. At the quarterly frequency, almost all correlations between nominal bond price indices and inflation are also negative, with two exceptions (see Table 3).

The monthly nominal returns on the ILBs are all negatively correlated with UK and German inflation and are all positively correlated with US inflation. The pattern for Dutch inflation is more mixed with five negative and three positive correlations. For the quarterly returns, the pattern is mixed for all countries, except for the US where the correlations all remain positive. As expected, the correlations with inflation of the nominal returns on ILBs are generally higher than those of the nominal returns on nominal bonds with inflation.

The correlations between monthly stock index returns and Dutch and US in-

Table 3: Correlations (in %) between nominal returns and CPI inflation rates

Index name	ticker	monthly correlations GE	UK	NL	US	quarterly correlations GE	UK	NL	US
S&P Eurozone Sov. Bond Index	SPBDEGIT	-31.1	-19.4	-17.4	13.6	-48.3	-31.2	-36.3	1.7
S&P France Sov. Bond Index	SPBDEFRT	-30.1	-19.6	-17.8	11.2	-44.0	-31.4	-36.0	-2.2
S&P Austria Sov. Bond Index	SPBDEATT	-31.4	-20.1	-18.6	10.1	-43.7	-31.0	-34.7	-3.5
S&P Belgium Sov. Bond Index	SPBDEBET	-30.7	-20.9	-17.1	10.3	-43.2	-33.4	-34.2	-2.7
S&P Finland Sov. Bond Index	SPBDEFIT	-29.6	-18.9	-18.6	13.1	-44.5	-30.9	-34.1	0.7
S&P Germany Sov. Bond Index	SPBDEDET	-27.4	-18.1	-17.0	12.6	-43.8	-29.2	-33.2	-0.5
S&P Netherlands Sov. Bond Index	SPBDENLT	-28.8	-18.9	-18.3	11.2	-44.4	-31.5	-35.8	-1.5
S&P U.K. Gilt Bond Index	SPFIGBT	-11.5	-15.5	-17.2	3.6	-20.9	-44.7	-25.5	-0.6
S&P U.S. Aggregate Bond Index	SPUSBMIT	-7.4	-19.6	-3.5	-14.8	-10.4	-22.7	5.5	-37.1
S&P Eurozone Sov. ILB Index	SPFID4IT	-14.4	-12.6	-3.8	28.2	-22.4	-14.2	-9.7	22.6
S&P U.K. Gilt ILB Index	SPFIGBIT	-10.3	-11.6	-17.6	13.4	-20.0	-24.2	-22.6	12.3
S&P 10 Year US TIPS Index	SPBDU1ST	-3.8	-19.3	4.6	7.7	-7.8	-16.5	13.3	4.2
S&P New Zealand ILB Index	SPBNILT	-8.4	-13.4	-6.6	11.6	-5.7	-12.8	-4.6	13.3
S&P Sweden Sov. ILB Index	SPFISEI	-6.1	-8.6	-2.3	29.6	-3.3	-3.6	-1.5	30.5
S&P Andean Sov. ILB Index	SPFIMLUT	-3.1	-17.7	-2.0	17.6	0.3	-1.1	4.8	20.0
S&P Pacific Allce. Sov. ILB Index	SPFIMPUT	-2.4	-14.3	2.5	26.9	4.3	12.7	18.9	32.9
S&P/BMV Mexico Sov. ILB Index	SPVIF0U	-1.8	-9.6	4.8	26.8	4.0	20.0	24.1	35.2
S&P 500	SPXT	-2.7	-7.8	4.5	22.0	2.3	6.9	3.1	34.0
MSCI World EUR	MSDEWIN	-2.5	-7.5	2.5	29.9	4.2	8.0	0.2	37.8
Dow Jones	DJITR	-2.9	-7.7	0.8	18.7	7.7	9.6	-2.3	31.1

Note: "GE" is Germany and "NL" is the Netherlands. The top panel contains nominal bond indices, the middle panel ILB indices and the lower panel stock indices.

flation rates are all positive, but all negative for the UK and German inflation rates. The correlations between quarterly stock market returns and inflation rates are all positive, except for the Dow Jones index-based return which is negatively correlated with Dutch inflation. Overall the table suggests that stocks can serve as a (partial) hedge against inflation, in particular at the quarterly frequency.

Next, we group all the indices into three broad asset classes: nominal bonds, ILBs, and stocks. Table 4 reports the average correlations between the nominal returns on all pairs of indices from different asset classes. The average correlations are positive and tend to be high or quite high, except for the average correlation between stocks and nominal bonds.

Table 5 reports in basis points the average monthly and quarterly returns of each asset class, as well as the volatility of these returns, measured as the average of the standard deviations of the instruments of the corresponding asset class. The nominal returns are expressed as percentage changes in euro denominated prices. The real returns are measured as changes in local currency prices adjusted for the corresponding CPI inflation. The average nominal returns are larger than the average real returns because the average inflation rate is positive in each country. In general, stocks have the highest average return and the highest volatility, while nominal bonds have the lowest average return and the lowest volatility.

Table 4: Average correlations (in %) between asset classes

	monthly nominal returns		
type of instruments	nominal bonds	ILBs	stocks
nominal bonds	76.7	35.0	9.7
ILBs	35.0	46.6	37.9
stocks	9.7	37.9	96.6
	quarterly nominal returns		
type of instruments	nominal bonds	ILBs	stocks
nominal bonds	78.2	33.8	-0.7
ILBs	33.8	47.1	31.6
stocks	-0.7	31.6	95.8

Note: Average nominal return correlations between index pairs based on the grouping of all assets into broad instrument classes.

2.3 Constructing mean-variance frontiers

Consider a portfolio of n instruments, with portfolio weights ω:

$$\omega' = \begin{bmatrix} \omega_1 & \omega_2 & \cdots & \omega_n \end{bmatrix}$$

The return on instrument i at time t is $R_{i,t}$. The vector of (empirical) average returns μ and the variance-covariance matrix Σ are, respectively:

$$\mu' = \begin{bmatrix} \mu_1 & \mu_2 & \cdots & \mu_n \end{bmatrix}$$

and

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_{1,2} & \cdots & \sigma_{1,n} \\ \sigma_{2,1} & \sigma_2^2 & \cdots & \sigma_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n,1} & \sigma_{n,2} & \cdots & \sigma_n^2 \end{pmatrix}$$

where $\mu_i = \frac{1}{T}\sum_{t=1}^{T} R_{i,t}$ and $\sigma_{i,j} = Cov(R_{i,t}, R_{j,t})$. The global minimum-variance portfolio solves the following minimization problem:

$$\min_{\omega} \sigma_m^2 = \omega'\Sigma\omega \text{ s.t. } \omega'\mathbf{1}_n = 1. \quad (1)$$

Table 5: Summary statistics of returns in basis points for each asset class

	monthly average					quarterly average				
	nominal	real				nominal	real			
type of instruments		GE	UK	NL	US		GE	UK	NL	US
nominal bonds	20.0	2.3	2.9	1.3	-3.8	61.0	8.2	7.9	4.7	-11.7
ILBs	43.2	25.4	25.1	24.4	18.6	128.2	74.5	69.8	70.8	51.5
stocks	88.0	70.1	67.9	69.0	61.6	264.1	209.2	196.2	206.1	184.7
	monthly volatility					quarterly volatility				
	nominal	real				nominal	real			
type of instruments		GE	UK	NL	US		GE	UK	NL	US
nominal bonds	153.8	166.7	270.6	167.1	253.0	283.2	320.2	476.7	313.2	438.8
ILBs	307.1	311.2	354.2	311.4	345.4	503.7	513.0	548.3	505.6	549.4
stocks	419.5	421.5	406.0	419.9	406.2	672.4	671.0	571.2	674.5	677.3

Note: The nominal returns are expressed as changes in euro denominated prices. The real returns are measured as changes in local currency prices adjusted for the corresponding CPI inflation. The average returns and the volatility of the returns of an asset class are measured by taking the averages of the means and standard deviations of the returns on the instruments within the corresponding asset class.

where $\mathbf{1}_n$ is an n-dimensional column vector of 1's. In the Appendix we show that solving Eq.(1) yields the following n-dimensional column vector of optimal portfolio weights ω_m:

$$\omega_m = (\mathbf{1}_n' \Sigma^{-1} \mathbf{1}_n)^{-1} \Sigma^{-1} \mathbf{1}_n. \tag{2}$$

The expected return on the global minimum-variance portfolio is $\mu_m = \omega_m' \mu$.

To construct the mean-variance frontiers we also need to derive the minimum-variance portfolio's subject to a given target rate of return μ_p. We do this by minimizing the same objective function, now subject to an additional constraint that pins down the target return μ_p:

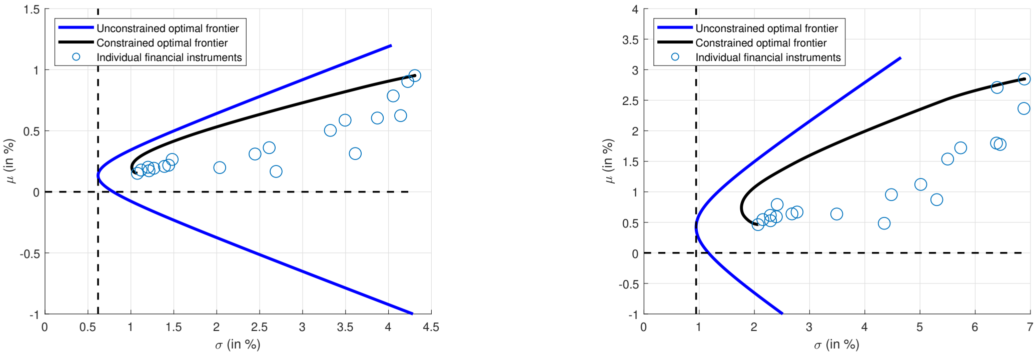
$$\min_{\omega} \sigma^2 = \omega' \Sigma \omega \tag{3}$$
$$\text{s.t. } \omega' \mu = \mu_p \text{ and } \omega' \mathbf{1}_n = 1.$$

In the Appendix we show that the solution is given by the vector of portfolio weights ω_p:

$$\omega_p = \Sigma^{-1} [\mu \; \mathbf{1}_n] \Theta^{-1} [\mu_p \; 1]' \tag{4}$$

where Θ is the two-by-two matrix:

$$\Theta = [\mu \; \mathbf{1}_n]' \; \Sigma^{-1} \; [\mu \; \mathbf{1}_n].$$



(a) Monthly nominal returns (b) Quarterly nominal returns

Figure 2: Mean-variance frontiers. *Note: mean-variance frontiers are expressed in percentages and constructed from 17 bond indices and three stock indices based on monthly nominal returns (left) and quarterly nominal returns (right). Returns are derived from prices in euros.*

Both the unconstrained and the constrained minimum-variance portfolio we just derived allow for portfolio weights that are negative or exceed 100%, i.e. they admit short-selling and borrowing. This is often not allowed in practice; therefore, we also consider minimum-variance portfolios that constrain the portfolio weights not to be negative or exceed 100%. These restrictions imply that $\omega_i \in [0, 1], \forall i$. In the following we solve the constrained optimization problem numerically .

3 Results

We use our $n = 20$ indices (17 bond indices and 3 stock indices) to construct mean-variance portfolios for nominal returns in Section 3.1 and real returns in Section 3.2. Section 3.3 presents a sensitivity analysis in which we explore a number of variations on the preceding setup. Our analysis is carried out for monthly and quarterly returns, in view of the fact that nominal returns may respond with a lag to inflation. Also, we present both portfolios in which short selling and leverage is allowed (labeled "unconstrained") and portfolio's where the portfolio shares ω_i are constrained to lie between zero and one, labeled "constrained".

3.1 Nominal returns

The left panel of Figure 2 shows the mean-variance frontier based on nominal monthly returns in the absence of any restrictions on portfolio weights ω_i, i.e. short selling ($\omega_i < 0$) and leverage ($\omega_i > 1$) are allowed. The minimum-variance portfolio corresponds to the most-left point on the frontier. Table 6 reports that the unconstrained global minimum-variance portfolio has a volatility of $\sigma = 61.8$

Table 6: Expected returns (μ_m) and volatility (σ_m) of the minimum-variance portfolios for nominal returns.

	monthly returns			quarterly returns		
minimum-variance portfolio	μ_m	σ_m	$\hat{\mu}_m$	μ_m	σ_m	$\hat{\mu}_m$
unconstrained	13.3	61.8	13.3	42.0	94.4	42.0
constrained	12.1	63.4	17.1	46.5	113.4	80.3

Note: Numbers are in basis points. Further, $\hat{\mu_m}$ is the expected return from the unconstrained mean-variance frontier when volatility equals that of the constrained minimum-variance portfolio.

Table 7: Portfolio allocations (in %) of the minimum-variance portfolios based on nominal returns.

	monthly		quarterly	
indices	unconstrained	constrained	unconstrained	constrained
nominal bonds	83.5	75.8	73.7	65.1
index-linked bonds	12.5	20.1	17.7	25.3
stocks	4.1	4.2	8.8	9.4

basis points (bps) and an expected return of $\mu = 13.3$ bps per month. The standard deviation associated with this portfolio is strictly positive, indicating that it is not possible to completely eliminate risk in nominal returns. Moving along the frontier to the right, both the expected return and the standard deviation of the portfolio increase. Table 7 presents the shares of the three asset categories in the minimum-variance portfolio. We see that by far the largest fraction of the portfolio is made up of nominal bond indices.

The unconstrained minimum-variance portfolio contains portfolio weights on some individual assets that exceed 100% or are negative, implying borrowing or short-selling. For example, the portfolio features an exposure of -214.7% to the Netherlands sovereign bond index and +236.5% to the German sovereign bond index.[7] We consider such weights unrealistic; therefore, in Figure 2 we also show the mean-variance frontier when we constrain the weights on all individual assets to lie in the $[0, 1]$ interval. As expected, given that the constraints bind, the constrained minimum-variance portfolio has somewhat higher volatility ($\sigma = 63.4$bps) and a marginally lower expected return ($\mu = 12.1$bps). Compared to the unconstrained minimum-variance portfolio, the fraction allocated to nominal indices falls but is still high. It is still dominated by the German sovereign bond index, which has a 75.5% allocation in the overall portfolio. Although the constrained minimum-variance portfolio unavoidably features a higher variance and a lower expected return, the differences with the unconstrained minimum-variance portfolio are limited. For the monthly data, the expected return from the unconstrained mean-variance frontier at the point where volatility equals that of the constrained minimum-variance portfolio is 17.1 bps, implying a nominal risk premium from the constraints of $17.1 - 12.1 = 5.0$ bps.

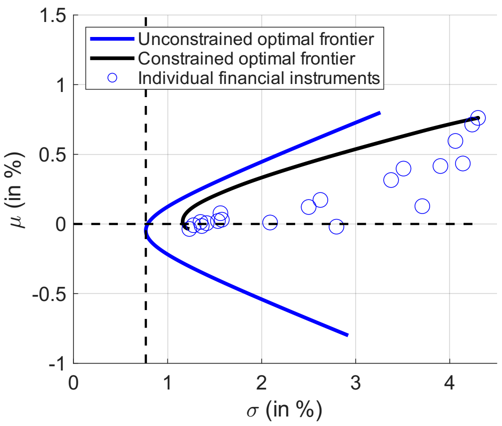
The right-hand panel of Figure 2 shows the mean-variance frontiers based on quarterly returns. These portfolios are also dominated by nominal bond indices, although less so than the portfolios based on monthly returns. Consequently, the weights of both ILBs and stocks are now higher. The implied risk premium of the portfolio weight constraints rises to $80.3 - 46.5 = 33.8$bps per quarter (see Table 6), which when annualized is higher than for portfolios based on monthly nominal returns: approximately 136 bps for the portfolios based on quarterly returns versus 60 basis points for the portfolios based on monthly returns.

In the remainder of this paper, we confine ourselves to portfolios that exclude borrowing and short-selling, to prevent the unrealistically extreme allocations to individual indices that characterize the unconstrained portfolios.

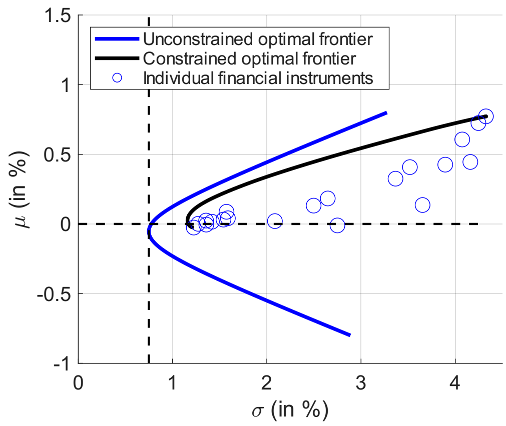
3.2 Real returns

So far, we have focused on mean-variance portfolios based on nominal returns. But because we are interested in the purchasing power of investment portfolios, hence

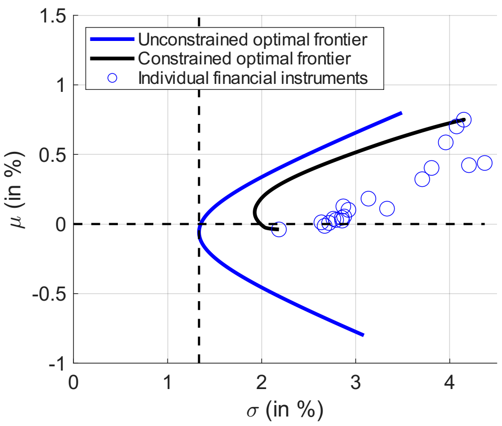
[7] The coexistence of highly-correlated indices in a portfolio frequently leads to extreme weights of opposite sign on these indices.



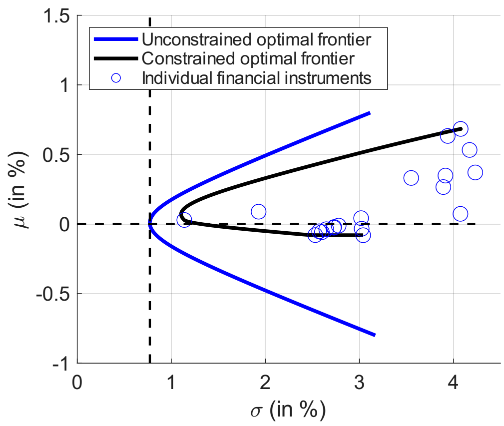
(a) Returns based on Dutch inflation



(b) Returns based on German inflation



(c) Returns based on UK inflation



(d) Returns based on US inflation

Figure 3: Mean-variance frontiers based on monthly real returns. *Note: real returns are obtained by subtracting CPI inflation from nominal returns.*

the hedging of inflation risks, we also construct the minimum-variance portfolio's based on real asset returns. The real returns are adjusted for the corresponding CPI inflation and the relevant currency (i.e. euros for NL and GE, dollars for the US, and pounds for the UK).[8]

Figure 3 depicts the mean-variance frontiers of portfolios based on monthly real returns for the aforementioned countries. Panel (a) shows that the unconstrained minimum-variance portfolio based on Dutch inflation has a volatility of $\sigma = 76.9$ bps and an expected return of $\mu = -4.7$ bps, while the corresponding figures for the constrained portfolio are $\sigma = 116.1$ bps and $\mu = 1.8$ bps, respectively. See also Table 8. Panels (b), (c), and (d) of Figure 3 depict the corresponding figures for real returns based on German, U.K. and U.S. inflation, respectively. The mean-variance frontiers based on Dutch and German inflation are highly similar.

[8]Appendix B describes the steps that we perform to construct the nominal and real returns from the data.

Table 8: Expected real returns (μ) and volatility (σ) of the minimum-variance portfolios.

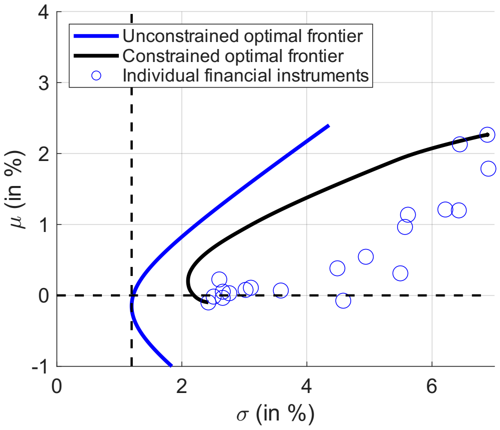
	monthly returns			quarterly returns		
country	μ	σ	$\hat{\mu}$	μ	σ	$\hat{\mu}$
NL unconstrained	-4.7	76.9		-14.9	119.7	
NL constrained	1.8	116.1	18.5	20.2	209.9	90.0
GE unconstrained	-5.4	74.8		-14.6	129.8	
GE constrained	2.6	115.9	18.3	26.4	217.1	91.4
UK unconstrained	-5.9	133.4		15.3	204.2	
UK constrained	8.4	192.6	31.1	59.5	297.9	146.1
US unconstrained	0.7	77.2		14.2	115.2	
US constrained	7.1	110.4	21.5	33.1	194.8	108.9

Note: Numbers are in basis points. Real returns are constructed by subtracting CPI inflation from the nominal returns. Borrowing and short-selling are excluded. Finally, ($\hat{\mu}$) is the expected return on the unconstrained mean-variance frontier when volatility equals that of the constrained minimum-variance portfolio.

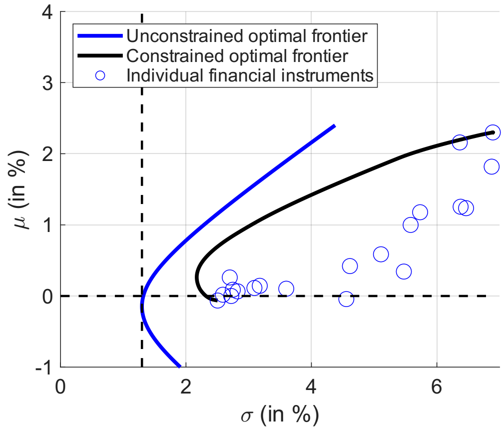
Table 9: Allocations of constrained minimum-variance portfolios based on real returns

	monthly returns				quarterly returns			
indices	NL	GE	UK	US	NL	GE	UK	US
nominal bonds	72.6	73.4	71.9	93.0	58.2	57.3	47.2	85.6
index-linked bonds	23.0	22.4	13.9	0.1	34.2	32.4	24.9	2.6
stocks	4.4	4.2	14.3	6.9	7.7	10.3	27.9	11.8

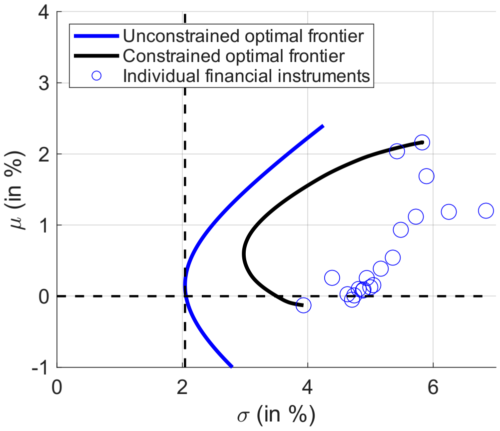
Note: allocations are in percent and exclude borrowing for and short-selling of individual indices. Real returns are obtained by subtracting CPI inflation from the nominal returns.



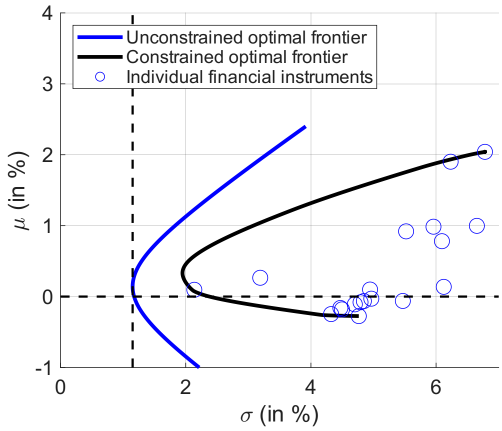
(a) Returns based on Dutch inflation



(b) Returns based on German inflation



(c) Returns based on UK inflation



(d) Returns based on US inflation

Figure 4: Mean-variance frontiers based on quarterly real returns. *Notes: allocations are in percent. Real returns are obtained by subtracting CPI inflation from nominal returns.*

The global minimum-variance portfolio based on U.K. inflation has a larger volatility than those based on Dutch, German and U.S. inflation. The constrained minimum-variance portfolio with the lowest volatility ($\sigma = 110.4$ bps) is based on U.S. inflation. The constrained portfolio based on U.K. inflation again features the highest volatility. The real-term volatilities of the minimum-variance portfolios constructed from the real returns are higher than the nominal-term volatilities of the minimum-variance portfolios based on nominal returns, as shown in Table 6. Most importantly, even though ILBs are part of the portfolios, it is not possible to create portfolios, unconstrained or constrained, that generate returns that are stable in real terms. In other words, inflation risk cannot be hedged away with an appropriate choice of portfolio composition.

Figure 4 depicts the mean-variance frontiers of portfolios based on the quarterly real returns. As before, the volatilities associated with the minimum-variance portfolios, unconstrained or constrained, are strictly positive; see also Table 8.

Table 9 presents the allocations of the constrained minimum-variance portfolios. Contrary to what one might have expected *a priori*, in all instances nominal bond indices, and not ILBs, again dominate the allocations, especially for the U.S. inflation. The weights on nominal bonds vary between 72% and 93% for the monthly real returns and between 47% and 86% for the quarterly real returns. Compared to constrained minimum-variance portfolios based on nominal returns, ILBs see their role increased for the quarterly real returns based on German and Dutch inflation, where they reach about one-third of the portfolio. Remarkably, however, for real returns based on U.S. inflation the minimum-variance portfolios contain hardly any allocation towards ILBs. Stocks feature a higher share in the portfolios based on the real returns for U.K. and the U.S. inflation, while for Dutch and German inflation the shares allocated to stocks are about the same as for the portfolios based on nominal returns.

Finally, Table 8 also reports $\tilde{\mu}$, the expected return on the unconstrained mean-variance frontier at the point where volatility equals that of the constrained minimum-variance portfolio. For the Netherlands, the implicit risk premium attributed to the constraints is $18.5 - 1.8 = 16.7$ bps per month for the monthly returns and $90.0 - 20.2 = 69.8$ bps per quarter for the quarterly returns. If both are brought to the annual basis, these are 200 bps and 279 bps, respectively. For German and U.S. inflation, the figures are similar, while for U.K. inflation, the implicit risk premia are larger than in the other cases, at 22.7 bps and 86.6bps for the monthly, respectively, quarterly returns.

3.3 Sensitivity analysis

This subsection explores to what extent our findings so far are affected by variations on the current setup. Henceforth, we confine ourselves to portfolios based on real returns.

Table 10: Expected return and volatility of minimum-variance portfolios excluding one asset class at a time.

	all instruments		excl. nominal bonds		excl. ILBs		excl. stocks	
	monthly	quarterly	monthly	quarterly	monthly	quarterly	monthly	quarterly
	expected returns (in bps)							
NL unconstrained	-4.7	-14.9	6.1	22.2	-3.2	-7.6	-6.0	-22.3
NL constrained	1.8	20.2	8.7	33.7	1.6	16.0	0.4	9.7
GE unconstrained	-5.4	-14.6	6.1	22.1	-3.8	-7.0	-5.6	-23.3
GE constrained	2.6	26.4	9.6	40.0	2.2	23.6	1.1	11.3
UK unconstrained	-5.9	15.3	14.2	56.0	-1.6	17.9	-8.0	-18.3
UK constrained	8.4	59.5	19.3	82.5	9.0	55.2	2.8	20.5
US unconstrained	0.7	14.2	15.5	58.1	2.2	13.7	-0.8	-3.1
US constrained	7.1	33.1	17.4	61.3	7.1	32.7	3.6	16.2
	volatility in returns (in bps)							
NL unconstrained	76.9	119.7	138.0	229.7	83.5	140.8	78.0	122.7
NL constrained	116.1	209.9	142.9	241.1	118.6	219.4	116.9	214.4
GE unconstrained	74.8	129.8	139.1	233.4	82.0	146.6	75.7	134.8
GE constrained	115.9	217.1	144.2	250.2	118.3	225.1	116.7	225.4
UK unconstrained	133.4	204.2	232.4	321.1	146.2	251.0	136.1	228.8
UK constrained	192.6	297.9	235.3	333.2	193.9	306.0	198.4	337.3
US unconstrained	77.2	115.2	167.3	235.7	82.6	135.8	78.9	122.4
US constrained	110.4	194.8	178.0	271.1	110.4	195.2	113.6	207.1

Note: real returns are constructed by subtracting CPI inflation from the nominal returns. Borrowing and short-selling are excluded.

3.3.1 Excluding one index at a time

In order to assess the contribution of individual asset classes to inflation hedging, we explore to what extent the properties of the minimum-variance portfolios change when we exclude one specific asset class at a time. Table 10 reports the expected returns and volatilities of the corresponding minimum-variance portfolios. The exclusion of the nominal bond indices has the biggest impact: excluding this asset class leads to significantly higher volatility of the minimum-variance portfolios, accompanied by a higher expected return.

3.3.2 Aligned volatilities

The relatively low risk of returns on nominal bond indices (see Table 5) may well explain why minimum-variance portfolios are dominated by this asset class. In order to explore this hypothesis, we counterfactually set the volatility of the real return on each index at the average volatility of the real returns across all indices, while rescaling the average real return on each index, so as to keep the Sharpe ratio at its original value.[9] In this way, the individual riskiness of each index becomes the same while maintaining the diversification benefits of the portfolio.

Figure 5 shows the mean-variance frontiers for the monthly real returns, while the first panel of Table 11 shows the asset allocations of the corresponding minimum-variance portfolios.[10] With the above scaling, the only way to reduce the riskiness of the portfolio is through diversification, as each individual index now has the same volatility. The minimum risk portfolio now contains only about 30% to 40% nominal bond indices. The allocation to ILB indices increases to 30%-45%, except for the monthly real returns based on German and Dutch inflation, where the allocation to ILB indices increases even further to 53%-55%. In addition, the weight of the stock indices increases. The large decrease in the portfolio share of nominal bonds, matched by an almost corresponding increase in the share allocated to ILBs, confirms that the high nominal bond shares that we found earlier are mostly related to the low volatility in the real returns on nominal bonds. If we eliminate this relative advantage of nominal bonds over ILBs, the dominant allocations to nominal bonds in the minimum-variance portfolio disappear, giving greater importance to diversification.

3.3.3 Including additional asset classes

The findings in Section 3.3.2 strongly indicate that diversification plays an important role in reducing portfolio risk. To see whether the diversification benefits can be raised even further, we explore the impact of adding additional asset classes. Specifically, we allow for exposure to global real estate markets and commodities by adding four real estate and three commodity indices to the portfolios.

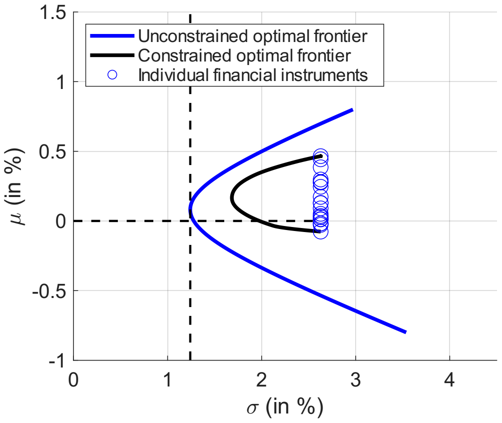
The correlations of the returns of these new indices with the CPI inflation are reported in Table 12. The nominal returns on the commodity indices are positively correlated with inflation in general, while the nominal returns on the real estate indices are positively correlated with US inflation, but weakly and mostly negatively correlated with CPI inflation of the other countries. This is in particular the case for monthly data.

Table 13 reports the average monthly and quarterly returns of these two asset classes, as well as the average standard deviation of the instruments of each of these asset classes. Similarly to the asset classes in Table 5, the average nominal returns are higher than the average real returns because the average inflation is positive in each country. Both the average return and its volatility are higher for real estate than for commodities.

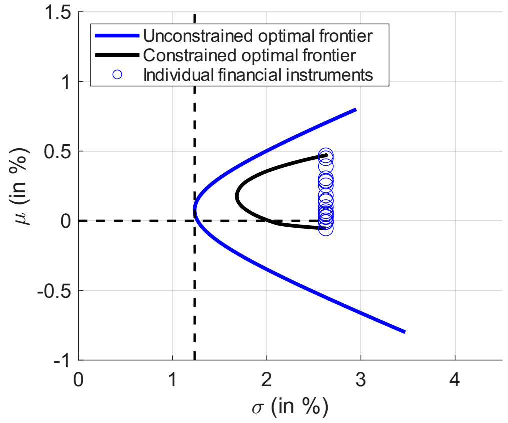
Figure 6 presents the optimal frontiers for monthly real returns when indices of real estate and commodities are included. The allocations of the corresponding constrained minimum-variance portfolio are reported in the second panel of Table

[9]The Sharpe ratio is the ratio of the average return divided by the standard deviation of the return.

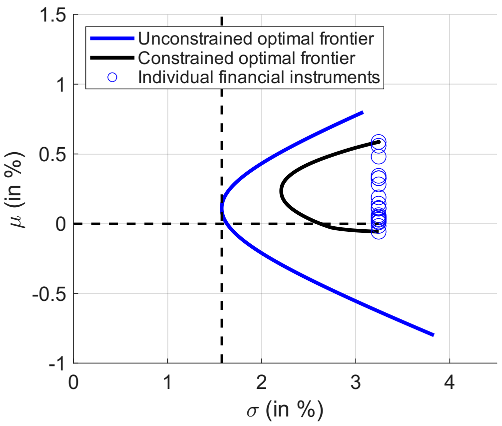
[10]Figure A1 in the Appendix shows the mean-variance frontiers for the quarterly real returns.



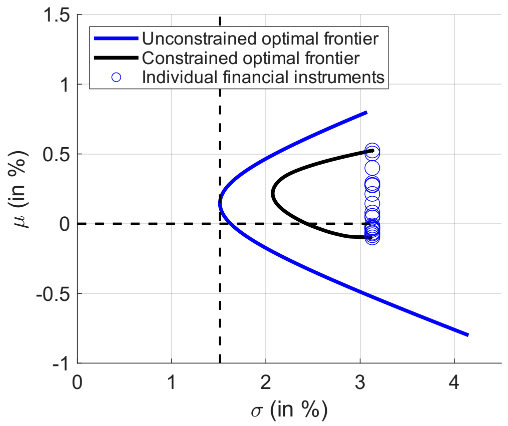
(a) Monthly real returns in terms of Dutch inflation



(b) Monthly real returns in terms of German inflation



(c) Monthly real returns in terms of U.K. inflation



(d) Monthly real returns in terms of U.S. inflation

Figure 5: Mean-variance frontiers based on monthly real returns after scaling the returns to align the volatilities. *Note: Real returns are obtained by subtracting CPI inflation from the nominal returns.*

Table 11: Allocations of the constrained minimum-variance portfolios – sensitivity analysis.

	monthly returns				quarterly returns			
indices	NL	GE	UK	US	NL	GE	UK	US
Section 3.3.2	scaled returns to align the volatilities							
nominal bonds	30.7	29.3	30.1	39.8	34.9	33.3	33.6	28.4
index-linked bonds	52.7	54.9	45.3	31.4	37.6	36.1	30.1	39.1
nominal bonds	16.7	15.8	24.5	28.9	27.5	30.6	36.3	32.5
Section 3.3.3	including real estate and commodities							
nominal bonds	77.0	78.4	76.0	90.5	74.9	74.4	58.0	87.3
index-linked bonds	13.2	11.5	6.4	1.1	14.0	15.5	25.7	2.5
stocks	0.5	0.2	5.1	1.1	2.8	5.7	17.7	2.5
real estate	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
commodities	9.8	10.0	17.5	8.4	11.2	10.0	16.3	10.2
Section 3.3.4	post COVID-19 (01-03-2020 to 18-04-2024)							
nominal bonds	57.1	62.2	51.7	97.4	28.5	17.2	6.8	74.8
index-linked bonds	42.9	37.8	48.2	2.6	63.1	62.9	70.3	23.8
stocks	0.0	0.0	0.0	0.0	8.4	19.9	22.9	1.4

Note: allocations are in percent and exclude short-selling of and borrowing for individual indices. Real returns are obtained by subtracting CPI from nominal returns.

Table 12: Correlations (in %) of the returns on real estate and commodity indices with CPI inflation rates.

		monthly correlations				quarterly correlations			
Index name	ticker	GE	UK	NL	US	GE	UK	NL	US
FTSE EPRA/NAREIT Developed	RNGL	-6.9	-10.2	-1.2	26.6	-1.1	11.3	-4.3	35.0
RMSG: MSCI US REIT Total Return	RMSG	-5.2	-7.9	1.7	20.4	1.3	16.6	-0.0	33.2
STOXX Europe 600 Real Estate	SX86P	-13.4	-13.0	-10.6	23.6	-11.0	-2.5	-18.5	25.9
Dow Jones U.S. Real Estate	DJUSRE	-5.5	-6.8	0.8	20.4	2.2	18.9	1.7	33.5
S&P GSCI Total return	SPGSCITR	23.7	19.0	19.1	65.7	35.8	37.4	42.0	79.8
Bloomberg Commodity Total return	BCOMTR	20.1	15.0	17.3	58.9	30.1	32.4	39.0	74.9
Dow Jones Commodity	DJCIT	19.2	12.4	15.7	61.7	28.3	29.2	35.7	74.8

Note: The top panel contains the correlations with the real estate indices and the bottom panel the correlations with the commodity indices.

Table 13: Summary statistics of returns in basis points for each asset class

	monthly average					quarterly average				
	nominal	real				nominal	real			
type of instruments		GE	UK	NL	US		GE	UK	NL	US
real estate	51.8	34.1	32.1	33.0	27.0	158.3	104.5	86.9	101.4	82.6
commodity	21.5	3.2	0.7	2.3	-3.5	59.1	3.2	-7.6	-0.6	-15.9
	monthly volatility					quarterly volatility				
	nominal	real				nominal	real			
type of instruments		GE	UK	NL	US		GE	UK	NL	US
real estate	583.6	586.4	576.9	585.2	592.2	1029.0	1030.4	932.5	1033.8	1056.9
commodity	530.1	522.5	507.9	523.1	542.8	952.2	923.9	890.8	919.6	975.8

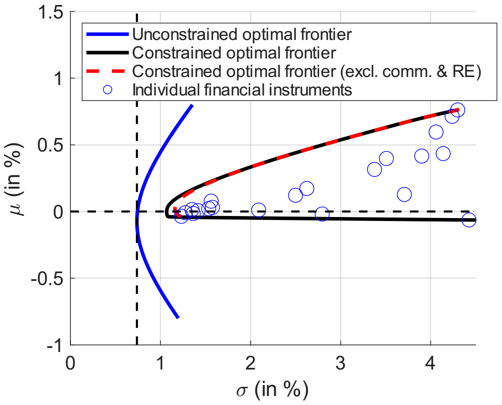
Note: The nominal returns are expressed as percentage changes in euro denominated prices. The real returns are measured as changes in local currency prices adjusted for the corresponding CPI inflation. The average returns and the volatility of the returns of an asset class are measured by taking the averages of the means and the standard deviations across the indices of the asset class.

11.

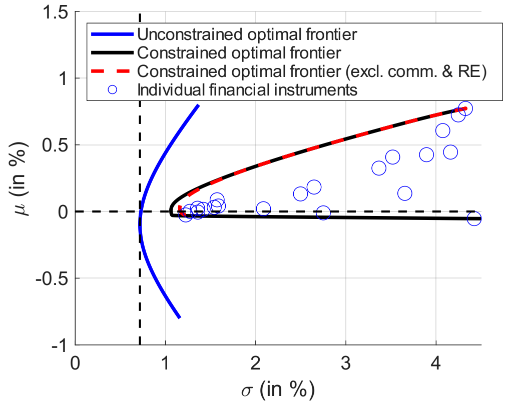
The allocations to the real estate indices are negligible, implying no change in the constrained minimum-variance portfolios. However, allocations to commodity indices range from 8.4% (monthly returns based on U.S. inflation) to 17.5% (monthly returns based on U.K. inflation) and are accompanied by smaller allocations to ILB and stock indices. When comparing these frontiers with the original frontiers excluding commodity and real estate indices, indicated by the red dashed lines, we observe a shift to the left, indicating that with the additional assets the minimum-variance portfolios have lower volatility. For monthly returns, the reduction in volatility ranges from 9 to 22 bps. The Appendix shows the results based on quarterly returns in Figure A2, where the reduction in volatility varies from 18 to 34 bps. A separate analysis, not explicitly reported, with only real estate included as an additional asset class does not result in reductions in the minimum variance. The conclusion is clear: adding real estate exposure does not yield additional diversification benefits, but adding commodity indices does help to further mitigate inflation risk.

3.3.4 The post COVID-19 crisis period

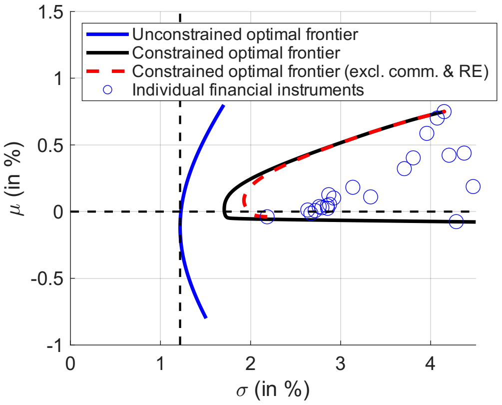
A priori, we would expect ILBs to be a particularly attractive asset class during periods of high inflation, such as that in the aftermath of the COVID-19 crisis. Therefore, we focus in this section on the subsample period since March 1, 2020. Figure 7 shows the corresponding mean-variance frontiers for the monthly real returns, while the corresponding asset class allocations are reported in the third panel of Table 11. In the case where real returns are based on U.K. CPI inflation,



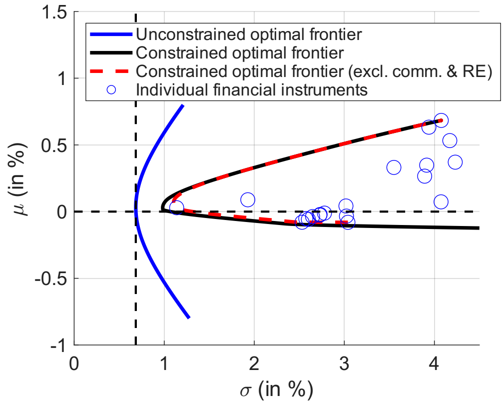
(a) Monthly real returns in terms of Dutch inflation



(b) Monthly real returns in terms of German inflation



(c) Monthly real returns in terms of U.K. inflation



(d) Monthly real returns in terms of U.S. inflation

Figure 6: Mean-variance frontiers based on monthly real returns when real estate and commodity indices are included. *Note: Real returns are obtained by subtracting CPI inflation from nominal returns.*

the risks associated with the constrained minimum-variance portfolios are lower than for the full sample period. However, the lower minimum risk comes at a substantial cost in terms of the expected return, which in all cases turns from positive to negative. Compared to the full sample, the asset allocations of the minimum-variance portfolios exhibit a substantial shift to ILB indices. For quarterly returns, except for U.S. inflation, ILBs are now the dominant class. Overall, these findings suggest that during periods of high inflation ILBs provide substantial, though still imperfect, inflation protection.

4 Evaluation of long-term inflation hedging through a variable annuity

So far, we have constructed optimal minimum-variance portfolios based on their performance over a short period of time, that is, a month or a quarter. This section makes optimal portfolio decisions more realistic in two directions.

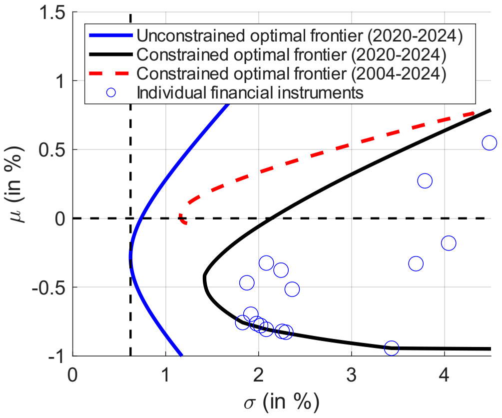
First, many investors tend to stay in the market for substantially longer, which requires evaluating performance over a longer time horizon. Portfolios that may be optimal in terms of the trade-off between risk and expected return over a short period may no longer be so when evaluated over a long horizon. Moreover, changes in inflation take time to affect nominal interest rates, while compensation for inflation through ILBs usually occurs with lags. For example, for U.S. TIPs the principal sum is uprated every half year with realized inflation over that half year. Through these lags, optimal portfolio compositions may be affected by the investment horizon, which compels us to analyze optimal portfolio compositions over a correspondingly longer evaluation period. Specifically, we analyze the performance of a variable annuity over a simulation period of 20 years. The length of the simulation period is roughly in line with the time the average worker can be expected to be in retirement.

Second, a variable annuity can be considered a financial product for people who enter retirement and face a trade-off between uncertainty in purchasing power and expected increases in purchasing power. Hence, we also no longer confine ourselves to minimum-variance portfolios, which result from extreme risk aversion, but consider portfolios that maximize utility of individual investors with finite risk aversion.

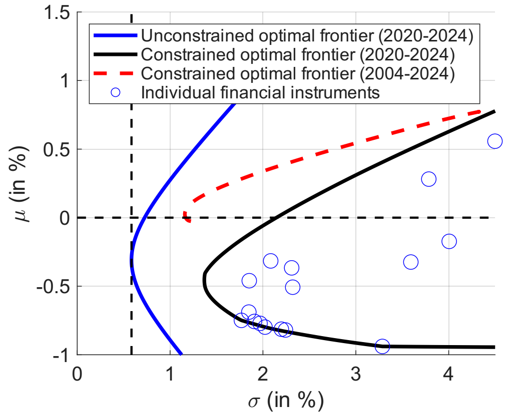
Overall, these changes allow us to evaluate the trade-off between risk and expected return over a long investment horizon.

4.1 Estimation

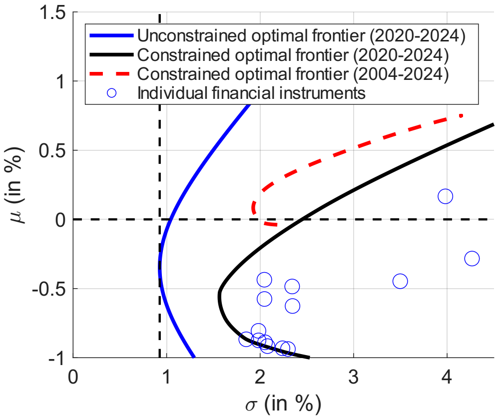
We jointly estimate the nominal returns of the $n = 20$ financial indices and the inflation rates. The nominal returns are modeled as:



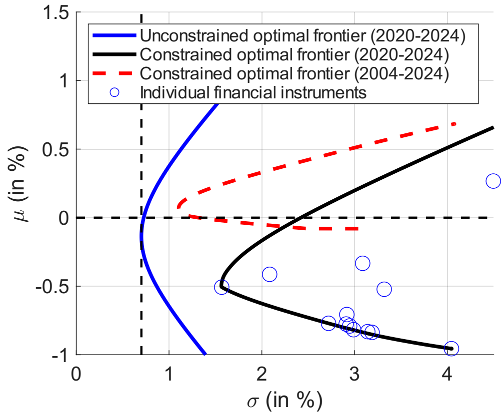
(a) Monthly real returns in terms of Dutch inflation



(b) Monthly real returns in terms of German inflation



(c) Monthly real returns in terms of U.K. inflation



(d) Monthly real returns in terms of U.S. inflation

Figure 7: Mean-variance frontiers based on monthly real returns over the post COVID-19 period. *Notes: Real returns are obtained by subtracting CPI inflation from the nominal returns. Sample period runs from 1 March 2020 to 18 April 2024.*

$$r^n_{i,t} = \mu_i + \sigma_i \varepsilon_{i,t}. \quad (5)$$

where the parameter μ is the same as that used in Section 2.3 to calculate the mean-variance frontiers based on the nominal returns. For each country, the inflation rate is assumed to obey the following process:

$$\pi_t = \alpha_0 + \sum_{l=1}^{p} \alpha_l \pi_{t-l} + \sigma_\pi \varepsilon_{\pi,t}. \quad (6)$$

This is a standard AR(p) approach that is traditionally used in the literature to model inflation, e.g. Stock and Watson (1999, 2007). Rygh et al. (2025) investigate the effectiveness of more advanced models, such as neural network models, and find that traditional models, including AR(p)-models, may offer more practical and accurate solutions. The long-run mean estimate is given by $\hat{\mu_\pi} = \frac{\hat{\alpha}_0}{1-\sum_{l=1}^{p} \hat{\alpha}_l}$.

As a final step in the estimation for each country, we construct the $(n+1) \times (n+1)$ covariance matrix Σ_ε of the residuals of the estimated processes of nominal returns ($\varepsilon_{i,t},\ i \in \{1, ..., n\}$) and inflation ($\varepsilon_{\pi,t}$). From this matrix, we obtain the volatility of each instrument σ_i, the volatility of inflation rates σ_π and the correlations.[11]

For each country, the estimates are reported in Table 14.[12] Return numbers apply to the time space between observations. The long-term annualized mean based on the full sample is around 3%. Estimates based on the period prior to COVID-19 indicate a lower average inflation rate (μ_π) and lower volatility of inflation rates (σ_π) than we obtain over the period after COVID-19. Not surprisingly, the estimates based on the full sample period lie in between the sub-sample estimates. The sum of the estimated coefficients of the lags varies from 0.38 to 0.82 based on the full sample period. This implies a stationary inflation process with moderate persistence. The model for quarterly U.S. inflation achieves a relatively low explanatory power ($R^2 = 10\%$), while the other models are able to explain 30% to 50% of the variation.

4.2 Simulation

Using parameter estimates and the variance-covariance matrix, we simulate $Q = 10,000$ scenarios of nominal returns and inflation rates. For consistency, the length

[11] The covariances between the asset returns and inflation in the 21-by-21 variance-covariance matrix are based on the empirical co-variances of the relevant residuals.

[12] Table A6 in the Appendix shows more details on these estimates, by presenting the estimated coefficients of all individual lags and the corresponding standard errors.

Table 14: Estimates of the parameters of the inflation process.

estimates	monthly ($\delta = \frac{1}{12}$) NL	GE	UK	US	quarterly ($\delta = \frac{1}{4}$) NL	GE	UK	US
	full sample period: 20-09-2004 to 18-04-2024							
$\hat{\alpha}_0$	0.07	0.04	0.06	0.07	0.23	0.16	0.30	0.40
$\sum_{l=1}^{p} \hat{\alpha}_l$	65.33	81.99	79.93	65.58	61.72	73.88	65.86	37.79
$\hat{\mu}_\pi$	0.20	0.20	0.28	0.21	0.61	0.62	0.88	0.64
$\hat{\sigma}_\pi$	0.34	0.26	0.31	0.20	0.70	0.65	0.77	0.72
R^2	45.95	49.03	41.42	52.89	25.36	37.43	33.77	9.94
	pre-COVID-19: 20-09-2004 to 01-03-2020							
$\hat{\alpha}_0$	0.10	0.06	0.10	0.12	0.35	0.21	0.48	0.64
$\sum_{l=1}^{p} \hat{\alpha}_l$	26.68	46.52	57.63	22.23	10.34	41.11	32.78	-31.19
$\hat{\mu}_\pi$	0.13	0.12	0.23	0.15	0.39	0.36	0.72	0.49
$\hat{\sigma}_\pi$	0.21	0.24	0.26	0.20	0.45	0.49	0.57	0.65
R^2	60.58	47.26	37.30	46.17	38.63	37.87	15.62	6.03
	post-COVID-19: 01-03-2020 to 18-04-2024							
$\hat{\alpha}_0$	0.37	0.22	0.12	0.13	1.36	1.37	1.02	1.14
$\sum_{l=1}^{p} \hat{\alpha}_l$	18.75	51.33	74.95	69.31	2.95	5.14	40.80	16.30
$\hat{\mu}_\pi$	0.45	0.46	0.48	0.42	1.40	1.44	1.73	1.36
$\hat{\sigma}_\pi$	0.52	0.26	0.39	0.17	0.85	0.85	1.03	0.70
R^2	46.83	62.57	55.94	67.98	42.52	40.67	55.53	26.33

Note: all values are multiplied by 100. The parameter estimates are from the following inflation process: $\pi_t = \alpha_0 + \sum_{l=1}^{p} \alpha_l \pi_{t-l} + \sigma_\pi \varepsilon_{\pi,t}$. *The estimated long-run mean is given by:* $\hat{\mu}_\pi = \frac{\hat{\alpha}_0}{1-\sum_{l=1}^{p} \hat{\alpha}_l}$. *The monthly data assumes order* $p = 12$ *and the quarterly data assumes order* $p = 4$.

of each period used in the scenario simulations is aligned with the frequency at which the data are estimated. Specifically, scenarios simulated at the monthly frequency use returns estimated at the monthly frequency, while scenarios simulated at the quarterly frequency use returns estimated at the quarterly frequency. Hence, differences in the optimal portfolio allocations and the welfare gains between the monthly and quarterly simulations can therefore be traced back to differences in the estimations at the two frequencies. In particular, the estimated returns processes and correlations between the indices differ substantially between the monthly and quarterly data, as documented in Table 3 and Table 4.

We simulate scenarios over a period of $T * \delta = 20$ years. That is, for the monthly (quarterly) simulation for each scenario, 240 (80) sets of shocks for the 20 assets and the inflation process are drawn using the matrix of residuals. These are substituted into the estimated return and inflation rate processes to simulate time paths for these variables. To simulate a time series for π_t using Eq.(6) we set the initial values π_{1-p} to π_0. We assume that these initial values in all cases are equal to the estimated long-term mean value $\hat{\mu_\pi}$. Finally, we obtain the real returns on the assets as $1 + r_{i,t} = \frac{1+r^n_{i,t}}{1+\pi_t}$.

4.3 Evaluation of welfare effects

For each scenario q, we evaluate a time-varying annuity that pays a constant benefit in each period based on a fixed discount factor β. Assume a fund $W_{q,t}$ from which the annuity is paid each period:

$$b_{q,t} = W_{q,t} / \sum_{s=t}^{T} \beta^{(T-s)\delta}, \text{ for } t \in \{0, 1, 2, ..., T\}. \tag{7}$$

for monthly simulations ($T = 20/\delta = 240$) and for quarterly simulations ($T = 20/\delta = 80$). The fund then evolves in scenario q with the vector of simulated real returns $r_{q,t}$ over the simulation period as:

$$W_{q,0} = 1, \tag{8}$$
$$W_{q,t+1} = (W_{q,t} - b_{q,t}) * (1 + \omega' r_{q,t+1}), \text{ for } t \in \{0, 1, 2, ..., T-1\}. \tag{9}$$

For simplicity, we do not consider dynamic portfolios. This is also in line with a commonly-shared strategy followed by long-term investors, such as pension funds, that tend to allocate fixed fractions or ranges of their portfolios to the different asset categories. Hence, we evaluate portfolios with fixed allocations ω assuming that an individual investor values annuity benefits according to the utility function $u(.)$ with a constant relative risk aversion parameter γ, i.e., $u(b) = \frac{b^{1-\gamma}}{1-\gamma}$, and with a time discount factor (β):

$$U(\omega,\gamma) \equiv \frac{1}{Q}\sum_{q=1}^{Q}\sum_{t=0}^{T}\beta^{t\delta}u(b_{q,t}(\omega)) \tag{10}$$

The implied certainty equivalent benefit (CE) is:

$$CE(\omega,\gamma) = \left(\frac{(1-\gamma)U(\omega,\gamma)}{\sum_{t=0}^{T}\beta^{t\delta}}\right)^{1/(1-\gamma)} \tag{11}$$

With a numerical solver, we can find the portfolio allocation ω_γ^* with the highest certainty-equivalent benefit, given γ. As a benchmark, we take the constrained minimum-risk portfolio based on real returns derived in Section 2.3, which we denote by ω_m, and calculate the welfare gain relative to this benchmark as:

$$Gain(\gamma) = \frac{CE(\omega_\gamma^*,\gamma)}{CE(\omega_m,\gamma)} - 1. \tag{12}$$

4.4 Results

We set the time discount factor β equal to $\frac{1}{1.03}$ in line with the risk-free rate of European swap rates for maturities of 10 to 30 years as of November 2025. First, we examine the optimal portfolio for an investor with extremely high risk aversion ($\gamma = 100$). While this level of risk aversion is not very realistic, it is nevertheless informative, as it highlights the characteristics of a portfolio when the investor seeks to hedge inflation risk as effectively as possible. Second, we consider lower values of the constant relative risk aversion parameter, reducing γ from 100 to 5.

We then compare the resulting welfare gains relative to the minimum-variance portfolios. For the highest level of risk aversion, the welfare gain primarily reflects the benefit of adopting a long-run optimal portfolio that closely approximates the minimum-risk portfolio. For lower levels of risk aversion, the welfare gains capture two components: (i) the benefit of adopting a multi-horizon perspective, and (ii) the gains from taking on a risk-return trade-off consistent with the investor's degree of risk aversion.

Table 15 reports the constrained portfolio allocations across asset classes that maximize welfare, while Table 16 presents the associated welfare gains for these optimal constrained portfolios.

4.4.1 Portfolio implications under extreme risk aversion

With extreme risk aversion ($\gamma = 100$), the optimal portfolio mainly allocates to equity indices and ILB indices. One exception arises in the case of monthly real returns in the U.S., where the optimal portfolio consists largely of nominal bonds alongside a low allocation to ILBs. Overall, these portfolios are well diversified.

Compared to the one-period minimum-variance portfolios (see Table 9), the long-horizon perspective leads to a noticeably higher allocation to ILBs and equities. Given these substantial differences in portfolio composition, the associated welfare gains are also considerable. Specifically, the welfare gains from adopting a multi-period optimal portfolio range from 25% (quarterly real returns in the U.S.) to 80% (monthly real returns in the U.K.).

4.4.2 Portfolio implications under varying risk aversion

With lower risk aversion, the optimal portfolios adjust as the investor trades off risk against expected return. When the constant relative risk aversion parameter γ decreases from 100 to 5, expected real returns become increasingly important relative to the volatility of the real returns.

The portfolio composition remains largely stable when γ varies between 20 and 100. However, as γ falls from 20 to 5, allocations to equities increase, while allocations to nominal bonds and ILBs decrease. For relatively low risk aversion ($\gamma = 5$), the optimal portfolio is heavily skewed toward stocks, although a substantial allocation towards ILBs remains, typically larger than in the one-period minimum-variance portfolios (see Table 9).

The welfare gains for the lowest level of risk aversion considered ($\gamma = 5$) range from 36% (quarterly real returns in the U.S.) to 61% (monthly real returns in Germany). These gains reflect both the benefits of adopting a multi-period perspective and selecting a portfolio that achieves the optimal risk-return trade-off.

Overall, the welfare gains are particularly high for investors with either very high risk aversion ($\gamma \geq 50$), due to the strong penalty for sub-optimal portfolios, or low risk aversion ($\gamma = 5$), as the investor favors more aggressive portfolios than the minimum-variance benchmark.

5 A framework to rationalize portfolio allocations

Our empirical results indicate a limited hedging effectiveness of inflation-linked bonds (ILBs), especially in the period before COVID-19, when inflation rates were relatively low and stable. In this section, we propose a possible explanation for the limited role of ILBs in minimum-variance portfolios. In addition, we provide intuition for the resulting portfolio allocations when risk aversion is finite and investment horizons are long. A key element is the presence of real basis risk, which arises from market imperfections in hedging inflation. This real basis risk reflects the limited availability of ILBs, illiquidity in their markets, and other imperfections. For example, not all desired maturities of ILBs are available, and for some inflation processes no ILBs exist at all. In particular, there is no ILB linked to Dutch inflation. As a result, investors seeking to hedge Dutch inflation

Table 15: Optimal portfolio allocations (in %) for the variable annuity including constraints that prevent borrowing and short-selling.

asset class	monthly returns				quarterly returns			
$\gamma = 100$	NL	GE	UK	US	NL	GE	UK	US
nominal bonds	10.4	2.2	5.0	65.5	22.4	25.6	0.1	0.2
index-linked bonds	46.8	42.6	39.6	4.0	60.4	58.0	31.3	44.7
stocks	42.8	55.2	55.4	30.5	17.2	16.3	68.6	55.2
$\gamma = 50$	NL	GE	UK	US	NL	GE	UK	US
nominal bonds	7.4	2.0	8.0	65.8	22.1	25.4	0.1	0.1
index-linked bonds	49.5	43.4	36.4	2.4	58.4	55.9	31.5	47.0
stocks	43.1	54.5	55.5	31.8	19.4	18.7	68.4	52.8
$\gamma = 20$	NL	GE	UK	US	NL	GE	UK	US
nominal bonds	4.2	1.5	13.9	63.0	17.9	24.9	0.1	5.3
index-linked bonds	54.7	48.8	28.8	1.9	54.0	48.0	28.7	45.9
stocks	41.2	49.7	57.3	35.1	28.2	27.1	71.2	48.8
$\gamma = 10$	NL	GE	UK	US	NL	GE	UK	US
nominal bonds	2.9	1.4	13.8	37.6	1.4	8.6	0.1	0.4
index-linked bonds	51.4	50.3	24.3	14.5	48.9	42.2	22.2	43.5
stocks	45.7	48.2	61.9	47.9	49.7	49.2	77.6	56.1
$\gamma = 5$	NL	GE	UK	US	NL	GE	UK	US
nominal bonds	1.1	1.0	0.7	0.7	0.2	0.1	0.1	0.1
index-linked bonds	22.6	21.9	13.0	20.2	18.5	15.6	7.2	16.6
stocks	76.2	77.1	86.3	79.1	81.3	84.3	92.7	83.3

Note: Simulations are based on real returns. Allocations are in percent.

Table 16: Welfare gain compared to the minimum-variance one-period portfolios including constraints that prevent borrowing and short-selling.

	monthly returns				quarterly returns			
γ	NL	GE	UK	US	NL	GE	UK	US
100	46.0	52.2	80.1	39.6	27.9	28.9	34.9	25.2
50	44.4	50.8	76.8	36.5	26.5	26.8	37.2	24.8
20	42.4	48.6	65.8	26.2	25.9	25.8	43.0	23.1
10	48.3	55.1	58.0	27.1	41.2	43.6	45.4	28.1
5	57.1	61.3	55.6	37.3	55.3	58.9	42.7	36.1

Note: numbers are expressed in percent. Welfare gains compare the certainty-equivalent benefit from the optimal portfolio (ω^*) with the certainty-equivalent benefit from the minimum-variance portfolio (ω_m).

can only rely on ILBs linked to foreign, non-Dutch CPI indices, which introduces imperfect inflation hedging.

5.1 Theoretical framework

We assume that the CPI evolves as:

$$d\Pi_t/\Pi_t = \mu_\pi dt + \sigma_\pi dZ_{\pi,t}. \tag{13}$$

We consider three assets. The first is a nominal bond, which serves as the market observable benchmark. It features a fixed nominal rate of return r:

$$dB_t^N/B_t^N = rdt. \tag{14}$$

The second asset is an index-linked bond. In an ideal market, this bond would perfectly hedge inflation. However, we allow for real basis risk, denoted by η, capturing the fact that ILBs do not perfectly hedge the relevant inflation risk. This reflects, for example, that ILBs may be linked to foreign CPI indices rather than the investor's domestic inflation, leading to imperfect inflation hedging. It also captures additional market frictions in inflation hedging, including limited ILB availability, market illiquidity and the absence of bonds at specific maturities.

The ILB nominal return consists of three components:

$$dB_t^R/B_t^R = (r - \mu_\pi)dt + d\Pi_t/\Pi_t + d\eta_t \tag{15}$$
$$= rdt + \sigma_\pi dZ_{\pi,t} + d\eta_t \tag{16}$$

The term $d\eta_t$ represents a stochastic "spread" between the nominal benchmark and the real asset. It captures the aforementioned imperfections and evolves as:

$$d\eta_t = \mu_\eta dt + \sigma_\eta dZ_{\eta,t}. \tag{17}$$

The parameter σ_η represents the uncertainty regarding the basis risk (i.e., the volatility of the spread). Consequently, the return on the index-linked bond is:

$$dB_t^R/B_t^R = (r + \mu_\eta)dt + \sigma_\pi dZ_{\pi,t} + \sigma_\eta dZ_{\eta,t}. \tag{18}$$

Finally, the third asset is a stock with nominal return, also following a geometric Brownian motion:

$$dS_t/S_t = \mu_S dt + \sigma_S dZ_{S,t}. \tag{19}$$

A representative agent has initial wealth W_0 and chooses portfolio allocations to maximize the expected utility of terminal wealth real wealth:

$$\max_{0 \le \omega_i \le 1, i \in \{S,R,N\}} E[\beta^T U(W_T/\Pi_T)]. \tag{20}$$
$$\text{s.t. } W_0 = 1 \tag{21}$$
$$dW_t/W_t = \omega_S \frac{dS_t}{S_t} + \omega_N \frac{dB_t^N}{B_t^N} + \omega_R \frac{dB_t^R}{B_t^R} \tag{22}$$
$$\omega_S + \omega_N + \omega_R = 1. \tag{23}$$

where W_T is terminal nominal wealth, and where we exclude short-selling and asset positions larger than total wealth. Using the above assumptions, we have:

$$dW_t/W_t = (r + \omega_S(\mu_S - r) + \omega_R \mu_\eta)dt + \omega_S \sigma_S dZ_{S,t} + \omega_R(\sigma_\pi dZ_{\pi,t} + \sigma_\eta dZ_{\eta,t}) \tag{24}$$

Applying Itô's Lemma to $f(\Pi_t) = 1/\Pi_t$ yields:

$$\frac{d(1/\Pi_t)}{1/\Pi_t} = (\sigma_\pi^2 - \mu_\pi)dt - \sigma_\pi dZ_{\pi,t} \tag{25}$$

Hence, the process for the return on real wealth ($w_t \equiv W_T/\Pi_t$) is as follows:

$$dw_t/w_t = (r - \mu_\pi + \omega_S(\mu_S - r) + (1-\omega_R)\sigma_\pi^2 + \omega_R\mu_\eta)dt \tag{26}$$
$$- (\omega_S\sigma_S\sigma_\pi\rho_{S,\pi} + \omega_R\sigma_\eta\sigma_\pi\rho_{\eta,\pi})dt \tag{27}$$
$$+ \omega_S\sigma_S dZ_{S,t} + (\omega_R - 1)\sigma_\pi dZ_{\pi,t} + \omega_R\sigma_\eta dZ_{\eta,t}, \tag{28}$$

where correlations between the Brownian motions are given by $\rho_{i,j} = corr(dZ_i, dZ_j)$ for $i, j \in \{S, \pi, \eta\}$.

With constant relative risk aversion (CRRA) utility and using $w_T/w_0 = \exp(\ln w_T - \ln w_0)$ and $E(d\ln w_t) = [E(dw_t/w_t) - \frac{1}{2}Var(dw_t/w_t)]dt$, we obtain:

$$E[U(w_T)]/U(w_0) = E[\exp((1-\gamma)((\ln w_T - \ln w_0))] \tag{29}$$
$$= E[\exp((1-\gamma)(\int_0^T E(dw_t/w_t) - \frac{\gamma}{2}Var(dw_t/w_t)dt))] \tag{30}$$

Next we solve analytically for the unconstrained asset allocation. As our stylized setup is equivalent to optimizing the mean-variance tradeoff, we can simplify our optimization problem analogously to Merton's optimal portfolio problem for CRRA utility (Merton, 1975):

$$\arg\max E[\beta^T U(W_T/\Pi_T)] = \arg\max[E(dw_t/w_t) - \frac{\gamma}{2}Var(dw_t/w_t)] \tag{31}$$

This implies that we need to solve:

$$\max \quad \omega_S(\mu_S - r) + (1-\omega_R)\sigma_\pi^2 + \omega_R\mu_\eta - \omega_S A - \omega_R C - ... \tag{32}$$
$$...\frac{\gamma}{2}(\omega_S^2\sigma_S^2 + (\omega_R - 1)^2\sigma_\pi^2 + \omega_R^2\sigma_\eta^2) - ... \tag{33}$$
$$...\gamma(\omega_S(\omega_R - 1)A + \omega_S\omega_R B + \omega_R(\omega_R - 1)C). \tag{34}$$

where $A = \sigma_S\sigma_\pi\rho_{S,\pi}$, $B = \sigma_S\sigma_\eta\rho_{S,\eta}$ and $C = \sigma_\eta\sigma_\pi\rho_{\eta,\pi}$. In Appendix A.3 we show that this leads to the following optimal portfolio allocations:

$$\omega_S^* = \frac{\mu_S - r - A}{\gamma\sigma_S^2} + \frac{A - \omega_R^*(A+B)}{\sigma_S^2} \tag{35}$$

$$\omega_R^* = \frac{\mu_\eta - \sigma_\pi^2 - C}{\gamma(\sigma_\pi^2 + \sigma_\eta^2)} + \frac{\sigma_\pi^2 - \omega_S^*(A+B) + (1 - 2\omega_R^*)C}{\sigma_\pi^2 + \sigma_\eta^2} \tag{36}$$

To provide some intuition for these expressions, consider the case in which all risk factors are mutually uncorrelated, i.e. $\rho_{S,\pi} = \rho_{S,\eta} = \rho_{\eta,\pi} = 0$, which implies $A = B = C = 0$. Then, these expressions simplify to:

$$\omega_S^* = \frac{\mu_S - r}{\gamma\sigma_S^2} \tag{37}$$

$$\omega_R^* = \frac{\mu_\eta + (\gamma - 1)\sigma_\pi^2}{\gamma(\sigma_\pi^2 + \sigma_\eta^2)} \tag{38}$$

The first line shows the well-known optimal stock allocation found by Merton (1975). The expression in the second line shows that even in the absence of real basis risk, i.e. $\mu_\eta = 0$, the optimal allocation to index-linked bonds is positive when risk aversion is greater than 1.

5.2 Rationalizing portfolio compositions

In the following we explore how far this simple setup can get us in replicating both the one-period minimum-variance portfolio allocations constructed earlier, corresponding to the extreme case of $\gamma \to \infty$, as well as the portfolio allocations resulting from our simulation exercise in the previous section in which we allowed for finite degrees of risk aversion and a long-term investment horizon.

The process for η is central here: as σ_η increases, the ILB becomes a less reliable instrument for hedging inflation risk, because the difference between the hedging instrument ILB and the evolution of inflation ~~the inflation target~~ becomes more uncertain.

Throughout, we assume that the volatility of inflation is equal to the average of the volatilities of our annualized inflation rates, i.e. $\sigma_\pi = 2.5\%$. Furthermore, the standard deviation of stocks is set at the average of the standard deviations of the returns on our three stock indices, i.e., $\sigma_S = 12.5\%$.

5.2.1 Replicating minimum-variance portfolio compositions

First, we turn to the composition of the minimum-variance portfolios. In Section 3.2 we showed that stock allocations range from 4% to 28% (and 11% on average) and ILB allocations from 0% to 34% (and 19% on average) in the minimum-variance portfolios. With investors interested only in hedging risk, i.e. $\gamma \to \infty$, the theoretical portfolio weights of stocks and indexed debt reduce to:

$$\omega_S^* = \frac{A - \omega_R^*(A+B)}{\sigma_S^2} \tag{39}$$

$$\omega_R^* = \frac{(\sigma_\pi^2 + C)\sigma_S^2 - A(A+B)}{(\sigma_\pi^2 + \sigma_\eta^2 + 2C)\sigma_S^2 - (A+B)^2} \tag{40}$$

We use these analytical expressions as well as a numerical solver to obtain the results for both interior and boundary solutions.

Our data show that the average correlation between the quarterly returns on the three stock indices and inflation varies from 0.3% to 34%, with an average of 12%.[13] Hence, we assume a positive correlation between the risk factors for the inflation rate and the returns on stocks, i.e. $\rho_{S,\pi} \geq 0$.

Zero correlations with real basis risk

For simplicity, we first consider the case where real basis risk is uncorrelated with both inflation ($\rho_{\eta,\pi} = 0$) and stock returns ($\rho_{S,\eta} = 0$). Relaxing these assumptions would create more degrees of freedom to replicate the asset allocations in the minimum-variance portfolios. However, little is known about the correlations of the real basis risk with these other risk factors, hence later on we will vary these correlations and explore how this affects portfolio allocations. The assumptions that $\rho_{S,\eta} = \rho_{\eta,\pi} = 0$ imply $B = C = 0$. The resulting weights of the minimum-variance portfolios are:

$$\omega_S^* = (1 - \omega_R^*)\rho_{S,\pi}\frac{\sigma_\pi}{\sigma_S} \tag{41}$$

$$\omega_R^* = 1 - \frac{\sigma_\eta^2}{\sigma_\pi^2(1-\rho_{S,\pi}^2) + \sigma_\eta^2} \tag{42}$$

The middle column of Figure 8 shows the optimal allocations as a function of the volatility of the basis risk (σ_η) for different correlations between inflation and stock returns $\rho_{S,\pi}$.

With zero basis risk ($\sigma_\eta = 0\%$), the ILB is a perfect hedge and it is optimal to invest the complete portfolio in ILBs. However, raising the volatility of the basis risk ($\sigma_\eta > 0$) produces a decreasing optimal share of ILBs. This is intuitive: as

[13] For Dutch inflation, the average correlation with stock indices is 0.3%. For German, U.K. and U.S. inflation, the average correlations are 4.7%, 8.2% and 34.3%, respectively.

basis risk increases, ILBs trade a reduction in inflation risk in the portfolio for an increase in more basis risk in the portfolio.

As the middle column of Figure 8 shows, the allocation to stocks is relatively small with a zero correlation between stock returns and the real basis risk, since the hedging properties of stocks are limited. Yet, this allocation increases with the correlation $\rho_{S,\pi}$ between inflation and stocks. The optimal allocation to stocks converges to $\rho_{S,\pi}\frac{\sigma_\pi}{\sigma_S}$ in the limit for $\sigma_\eta \to \infty$. The highest correlation between inflation and stocks that we consider in Figure 8 is 0.5, which is already quite large given that the highest correlation in the data (between US CPI and MSCI World) is 38%. Under this assumption, the highest possible allocation to stocks is only 10%.[14]. Hence, to generate higher allocations to stocks we need to relax our assumption of a zero correlation between the basis risk and other risk factors.

Non-zero correlation between inflation rate and basis risk

We next vary the correlation between basis risk and the inflation process from $\rho_{\eta,\pi} = -0.5$ to $\rho_{\eta,\pi} = 0.5$ – see Figure 8. Raising the correlation $\rho_{\eta,\pi}$ leads to a higher allocation towards index-linked bonds when real basis risk σ_η is sufficiently large. An increase in the share of ILBs reduces inflation risk in the investment portfolio, but it also injects basis risk into the portfolio. A higher correlation $\rho_{\eta,\pi} = 0.5$ implies that, for given σ_η, a larger fraction of the basis risk that is injected is effectively hedged away.

Based on the allocations in Figure 8, we argue that the correlation between stock returns and inflation should be at least 0.2 ($\rho_{S,\pi} > 0.2$) to align with the stock allocations obtained in Section 3.2. This exceeds the average correlation from our data, but is still well within the empirically observed range. Moreover, the volatility of the real basis risk should be at least 2% ($\sigma_\eta > 2\%$) to match the portfolio weights allocated to ILBs.

Non-zero correlation between stock return and basis risk

Figure 9 presents the optimal minimum-variance portfolio weights with a positive correlation between stock returns and the basis risk ($\rho_{S,\eta} = 0.5$), while Figure 10 analogously assumes a negative correlation $\rho_{S,\eta} = -0.5$. In Figure 9 allocations towards stocks are lower than before, because the stock returns move in tandem with the basis risk, making stocks less attractive in diversifying away basis risk. The opposite is the case in Figure 10. The combination of ILBs and stocks in the portfolio becomes more attractive, as stocks are effective in diversifying away basis risk. This becomes more valuable when the volatility of basis risk rises. Hence, the portfolio fraction allocated to stocks is increasing in basis risk.

[14]This is calculated as $\rho_{S,\pi}\frac{\sigma_\pi}{\sigma_S} = 0.5 * 0.025/0.125 = 10\%$.

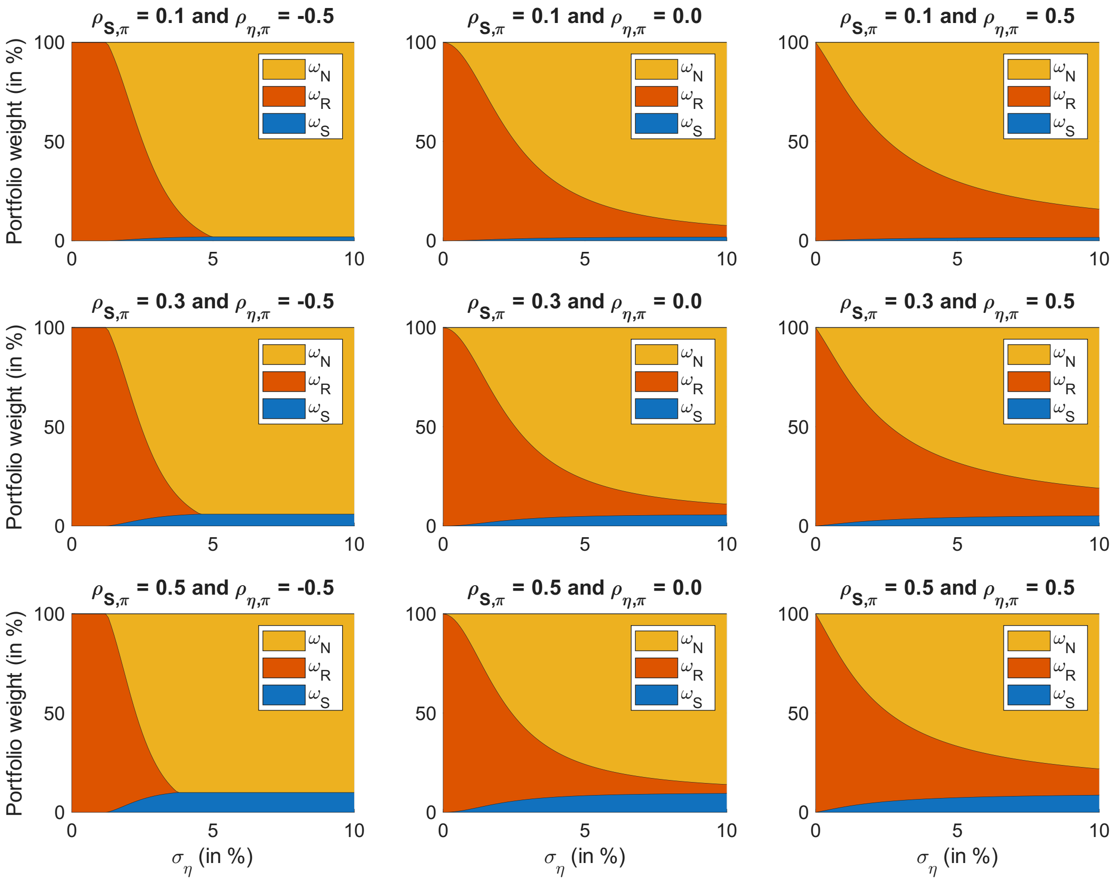


Figure 8: Optimal portfolio weights for different volatilities of real basis risk (σ_η), different correlations between stock returns and inflation ($\rho_{S,\pi}$) and different correlations between real basis risk and inflation $\rho_{\eta,\pi}$, for a correlation between stock returns and real basis risk of $\rho_{S,\eta} = 0$. *Note: the portfolio weights add up to 100%, where ω_N refers to the nominal bond weight, ω_R to the index-linked bond weight and ω_S to the stock weight.*

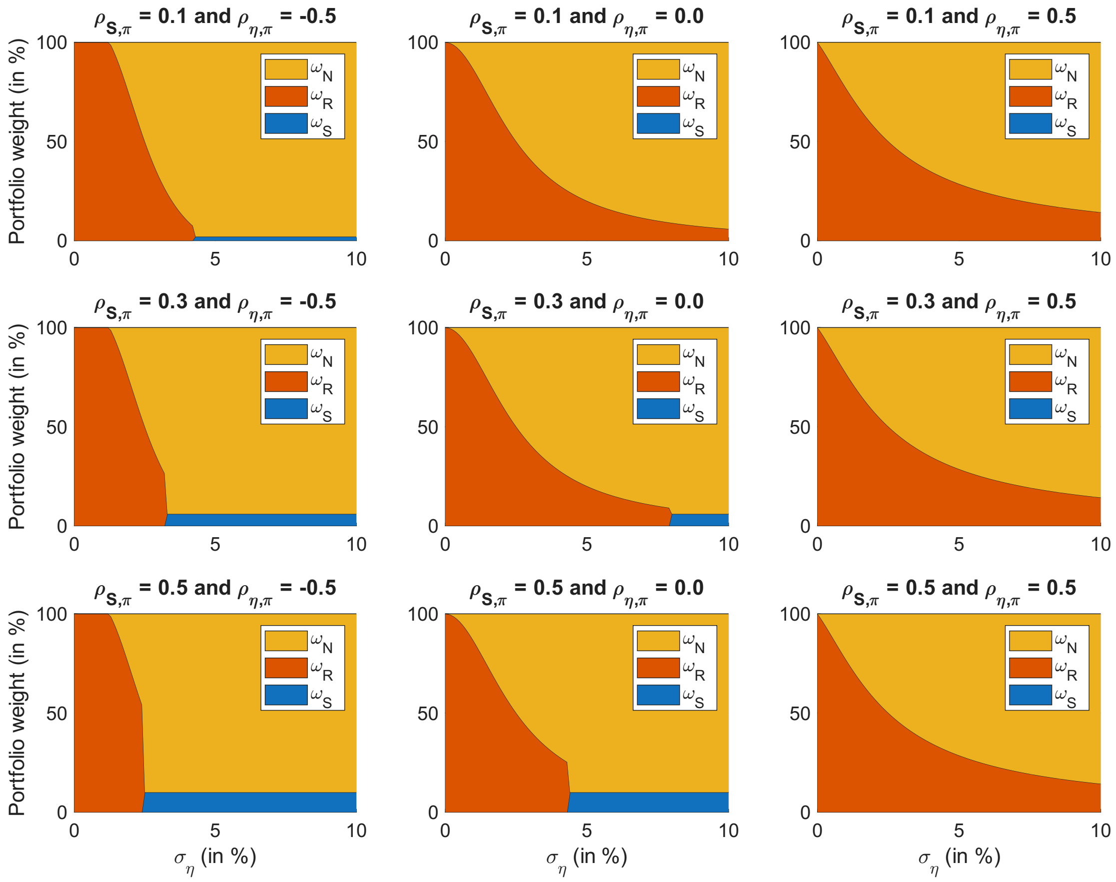


Figure 9: Optimal portfolio weights for different volatilities of real basis risk (σ_η), different correlations between stock returns and inflation ($\rho_{S,\pi}$) and different correlations between real basis risk and inflation $\rho_{\eta,\pi}$, for a correlation between stock returns and real basis risk of $\rho_{S,\eta} = 0.5$. *Note: the portfolio weights add up to 100%, where ω_N refers to the nominal bond weight, ω_R to the index-linked bond weight and ω_S to the stock weight.*

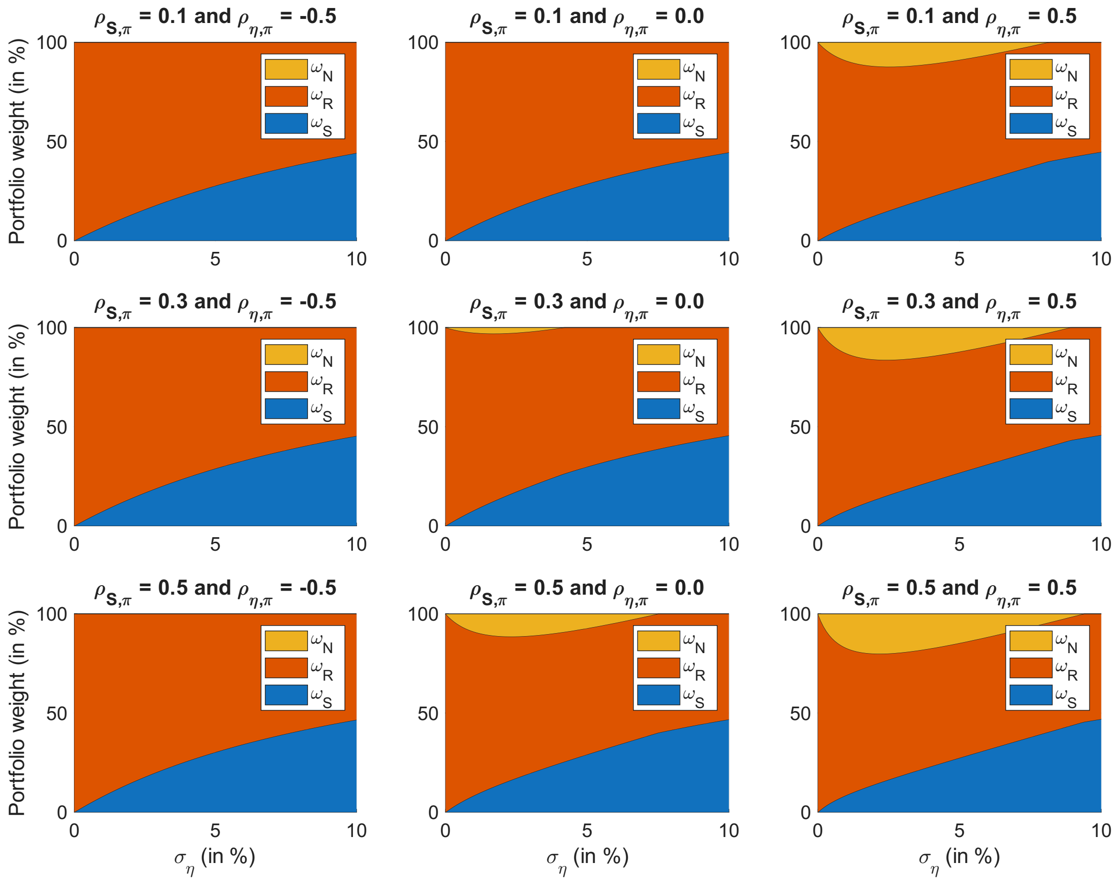


Figure 10: Optimal portfolio weights for different volatilities of real basis risk (σ_η), different correlations between stock returns and inflation ($\rho_{S,\pi}$) and different correlations between real basis risk and inflation $\rho_{\eta,\pi}$, for a correlation between stock returns and real basis risk of $\rho_{S,\eta} = -0.5$. *Note: the portfolio weights add up to 100%, where* ω_N *refers to the nominal bond weight,* ω_R *to the index-linked bond weight and* ω_S *to the stock weight.*

5.2.2 Replicating portfolio compositions with finite risk aversion

We investigate now to what extent our simple model is able to reproduce the composition of the optimal portfolios found in Section 4.4, where portfolio performance was evaluated over a longer horizon, trading off risk and return based on a finite value for the constant relative risk aversion parameter γ. We proceed as follows. First, we demonstrate numerically how far we get in attaining portfolio allocations under the parameter assumptions that we make. Second, we explain what assumptions would be implied about the model parameters to generate the portfolio allocations found in Section 4.4. Finally, we discuss to which extent these parameter assumptions are supported by available empirical evidence.

Attainable portfolio allocations

The composition of the optimal portfolios found in Section 4.4 for finite risk aversion and a long-term investment horizon indicated that the allocation to stock indices varied between 19% and 68%, being 43% on average, while the allocation to the ILBs varied between 2% and 58%, being 41% on average. Hence, overall these findings point to quite a well diversified optimal portfolios.

Portfolios that are well diversified across all three asset classes arise only for specific parameter configurations. When $\rho_{S,\eta} = 0$ (Figure 8), diversification occurs for high basis risk volatility combined with positive correlations $\rho_{S,\pi}$ and $\rho_{\eta,\pi}$, although the allocations towards stocks remain limited. When $\rho_{S,\eta} = -0.5$ (Figure 10), diversified portfolios require a non-negligible amount of basis risk volatility and positive correlations $\rho_{S,\pi}$ and $\rho_{\eta,\pi}$. Still allocations towards nominal bonds remain limited. Hence, overall it is not easy to find parameter combinations that combine substantial allocations towards stocks and index-linked bonds with non-negligible allocations towards nominal bonds.

5.3 Empirical plausibility of implied parameter assumptions

Bringing the numerical results from our theoretical framework in line with the portfolios in Section 3.2 and Section 4.4 requires assumptions about key parameters, including about basis risk η of which the properties are closely related to what the literature often describes as the inflation risk premium and liquidity premia.

The empirical literature documents a positive link between the level and the variability of inflation. Using historical international CPI data, Tsyplakov (2010) finds that higher trend inflation is associated with higher longer-horizon inflation uncertainty. Related work by Ball et al. (1990) finds that increases in inflation tend to raise uncertainty about future inflation. Alesina and Summers (1993) show that greater central bank independence is associated with lower average inflation and, indirectly, with more stable inflation outcomes. These findings support the view that periods of high inflation are also periods of increased inflation volatility.

Our data also support the positive link between inflation and inflation variability. In Table 17 we report average inflation and its standard deviation for the full sample period and four sub-periods of equal length. Average inflation is highest in the latest sub-period, which also exhibits the highest inflation volatility for all countries. The third sub-period features the lowest average inflation rate in all cases. The third sub-period also exhibits the lowest inflation volatility, except for the U.K.

In asset pricing higher inflation uncertainty would typically have investors demand more compensation for bearing inflation risk. Hence, the positive association between inflation and inflation uncertainty would imply a positive correlation between inflation and the inflation risk premium.

Not much is known empirically about the *volatility* of the inflation risk premium, although some papers report a standard deviation of the estimated average inflation risk premium. For example, Azoulay et al. (2014) report a standard deviation of 7 basis points and an average risk premium of 25 basis points. Regarding the volatility of the basis risk (σ_η), our results suggest it must be at least 2% to match observed ILB weights. This aligns with the view of Bekaert and Wang (2010), who find that the inflation risk premium tends to be large and to vary substantially over time. They argue that measuring the inflation risk premium is complex because there is little reliable data on expected inflation and real returns. However, based on the literature estimates of the size the inflation risk premium vary between 50 and over 200 basis points at the 10-year horizon.

Our data reveal a strictly positive correlation between stock returns and the inflation rate, averaging 12% and reaching values as high as 34%. Such a positive correlation is needed to generate a substantial allocation towards stocks. A substantial body of literature supports this positive relationship. For example, Lintner (1975) and Bodie (1976) argue theoretically that equities should provide inflation protection as real assets.

Finally, empirical evidence regarding the correlation between stock returns and real basis risk remains scarce. Nevertheless, several recent studies establish inflation risk as a priced factor in financial markets, providing a robust foundation for our parameter assumptions. Boons et al. (2020) find that inflation risk is time-varying and significantly correlated with stock returns, particularly for equities with high exposure to inflation shocks. Furthermore, Fang et al. (2025) argue that inflation carries a negative risk premium and that stocks exhibit negative inflation betas. This implies that an increase in the inflation risk premium, which is a key driver of our basis risk, is associated with higher required stock returns and, consequently, lower contemporaneous stock returns due to a decline in equity valuations. Cieslak and Pflueger (2023) demonstrate that inflation and inflation risk are priced across asset classes, including equities. Collectively, these studies suggest that a negative correlation between stock returns and real basis risk is empirically plausible.

Table 17: Mean and standard deviation of annual inflation rates for different sample periods.

mean (standard deviation)	NL	GE	UK	US
full sample period	2.28 (2.81)	2.10 (2.32)	3.78 (3.00)	2.57 (2.05)
subperiod 1	1.83 (1.11)	2.49 (0.86)	3.79 (0.92)	3.41 (1.29)
subperiod 2	2.47 (0.56)	2.00 (0.46)	4.01 (1.04)	2.02 (1.20)
subperiod 3	1.12 (0.99)	0.78 (0.79)	2.39 (1.24)	1.42 (1.13)
subperiod 4	4.51 (3.54)	4.04 (3.73)	6.82 (5.44)	4.50 (3.50)

Note: all values are expressed in percentages. The full sample period is from 20-09-2004 to 18-04-2024. The four subperiods are equal subperiods of the full sample.

6 Conclusion

This paper has investigated the effectiveness of existing financial instruments in hedging real portfolio returns against inflation, a question of growing relevance for long-term investors such as pension funds. Using mean-variance frontiers, we have assessed the hedging effectiveness of nominal bonds, inflation-linked bonds (ILBs), and equities during the period 2004-2024. We found that it is not possible to construct portfolios of these asset categories that completely eliminate the effect of inflation risk on real returns. Introducing realistic market constraints, such as no borrowing or short-selling, further limits the scope for protecting portfolio returns against inflation risk. Although ILBs are commonly believed to offer the best protection against inflation risk, our findings actually show that nominal bond indices have been a more effective hedge against inflation risk during this period.

The benefits of diversification are confirmed by several sensitivity analyses. Once we (counterfactually) eliminate the comparative advantage of nominal bonds in terms of the volatility in their real returns, the minimum-variance portfolio becomes more balanced across asset classes. In addition, focusing on the recent high-inflation period in the aftermath of the Covid-19 crisis creates a more substantial role for ILBs in hedging inflation risks.

To extend the short-term mean-variance analysis to a long-term investment setting, we construct optimal portfolios for an investor receiving a variable annuity over a 20-year horizon while allowing for finite risk aversion. The results show that the optimal portfolio composition depends crucially on the investment horizon and the degree of risk aversion. Under extreme risk aversion, the optimal allocation is characterized by substantial exposure to both equities and ILBs, and the associated welfare gains primarily reflect the benefits of adopting a multi-period perspective. As risk aversion declines, investors increasingly trade off risk against expected return, leading to higher equity allocations and reduced, but still meaningful, exposure to ILBs. Across all specifications, the welfare gains relative to one-period minimum-variance portfolios are economically large, reflecting both the value of long-horizon optimization and improved risk-return trade-offs. Overall, while ILBs play an important role in hedging inflation risk, they do not dominate optimal portfolios, underscoring both the benefits of diversification and the limits of inflation hedging.

Finally we develop a simple theoretical framework that helps to clarify our empirical allocations. We allow for time-varying real basis risk, implying that investors in inflation-linked bonds face a trade-off between hedging inflation and being exposed to fluctuations in real basis risk. A substantial allocation towards nominal bonds as in the one-period minimum-variance portfolios requires substantial uncertainty about basis risk and a not too low correlation between stock returns and basis risk. However, to generate a substantial allocation towards stocks, as was optimal for our long-horizon portfolios with finite risk aversion, requires a negative correlation between stock returns and basis risk. Existing evidence suggests that the inflation risk premium can be substantial and vary considerably over time, which supports the theoretical framework in rationalizing the optimal portfolio allocations we found. Yet, more research is needed to empirically identify and quantify basis risk and its correlation with other risk factors. Also, more sophisticated theoretical frameworks may be needed, with the flexibility to simultaneously rationalize the different optimal portfolios found in our empirical analysis.

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Online Appendix - Not for Publication

A Solving for the optimal portfolio analytically

This section closely follows the discussion of mean-variance frontiers in Gale (2005).

A.1 Global minimum portfolio

The global minimum-variance portfolio is the solution to the following minimization problem:

$$\min_{\omega} \sigma_m^2 = \omega' \Sigma \omega \quad \text{s.t. } \omega' \mathbf{1}_n = 1. \tag{A1}$$

In order to solve this, we construct the Lagrange function:

$$L(\omega, \lambda,) = \omega' \Sigma \omega + \lambda(\omega' \mathbf{1}_n - 1). \tag{A2}$$

The corresponding first-order conditions are

$$\frac{\partial L(\omega, \lambda)}{\partial \omega} = 2\Sigma\omega + \lambda \mathbf{1}_n = \mathbf{0} \tag{A3}$$

$$\frac{\partial L(\omega, \lambda)}{\partial \lambda} = \omega' \mathbf{1}_n - 1 = 0. \tag{A4}$$

Then we can derive the global minimum portfolio ω_m as follows:

$$\mathbf{1}_n' \omega_m = -\frac{1}{2} \lambda \mathbf{1}_n' \Sigma^{-1} \mathbf{1}_n = 1 \tag{A5}$$

$$\Rightarrow \lambda = -2(\mathbf{1}_n' \Sigma^{-1} \mathbf{1}_n)^{-1} \tag{A6}$$

$$\Rightarrow \omega_m = (\mathbf{1}_n' \Sigma^{-1} \mathbf{1}_n)^{-1} \Sigma^{-1} \mathbf{1}_n. \tag{A7}$$

A.2 Mean-variance portfolio

The mean-variance frontier is built by solving the optimal portfolio problem for given target expected returns μ_p. Such a mean-variance frontier portfolio is the solution of the following minimization problem:

$$\min_{\omega} \ \sigma_p^2 = \omega' \Sigma \omega \quad \text{s.t. } \mu_p = \omega' \mu \text{ and } \omega' \mathbf{1}_n = 1. \tag{A8}$$

In order to solve this, we construct the Lagrange function:

$$L(\omega, \lambda_1, \lambda_2) = \omega' \Sigma \omega + \lambda_1(\omega' \mu - \mu_p) + \lambda_2(\omega' \mathbf{1}_n - 1). \tag{A9}$$

The corresponding first-order conditions are

$$\frac{\partial L(\omega,\lambda_1,\lambda_2)}{\partial\omega} = 2\Sigma\omega + \lambda_1\mu + \lambda_2\mathbf{1}_n = \mathbf{0}_n \tag{A10}$$

$$\frac{\partial L(\omega,\lambda_1,\lambda_2)}{\partial\lambda_1} = \omega'\mu - \mu_p = 0 \tag{A11}$$

$$\frac{\partial L(\omega,\lambda_1,\lambda_2)}{\partial\lambda_2} = \omega'\mathbf{1}_n - 1 = 0. \tag{A12}$$

Then we can derive the mean-variance portfolio ω_p as follows:

$$\omega_p = -\frac{1}{2}\lambda_1\Sigma^{-1}\mu - \frac{1}{2}\lambda_2\Sigma^{-1}\mathbf{1}_n = -\frac{\lambda'}{2}\Sigma^{-1}\left[\mu\ \mathbf{1}_n\right] \tag{A13}$$

with $\lambda = \begin{bmatrix}\lambda_1\\ \lambda_2\end{bmatrix}$. Then we can use Eq.(A13) to obtain:

$$\mu_p = \mu'\omega_p = -\mu'\frac{\lambda'}{2}\Sigma^{-1}\left[\mu\ \mathbf{1}_n\right] \tag{A14}$$

$$1 = \mathbf{1}'\omega_p = -\mathbf{1}_n'\frac{\lambda'}{2}\Sigma^{-1}\left[\mu\ \mathbf{1}_n\right]. \tag{A15}$$

We can solve for λ:

$$\lambda = -2\Theta^{-1}\begin{bmatrix}\mu_p\\ 1\end{bmatrix} \tag{A16}$$

with Θ defined as the two-by-two matrix:

$$\Theta = \begin{pmatrix}\mu'\Sigma^{-1}\mu & \mu'\Sigma^{-1}\mathbf{1}_n\\ \mathbf{1}_n'\Sigma^{-1}\mu & \mathbf{1}_n'\Sigma^{-1}\mathbf{1}_n\end{pmatrix} \tag{A17}$$

$$= \left[\mu\ \mathbf{1}_n\right]'\ \Sigma^{-1}\ \left[\mu\ \mathbf{1}_n\right] \tag{A18}$$

which we can then use in Eq.(A13) to obtain the following expression for the optimal portfolio weights ω_p:

$$\omega_p = \Sigma^{-1}[\mu\ \mathbf{1}_n]\Theta^{-1}\begin{bmatrix}\mu_p\\ 1\end{bmatrix} \tag{A19}$$

A.3 Optimal portfolio terminal wealth problem

The expected return on real wealth (Eq.(26)) and its variance are as follows.

$$E(dw_t/w_t) = r - \mu_\pi + \omega_S(\mu_S - r) + (1-\omega_R)\sigma_\pi^2 + \omega_R\mu_\eta - \tag{A20}$$

$$...\omega_S\sigma_S\sigma_\pi\rho_{S,\pi} - \omega_R\sigma_\eta\sigma_\pi\rho_{\eta,\pi} \tag{A21}$$

$$Var(dw_t/w_t) = \omega_S^2\sigma_S^2 + (\omega_R-1)^2\sigma_\pi^2 + \omega_R^2\sigma_\eta^2 + \tag{A22}$$

$$...2\omega_S(\omega_R-1)\sigma_S\sigma_\pi\rho_{S,\pi} + \tag{A23}$$

$$...2\omega_S\omega_R\sigma_S\sigma_\eta\rho_{S,\eta} + \tag{A24}$$

$$...2\omega_R(\omega_R-1)\sigma_\eta\sigma_\pi\rho_{\eta,\pi}. \tag{A25}$$

The first-order conditions of Eq.(32) with respect to ω_S and ω_R are, respectively:

$$0 = (\mu_S - r) - A - \gamma * [\omega_S^* \sigma_S^2 + (\omega_R^* - 1)A + \omega_R^* B] \tag{A26}$$

$$0 = -\sigma_\pi^2 + \mu_\eta - C - \gamma * [(\omega_R^* - 1)\sigma_\pi^2 + \omega_R^* \sigma_\eta^2 + \omega_S^*(A + B) + (2\omega_R^* - 1)C]. \tag{A27}$$

We can rewrite these equations as follows.

$$\omega_S^* = \frac{\mu_S - r - A}{\gamma \sigma_S^2} + \frac{A - \omega_R^*(A + B)}{\sigma_S^2} \tag{A28}$$

$$\omega_R^* = \frac{\mu_\eta - \sigma_\pi^2 - C}{\gamma(\sigma_\pi^2 + \sigma_\eta^2)} + \frac{\sigma_\pi^2 - \omega_S^*(A + B) + (1 - 2\omega_R^*)C}{\sigma_\pi^2 + \sigma_\eta^2} \tag{A29}$$

Substituting the expression for ω_S^* in the last line and solving gives

$$\omega_R^* = \frac{(\mu_\eta - \sigma_\pi^2 - C)\sigma_S^2 + (A + B)(A - (\mu_S - r))}{\gamma((\sigma_\pi^2 + \sigma_\eta^2 + 2C)\sigma_S^2 - (A + B)^2)} + \frac{(\sigma_\pi^2 + C)\sigma_S^2 - A(A + B)}{(\sigma_\pi^2 + \sigma_\eta^2 + 2C)\sigma_S^2 - (A + B)^2} \tag{A30}$$

B Constructing nominal and real returns

The nominal returns are in euros, while the real returns are adjusted for the corresponding CPI inflation and the relevant currency (i.e. euros for NL and GE, dollars for the US, and pounds for the UK). We perform the following steps to construct these nominal and real returns:

- For each index, we take the prices in euros. The indices are total return indices that reinvest coupon and dividend payments.
- The nominal returns in euros are derived by taking the percentage changes in these euro prices.
- The real returns for Germany and for the Netherlands are derived (i) by dividing the euro prices by the German CPI and the Dutch CPI, respectively, and (ii) by taking the percentage changes in these CPI adjusted price indices.
- The real returns for the UK and for the US are derived (i) by converting the euro prices to pounds and dollars, respectively, (ii) by dividing the resulting converted prices by, respectively, the UK CPI and the US CPI, and (iii) by taking the percentage changes in these CPI adjusted price indices.

C Additional results

C.1 Results for quarterly returns of the sensitivity analysis

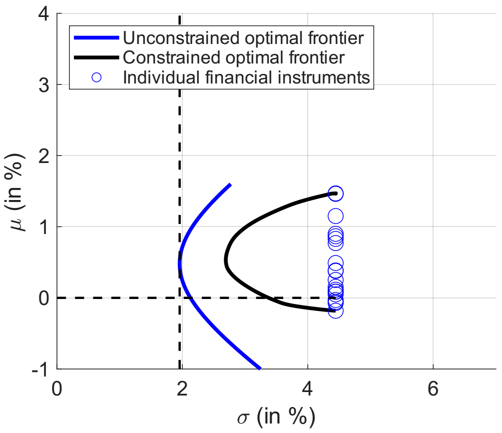
Figure A1 and Figure A2 show the mean-variance frontiers of the sensitivity analysis in Section 3.3 based on quarterly returns. The mean-variance frontiers based on quarterly returns during the period after the COVID-19 outbreak are not shown, as our sample data have too few observations. to construct those frontiers.

C.2 Optimal portfolio allocations for the minimum risk portfolios

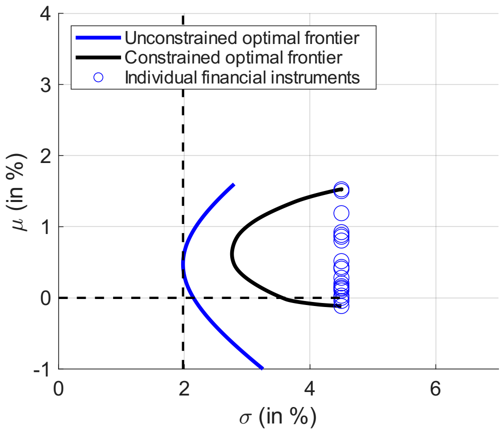
The tables Section 3 provide the minimum risk portfolios per asset class. The corresponding portfolio allocation per index is presented in Table A1 to Table A5, based on real returns with respect to the inflation of the Netherlands, Germany, the U.K., and the U.S, respectively.

C.3 Estimates of the parameters of the inflation process

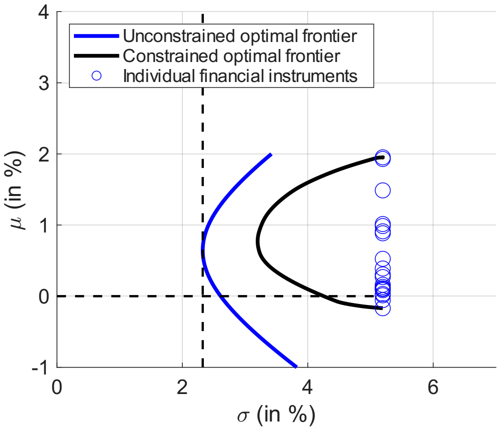
Table 14 presents a summary of the estimates of the parameters of the inflation process. Table A6 provides more details, by also presenting the coefficients of all individual lags and the corresponding standard error for the full sample period.



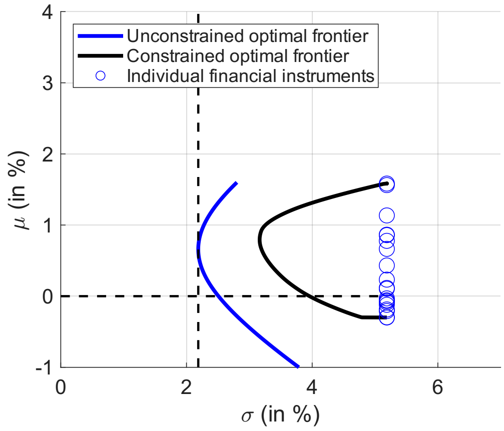
(a) Quarterly real returns in terms of Dutch inflation



(b) Quarterly real returns in terms of German inflation

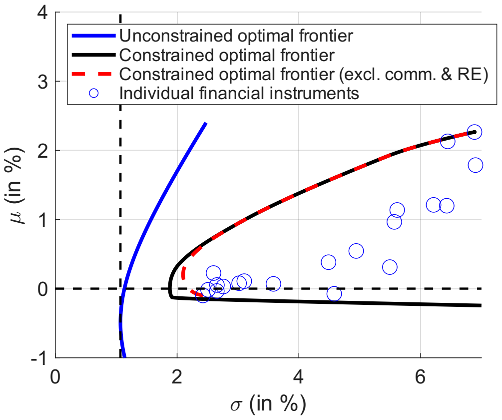


(c) Quarterly real returns in terms of U.K. inflation

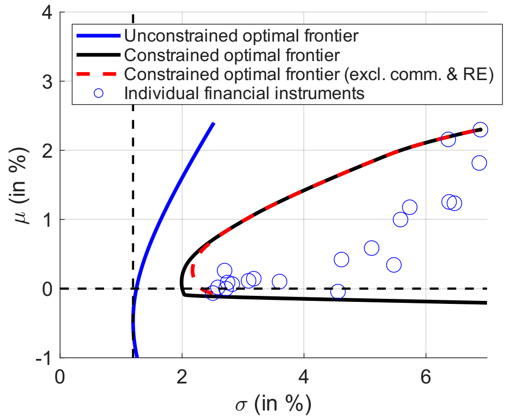


(d) Quarterly real returns in terms of U.S. inflation

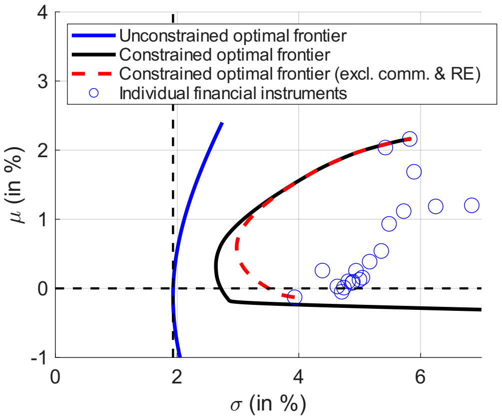
Figure A1: Mean-variance frontiers based on quarterly real returns after scaling the returns to align the volatilities. *Note: Real returns are obtained by subtracting CPI from nominal returns.*



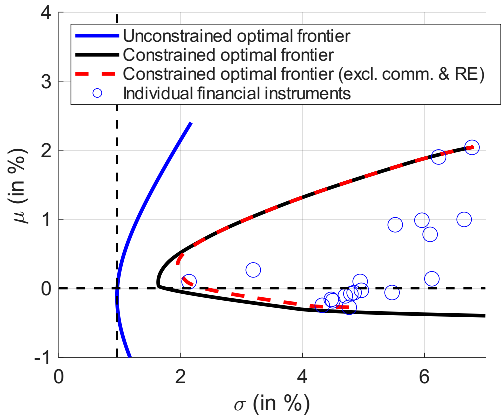
(a) Quarterly real returns in terms of Dutch inflation



(b) Quarterly real returns in terms of German inflation



(c) Quarterly real returns in terms of U.K. inflation



(d) Quarterly real returns in terms of U.S. inflation

Figure A2: Mean-variance frontiers based on quarterly real returns, also including real estate (RE) and commodity (comm) indices. *Note: allocations are in percent. Real returns are obtained by subtracting CPI from nominal returns.*

Table A1: Portfolio allocations (in %) of the minimum-variance portfolios based on nominal returns.

	monthly		quarterly	
index ticker	unconstrained	constrained	unconstrained	constrained
SPBDEGIT	40.2	0.1	36.9	0.0
SPBDEFRT	32.9	0.0	37.3	0.0
SPBDEATT	-41.2	0.0	-95.1	0.0
SPBDEBET	-61.5	0.0	-51.7	0.0
SPBDEFIT	80.0	0.1	87.8	0.0
SPBDEDET	236.5	75.5	194.8	65.1
SPBDENLT	-214.7	0.0	-136.3	0.0
SPFIGBT	-2.8	0.0	-1.8	0.0
SPUSBMIT	14.1	0.1	1.8	0.0
SPFID4IT	19.1	14.5	23.2	20.1
SPFIGBIT	-1.7	0.0	-1.4	0.0
SPBDU1ST	-15.9	0.0	-11.7	0.0
SPBNILT	-2.1	0.0	-3.4	0.0
SPFISEI	10.7	5.2	6.8	5.2
SPFIMLUT	1.9	0.3	-25.4	0.0
SPFIMPUT	-6.2	0.1	61.2	0.0
SPVIF0U	6.7	0.0	-31.6	0.0
SPXT	-7.4	0.0	-7.8	0.0
MSDEWIN	8.0	2.5	10.4	3.6
DJITR	3.5	1.7	6.2	5.8

Note: The upper part contains nominal bond indices, the middle part index-linked bond indices, and the lower part stock indices.

Table A2: Portfolio allocations of the minimum-variance portfolios including constraints that prevent borrowing and short-selling based on real returns

	monthly returns				quarterly returns			
index ticker	NL	GE	UK	US	NL	GE	UK	US
SPBDEGIT	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0
SPBDEFRT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPBDEATT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPBDEBET	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPBDEFIT	0.1	0.1	0.1	0.0	0.0	0.0	0.0	0.0
SPBDEDET	72.2	73.1	0.2	0.0	58.1	57.2	0.0	0.0
SPBDENLT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPFIGBT	0.0	0.0	60.5	0.0	0.0	0.0	47.2	0.0
SPUSBMIT	0.3	0.1	10.9	92.9	0.0	0.0	0.0	85.6
SPFID4IT	16.6	15.4	2.7	0.0	22.6	22.1	17.7	0.0
SPFIGBIT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPBDU1ST	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPBNILT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPFISEI	5.8	6.4	10.9	0.0	7.7	9.2	0.8	0.0
SPFIMLUT	0.1	0.3	0.0	0.0	0.0	0.1	0.0	0.1
SPFIMPUT	0.4	0.2	0.1	0.0	0.1	1.0	6.2	1.0
SPVIF0U	0.1	0.1	0.1	0.0	3.7	0.0	0.1	1.4
SPXT	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
MSDEWIN	3.3	2.4	11.2	0.0	1.9	0.2	8.7	0.0
DJITR	1.0	1.7	3.0	6.8	5.7	10.0	19.2	11.8

Note: allocations are in percent. Real returns are obtained by subtracting CPI from nominal returns. The upper part contains nominal bond indices, the middle part index-linked bond indices, and the lower part stock indices.

Table A3: Portfolio allocations of the minimum-variance portfolios including constraints that prevent borrowing and short-selling, based on real returns after scaling the returns to align the volatilities.

	monthly returns				quarterly returns			
index ticker	NL	GE	UK	US	NL	GE	UK	US
SPBDEGIT	0.1	0.1	0.1	0.1	0.1	0.1	0.0	0.0
SPBDEFRT	0.0	0.0	0.0	0.1	0.1	0.1	0.0	0.0
SPBDEATT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPBDEBET	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPBDEFIT	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0
SPBDEDET	12.4	10.9	0.7	11.6	27.3	27.5	9.4	4.0
SPBDENLT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPFIGBT	0.5	0.9	17.7	5.8	0.0	0.0	23.9	0.0
SPUSBMIT	17.5	17.2	11.4	22.1	7.3	5.5	0.1	24.3
SPFID4IT	14.9	15.1	5.0	0.1	0.1	0.1	0.0	0.0
SPFIGBIT	7.1	9.2	14.1	2.9	0.0	0.1	0.0	0.0
SPBDU1ST	0.0	0.0	0.0	6.9	0.0	0.0	6.1	12.6
SPBNILT	1.7	1.8	1.6	0.0	0.0	0.0	0.0	0.0
SPFISEI	9.5	9.5	4.6	0.1	14.3	14.7	4.6	3.9
SPFIMLUT	8.6	9.4	6.5	10.2	12.6	15.6	12.6	14.5
SPFIMPUT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPVIF0U	10.9	9.9	13.6	11.0	10.4	5.7	6.7	8.1
SPXT	0.3	0.1	5.9	1.0	0.0	0.0	0.0	0.0
MSDEWIN	10.5	8.2	3.5	0.0	17.6	13.7	23.8	0.0
DJITR	5.8	7.5	15.1	27.8	9.8	16.8	12.5	32.4

Note: Allocations are in percent. Real returns are obtained by subtracting CPI from nominal returns. The upper part contains nominal bond indices, the middle part index-linked bond indices, and the lower part stock indices.

Table A4: Portfolio allocations of the minimum-variance portfolios including constraints that prevent borrowing and short-selling, based on real returns also including real estate and commodity indices.

	monthly returns				quarterly returns			
index ticker	NL	GE	UK	US	NL	GE	UK	US
SPBDEGIT	0.1	0.1	0.2	0.0	0.0	0.0	0.0	0.0
SPBDEFRT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPBDEATT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPBDEBET	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPBDEFIT	0.1	0.1	0.2	0.0	0.0	0.0	0.0	0.0
SPBDEDET	76.6	78.1	3.5	0.0	74.8	74.3	0.1	0.0
SPBDENLT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPFIGBT	0.0	0.0	62.0	0.0	0.0	0.0	57.0	0.0
SPUSBMIT	0.2	0.1	10.1	90.5	0.0	0.0	0.9	87.3
SPFID4IT	11.7	9.9	0.1	0.0	8.8	8.0	6.8	0.0
SPFIGBIT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPBDU1ST	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPBNILT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPFISEI	0.9	1.3	1.1	0.0	0.2	1.7	0.0	0.0
SPFIMLUT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPFIMPUT	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0
SPVIF0U	0.0	0.0	0.0	0.0	2.2	0.0	1.1	0.0
SPXT	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
MSDEWIN	0.0	0.0	0.1	0.0	0.2	0.0	0.5	0.0
DJITR	0.3	0.2	5.0	1.0	2.5	5.7	17.2	2.5
RNGL	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
RMSG	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SX86P	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
DJUSRE	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPGSCITR	2.1	2.1	8.4	5.8	6.1	6.3	7.9	8.6
BCOMTR	0.6	0.2	6.6	0.2	4.5	1.6	0.3	1.5
DJCIT	7.2	7.8	2.6	2.3	0.5	2.1	8.1	0.0

Note: allocations are in percent. Real returns are obtained by subtracting CPI from nominal returns. The first part contains nominal bond indices, the second part index-linked bond indices, the third part stock indices, the fourth part real estate indices and the fifth part commodity indices.

Table A5: Portfolio allocations of the minimum-variance portfolios including constraints that prevent borrowing and short-selling, based on real returns over the post COVID-19 period.

	monthly returns				quarterly returns			
index ticker	NL	GE	UK	US	NL	GE	UK	US
SPBDEGIT	0.1	0.1	0.0	0.0	0.0	0.0	0.0	0.0
SPBDEFRT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPBDEATT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPBDEBET	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPBDEFIT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPBDEDET	3.3	11.3	8.1	0.0	0.0	0.0	0.0	0.0
SPBDENLT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPFIGBT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPUSBMIT	53.7	50.7	43.5	97.4	28.4	17.1	6.7	74.8
SPFID4IT	12.9	5.7	22.4	0.0	31.1	11.4	42.2	0.0
SPFIGBIT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SPBDU1ST	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.8
SPBNILT	0.0	0.0	0.0	0.0	5.1	8.7	0.0	0.0
SPFISEI	17.6	18.4	19.6	0.0	8.2	15.1	0.0	0.0
SPFIMLUT	12.4	13.6	6.1	2.5	18.6	27.7	28.0	0.0
SPFIMPUT	0.0	0.0	0.0	0.0	0.1	0.0	0.0	2.6
SPVIF0U	0.0	0.0	0.0	0.0	0.0	0.0	0.0	20.4
SPXT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
MSDEWIN	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
DJITR	0.0	0.0	0.0	0.0	8.4	19.9	22.9	1.4

Notes: allocations are in percent. The real returns are obtained by subtracting CPI from the nominal returns. The sample period is 01-03-2020 to 18-04-2024. The upper part contains nominal bond indices, the middle part index-linked bond indices, and the lower part stock indices.

Table A6: Estimates of the parameters of the inflation process.

monthly ($\delta = \frac{1}{12}$)	NL		GE		UK		US	
$\hat{\alpha}_0$	0.07	(0.04)	0.04	(0.03)	0.06	(0.04)	0.07	(0.03)
$\hat{\alpha}_1$	67.96	(5.76)	46.41	(4.84)	40.75	(5.72)	90.95	(5.16)
$\hat{\alpha}_2$	-34.19	(9.86)	-19.01	(7.91)	3.50	(6.69)	-44.79	(6.26)
$\hat{\alpha}_3$	0.33	(11.49)	16.77	(7.78)	5.54	(6.69)	12.83	(9.14)
$\hat{\alpha}_4$	-1.68	(8.21)	-3.40	(10.48)	9.99	(7.70)	10.72	(10.79)
$\hat{\alpha}_5$	9.31	(6.96)	-2.38	(11.12)	-3.19	(7.46)	-14.14	(12.19)
$\hat{\alpha}_6$	-3.04	(6.01)	25.53	(8.88)	25.80	(8.19)	10.99	(10.95)
$\hat{\alpha}_7$	15.11	(7.96)	-3.54	(7.65)	-11.04	(6.74)	-0.75	(9.33)
$\hat{\alpha}_8$	-16.41	(7.61)	-2.56	(7.72)	-8.51	(6.44)	1.57	(10.44)
$\hat{\alpha}_9$	3.16	(7.33)	-15.53	(7.02)	-10.19	(7.62)	-2.85	(10.14)
$\hat{\alpha}_{10}$	7.66	(10.00)	4.77	(8.15)	4.47	(8.15)	7.21	(9.29)
$\hat{\alpha}_{11}$	-0.93	(9.10)	9.19	(9.64)	-9.92	(5.81)	11.00	(7.46)
$\hat{\alpha}_{12}$	18.06	(8.05)	25.72	(6.16)	32.73	(5.86)	-17.16	(5.83)
$\sum_{l=1}^{12} \hat{\alpha}_l$	65.33		81.99		79.93		65.58	
$\hat{\mu}_\pi$	0.20		0.20		0.28		0.21	
$\hat{\sigma}_\pi$	0.34		0.26		0.31		0.20	
R^2	45.95		49.03		41.42		52.89	
quarterly ($\delta = \frac{1}{4}$)	NL		GE		UK		US	
$\hat{\alpha}_0$	0.23	(0.12)	0.16	(0.11)	0.30	(0.18)	0.40	(0.19)
$\hat{\alpha}_1$	38.60	(8.93)	17.56	(11.74)	43.58	(13.83)	28.79	(10.29)
$\hat{\alpha}_2$	-21.68	(10.25)	32.06	(11.98)	24.95	(11.38)	-1.58	(11.89)
$\hat{\alpha}_3$	25.39	(12.41)	-14.98	(10.63)	-25.70	(16.47)	12.85	(17.12)
$\hat{\alpha}_4$	19.41	(11.27)	39.25	(12.50)	23.03	(14.11)	-2.28	(17.57)
$\sum_{l=1}^{4} \hat{\alpha}_l$	61.72		73.88		65.86		37.79	
$\hat{\mu}_\pi$	0.61		0.62		0.88		0.64	
$\hat{\sigma}_\pi$	0.70		0.65		0.77		0.72	
R^2	25.36		37.43		33.77		9.94	

Note: all values are multiplied by 100. The sample period is 20-09-2004 to 18-04-2024. The parameter estimates are from the following inflation process: $\pi_t = \alpha_0 + \sum_{l=1}^{p} \alpha_l \pi_{t-l} + \sigma_\pi \varepsilon_{\pi,t}$. *The estimated long run mean is given by:* $\hat{\mu}_\pi = \frac{\hat{\alpha}_0}{1-\sum_{l=1}^{p} \hat{\alpha}_l}$. *The monthly data assumes order* $p = 12$ *and the quarterly data assumes order* $p = 4$.

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