

The impact of carbon pricing and a CBAM on EU  
competitiveness: Technical appendix

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# 1. Introduction

We aim to calculate the impact on the European economy of energy policy scenarios, at the level of industries (1 or 2-digit). More in particular, we use a multiregional Input-Output (IO) model to calculate the sectoral price effects of a CO<sub>2</sub> tax and a Carbon Border Adjustment Mechanism (CBAM).

The standard IO price model can be used for scenario analyses, i.e. imposing a CO<sub>2</sub> tax. Calculations are typically based on fixed IO data for a recent year. Such an approach is very useful especially for short horizons in which the mixture of capital, labour and energy in production cannot be changed.

The remainder of this technical report is as follows. In Section 2 we describe how conventional IO analysis can be used to evaluate the sectoral quantity and price effects of a CO<sub>2</sub> tax. In Section 3 we describe the multiregional IO model and how we constructed country level price aggregates to summarize IO results at the macro (country) level.

## 2. basic Input-Output model and pricing CO<sub>2</sub> emissions

### 2.1. quantity model

Consider an economy of  $n$  sectors. Denote with  $x_i$  total output (production) for sector  $i$ . The following equation<sup>1</sup> describes how sector  $i$  distributes its product through intermediate sales to other sectors ( $z_{ij}$ ) and final demand ( $f_i$ ):

$$x_i = z_{i1} + z_{i2} + \dots + z_{in} + f_i, \quad i = 1, \dots, n. \quad (2.1)$$

A fundamental assumption in conventional IO models is that  $z_{ij}$ , i.e. the interindustry flow from sector  $i$  to  $j$ , is entirely determined by the total output of sector  $j$ . In other words, the technical coefficients defined as:

$$a_{ij} = \frac{z_{ij}}{x_j}, \quad (2.2)$$

measure fixed relationships between a sector's output  $x_j$  and its inputs  $z_{ij}$ . In other words, IO analysis requires that a sector use inputs in fixed proportions. Consider, for example, the case of two inputs. Once the proportion  $z_{1j}/z_{2j}$  of inputs 1 and 2 is known, then additional amounts of input 1 or input 2 separately are useless for increasing output of sector  $j$ .

These fixed input-output ratios imply zero elasticity of substitution between inputs in the production function. Ignoring the contribution of value added, the implicit form of the production function used in IO analysis is the Leontief production function:

$$x_j = \min \left\{ \frac{z_{1j}}{a_{1j}}, \dots, \frac{z_{nj}}{a_{nj}} \right\}. \quad (2.3)$$

This mathematical representation reflects the property of fixed proportions: increasing one input, while leaving the other inputs unchanged, will not increase output. Under this assumption of fixed technical coefficients (2.2), equation (2.1) can be expressed as:

$$x_i = a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n + f_i, \quad i = 1, \dots, n. \quad (2.4)$$

The matrix expression for (2.4) is:

$$x = Ax + f. \quad (2.5)$$

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<sup>1</sup>This exposition uses notation from Miller and Blair (2009).

Solving for  $x$  leads to the familiar expression:

$$x = Lf, \quad (2.6)$$

where  $L = (I - A)^{-1}$  is known as the Leontief inverse. A typical element  $l_{ij}$  of  $L$  measures how total output for sector  $i$  ( $x_i$ ) depends on final demand for product  $j$  ( $f_j$ ).

## 2.2. price model

A second set of  $n$  equations, which is closely related to (2.1), describes how sectoral output  $x_j$  is divided among value added  $v_j$  and intermediate inputs  $z_{1j}, \dots, z_{nj}$ :

$$x_j = z_{1j} + z_{2j} + \dots + z_{nj} + v_j, \quad j = 1, \dots, n. \quad (2.7)$$

Note the transpose of the subscripts, i.e. output is now defined as the sum of the column inputs and value added. Value added is produced by the primary inputs capital and labor. Dividing all elements in (2.7) by sectoral total output  $x_j$ , we have:

$$1 = a_{1j} + a_{2j} + \dots + a_{nj} + \frac{v_j}{x_j}, \quad j = 1, \dots, n, \quad (2.8)$$

where we exploited the definition of the technical coefficients (2.2). The elements on the right-hand side reflect how much of each input is used to produce a single unit of output from industry  $j$ . The term  $\frac{v_j}{x_j}$  is the value added content of output for sector  $j$ . The model (2.7) is expressed in monetary terms, hence it can be split into separate price and quantity<sup>2</sup> components:

$$x_j p_j = z_{1j} p_1 + z_{2j} p_2 + \dots + z_{nj} p_n + v_j, \quad j = 1, \dots, n. \quad (2.9)$$

Dividing all elements in (2.9) by sectoral total output  $x_j$ , we have:

$$p_j = a_{1j} p_1 + a_{2j} p_2 + \dots + a_{nj} p_n + \frac{v_j}{x_j}, \quad j = 1, \dots, n. \quad (2.10)$$

Output prices  $p_j$  are equal to the cost of production and (using matrix notation) the IO price model becomes:

$$p = A'p + v_c, \quad (2.11)$$

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<sup>2</sup>There is a slight abuse of notation in the sense that we don't use separate symbols for quantities compared with the value transactions before.

where  $v_c = \left( \frac{v_1}{x_1}, \dots, \frac{v_n}{x_n} \right)'$  is the vector of value added content of output. Solving for  $p$  leads to:

$$p = L'v_c, \quad (2.12)$$

which describes how output prices depend on primary input prices. This structure can be used to evaluate how changes in value added lead to changes in sectoral unit costs and therefore output prices. The price model (2.12) is known as the cost-push IO model as opposed to the demand-pull quantity model in (2.6).

### 2.3. price effects of CO<sub>2</sub> taxation

We use the IO price model (2.12) to calculate direct and indirect price effects of a carbon tax.<sup>3</sup> In the IO price model the carbon tax can be modeled as a tax on intermediate inputs or value added. Although environmental corporate income taxes do exist, most of the environmental taxes apply to the purchase of an intermediate input. Typical examples are coal, oil and gas. Fullerton (1995) gives an overview of environmental taxes for the US as well as an unifying framework for analyzing their effects in the IO price model. Assuming that each intermediate input has its own tax rate we rewrite (2.10) as:

$$p_j = a_{1j}(1 + \tau_1)p_1 + a_{2j}(1 + \tau_2)p_2 + \dots + a_{nj}(1 + \tau_n)p_n + v_{cj}, \quad j = 1, \dots, n, \quad (2.13)$$

where  $\tau_j$ ,  $j = 1, \dots, n$ , are tax rates. Defining

$$T = \begin{bmatrix} 1 + \tau_1 & 0 & \dots & 0 \\ 0 & 1 + \tau_2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \dots & 0 & 1 + \tau_n \end{bmatrix}, \quad (2.14)$$

we can then express the IO price model including taxes as:

$$p = (I - A'T)^{-1}v_c. \quad (2.15)$$

Instead, if the tax applies to value added we rewrite (2.10) as:

$$p_j = a_{1j}p_1 + a_{2j}p_2 + \dots + a_{nj}p_n + v_{cj}(1 + \tau_j), \quad j = 1, \dots, n, \quad (2.16)$$

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<sup>3</sup>Due to (2.8) all baseline prices are set equal to 1.

resulting in:

$$p = (I - A')^{-1} T v_c. \quad (2.17)$$

Summarizing, a tax on intermediate inputs can be seen as changing  $A$ , while an income tax changes  $v_c$ . In general the two types of taxes will lead to different cost increases in the IO price model. We will focus on the latter in the remaining of the analysis.

To determine the sectoral tax rate  $\tau_j$  we first calculate the total tax revenues, which are the product of the uniform CO<sub>2</sub> price  $tr$  (euro per kg) and the sectoral CO<sub>2</sub> emissions  $co2_j$  (kg):

$$t_j = tr \times co2_j, \quad j = 1, \dots, n. \quad (2.18)$$

The sectoral tax rate is then defined as the total tax revenue per unit value added:

$$\tau_j = \frac{t_j}{v_j}, \quad j = 1, \dots, n, \quad (2.19)$$

which is the relative change in value added as a result of the CO<sub>2</sub> tax.

## 2.4. discussion

The conventional IO model assumes fixed technical coefficients. Output changes are solely due to changes in final demand (income effect) and output changes are independent of price changes. There are a number of alternative ways to relax the restrictive assumption of a zero elasticity of substitution. First, the Leontief production function (2.3) can be replaced with another production function, which explicitly allows for substitution. Examples are generalized Leontief, Cobb-Douglas and CES production functions. Klijs et al. (2015) apply a non-linear IO model for economic impact analysis in the region Zeeland. Second, conventional IO analysis can be combined with a Computational General Equilibrium (CGE) model. The IO analysis then provides volume effects, while the CGE model quantifies price effects. For example, in the EXIOMOD model developed by Bulavskaya et al. (2016) the production technology is modeled as a nested CES production function. In particular, energy can be substituted to the aggregate labor-capital input. Also there is substitution possible between energy types (electricity and petroleum products).

An advantage of non-linear IO or CGE models is that substitution is endogenously determined. A major disadvantage of non-linear IO models is that solving a large number of

nonlinear equations is numerically challenging. In order to maintain the sectoral aggregation level and to avoid computational difficulties due to non-linearities, we maintain the linear conventional IO framework. Earlier DNB analysis (Hebbink et al., 2018) also has shown that, within the IO framework, relaxing the assumption of a zero elasticity of substitution does not lead to markedly different empirical results.

### 3. multiregional IO model

In our multiregional IO model we have  $r = 1, \dots, c$  countries and  $i = 1, \dots, n$  sectors. Compared to the standard IO model introduced earlier the multiregional IO set up needs some additional notation.<sup>4</sup> In the standard IO model the basic equation for the distribution of the product of sector  $i$  is:

$$x_i = z_{i1} + z_{i2} + \dots + z_{in} + f_i, \quad i = 1, \dots, n. \quad (3.1)$$

In a multicountry set up this is generalized to:

$$x_i^r = \sum_{s=1}^c \sum_{j=1}^n z_{ij}^{rs} + f_i^r, \quad s = 1, \dots, c; i = 1, \dots, n, \quad (3.2)$$

where superscripts  $(r, s)$  denote the region and subscripts  $(i, j)$  sectors. The element  $z_{ij}^{rs}$  is intermediary sales from sector  $i$  in country  $r$  to sector  $j$  in country  $s$ , while  $f_i^r$  is final demand in sector  $i$  of country  $r$ . Total production in sector  $i$  of country  $r$  is denoted by  $x_i^r$ .

The sectoral CO<sub>2</sub> tax rate  $\tau_i^r$  is calculated according to (2.19). Regarding the CO<sub>2</sub> tax the regional coverage is the EU, while we model the CBAM as a world wide CO<sub>2</sub> tax. Focusing on the effects for the EU only, the additional effects of the CBAM can be calculated on EU import prices from non-EU countries.

Prices can be summarized in various ways. We have an industry and country specific  $nc \times 1$  price vector  $p$ . A typical element  $p_i^r$  in this column vector is the price in sector  $i$  of country  $r$ . Both at the sectoral and country level we define different prices: (1) export price; (2) consumer price; (3) import price.

The exports for industry  $i$  in country  $r$  consists of two components, i.e. exports for final

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<sup>4</sup>We use notation from Miller and Blair (2009).

demand and intermediate exports. Define:

$$K_f = I_c \otimes \iota_n, \quad (3.3)$$

$$E_1 = (\iota_n \otimes \iota_c \iota'_c - K_f) \circ F, \quad (3.4)$$

where  $F$  is the  $nc \times c$  matrix of final demand with typical element  $f_i^{rs}$ , i.e. sales of sector  $i$  in country  $r$  to final demand of sector  $i$  in country  $s$ . Note that total final demand of sector  $i$  in country  $r$  is defined as the row sum of  $F$ , i.e.  $f_i^r = \sum_{s=1}^c f_i^{rs}$ . Furthermore, define:

$$K_\iota = I_c \otimes \iota_n \iota'_n, \quad (3.5)$$

$$E_2 = (\iota_{nc} \iota'_{nc} - K_\iota) \circ Z, \quad (3.6)$$

where the typical element of  $Z$  is  $z_{ij}^{rs}$ . Then exports for final demand and intermediate exports are calculated as  $e1 = E_1 \iota_c$  and  $e2 = E_2 \iota_{nc}$  respectively, with typical elements:

$$\begin{aligned} e1_i^r &= \sum_{s=1}^c f_i^{rs} - f_i^{rr} \\ &= \sum_{s \neq r} f_i^{rs}, \end{aligned} \quad (3.7)$$

$$\begin{aligned} e2_i^r &= \sum_{j=1}^n \sum_{s=1}^c z_{ij}^{rs} - \sum_{j=1}^n z_{ij}^{rr} \\ &= \sum_{j=1}^n \sum_{r \neq s} z_{ij}^{rs}. \end{aligned} \quad (3.8)$$

The quantity  $e1_i^r$  is total exports of sector  $i$  in country  $r$  due to final demand in the rest of the world. The quantity  $e2_i^r$  is total exports of sector  $i$  in country  $r$  due to intermediate sales to all sectors in the rest of the world. Total exports of sector  $i$  in country  $r$  therefore is simply the aggregate of intermediate exports and final demand exports:

$$e_i^r = (e1_i^r + e2_i^r). \quad (3.9)$$

Quantities are fixed in the IO model, hence changes in exports are due to price changes. The relative price change for exports of country  $r$  is then calculated as:

$$EXP^r = \frac{\sum_{i=1}^n e_{i1}^r}{\sum_{i=1}^n e_{i0}^r}, \quad (3.10)$$



where  $e_{i1}^r$  and  $e_{i0}^r$  are the value of exports before and after tax.

To measure the price competitiveness of sector  $i$  in country  $j$  we calculate proceed as follows. Define the change in the relative export price for country  $r$  as

$$REP^r = EXP^r - \sum_{s \neq r} \omega^{rs} EXP^s, \quad (3.11)$$

with  $\omega^{rs}$  the share of exports of country  $r$  to country  $s$ . Bilateral exports and export shares from country  $r$  to country  $s$  are calculated as:

$$e^{rs} = \sum_{i=1}^n f_i^{rs} + \sum_{i=1}^n \sum_{j=1}^n z_{ij}^{rs}, \quad (3.12)$$

$$\omega^{rs} = \frac{e^{rs}}{\sum_{s=1}^c e^{rs}}, \quad (3.13)$$

where the denominator is total exports for country  $r$ . Note that  $\sum_{s=1}^c e^{rs} = \sum_{i=1}^n e_i^r$  and  $\sum_{s=1}^c \omega^{rs} = 1$  by definition. An increase of  $REP^r$  means a decrease in the competitiveness of country  $r$ .

The consumer price of country  $r$  is defined as a weighted average its production prices:

$$pc^r = \sum_{i=1}^n w_i^r p_i^r, \quad (3.14)$$

where the weights  $w_i^r$  represent the share of consumption for each good  $i$  with respect to total final demand in country  $r$ :

$$w_i^r = \frac{f_i^r}{\sum_{i=1}^n f_i^r}. \quad (3.15)$$

The relative price change for the consumer price of country  $r$  is then calculated as:

$$PC^r = \frac{pc_1^r}{pc_0^r}. \quad (3.16)$$

We define the import price of sector  $i$  in country  $r$  as a weighted average of the production prices of all other countries:

$$pi_i^r = \sum_{s=1, s \neq r}^c wi_i^s p_i^s, \quad (3.17)$$

where the weights  $wi_i^s$  represent the share of imports from country  $s$  with respect to total imports of sector  $i$  in country  $r$ . The import price of country  $r$  is defined as a weighted average of its sectoral import prices:

$$pi^r = \sum_{i=1}^n w_i^r pi_i^r, \quad (3.18)$$

where the weights  $w_i^r$  are defined in (3.15). The relative price change for the import price of country  $r$  is then calculated as:

$$PI^r = \frac{p_i^r}{p_i^0}. \quad (3.19)$$

Similar calculations lead to export, import and consumer prices at the sectoral level or for groups of sectors. Finally, the difference between the increase in the import price and the rise in domestic production costs on a sectoral level is referred to as ‘domestic market competitiveness’.

## References

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