Indeterminacy and imperfect information by Lubik, Matthes & Mertens - Discussion

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Impressive paper

Evaluates rational expectations equilibria when some agents suffer from informational frictions.

Partial information leads to self-confirming inference.

"Sentiments are akin to sunspots, but operate in unique equilibrium economies." [Angeletos & La'O, 2013]

Imperfect information in rational expectations models was initiated by Minford & Peel (1983), Pearlman, Currie & Levine (1986), Collard & Dellas (2003)...

... also related to Preston's (2005) gradual learning, rational inattention (Sims, 2003, Mackowiak & Wiederholt, 2009), Andolfatto & Gomme (2003), sparsity (Gabaix, 2014) and related forms of bounded rationality.

None of these have found multiplicity but Pearlman, Currie & Levine (1986) were concerned.

New: simultaneous presence of expectations that face different access to information i.e. full and imperfect.

Introduced as a central bank doesn't fully know the current real rate.

Interaction of asymmetric expectations about the economy leads to an indeterminacy.

Imperfect informed central bank slow in reacting - has to filter information.

Nobody's violating the Taylor principle and everybody's forming rational expectations!

Some thoughts on what's (not) going on.

Indeterminacy but not of the "usual" kind.

Simple model with two expectations and $|a| < 1$

$$p_t = a\lambda E_t p_{t+1} + a(1-\lambda) p_{t+1|t} + m_t \qquad 0 < \lambda < 1$$

$p_{t+1|t}$ expectations not at the fullest.

Extreme way of throwing sand at model

$p_{t+1|t}$ replaced by *static expectations* (and drop shocks)

$$p_t = a\lambda E_t p_{t+1} + a(1-\lambda)p_t$$

Yields

$$p_{t+1} = \frac{1-(1-\lambda)a}{a\lambda} p_t + \eta_{t+1}$$

More naive people make indeterminacy less likely

$$\partial \left(\frac{1-(1-\lambda)a}{a\lambda} \right) / \partial\lambda < 0$$

If unique it stays unique.

(That's reminiscent to Cochrane's take on Gabaix' behavioral NKM model.)

Now form *adaptive expectations*

$$\begin{aligned} p_t &= a\lambda E_t p_{t+1} + a(1-\lambda)p^e_{t+1} + m_t \\ p^e_{t+1} &= p^e_t + \kappa\left(p_t - p^e_{t-1}\right) \end{aligned}$$

Maintaining $|a| < 1$

$$\left[\begin{array}{c} p_{t+1} \\ p^e_{t+1} \end{array}\right] = \left[\begin{array}{cc} \frac{1-a(1-\lambda)\kappa}{a\lambda} & -\frac{a(1-\lambda)(1-\kappa)}{a\lambda} \\ \kappa & 1-\kappa \end{array}\right] \left[\begin{array}{c} p_t \\ p^e_t \end{array}\right] + \left[\begin{array}{c} \eta_{t+1} \\ 0 \end{array}\right] + \left[\begin{array}{c} \frac{1}{a\lambda} m_t \\ 0 \end{array}\right]$$

One stable root.

Need more "sophisticated" learning to get (inter-) action. *Kalman filter.*

Simple model and imperfect information (sets)

$$
\begin{aligned}
p_t &= a\lambda E_t p_{t+1} + a(1-\lambda)p_{t+1|t} + m_t \\
m_{t+1} &= \rho m_t + \varepsilon_{t+1}
\end{aligned}
$$

If $\lambda = 0$

$$p_t = \sum_{j=0}^{\infty} a^j E_t m_{t+j} = \frac{1}{1-a\rho} m_t$$

If $\lambda = 1$

$$p_{t|t} = \sum_{j=0}^{\infty} a^j m_{t+j|t} = \frac{1}{1-a\rho} m_{t|t}.$$

It's all about the interaction.

$$p_t = a\lambda E_t p_{t+1} + a(1-\lambda)p_{t+1|t} + m_t$$

Solution for imperfect people, projection must follow

$$p_{t|t} = a\lambda p_{t+1|t} + a(1-\lambda)p_{t+1|t} + m_{t|t} = ap_{t+1|t} + m_{t|t}$$

$$\Rightarrow p_{t|t} = \frac{1}{1-a\rho}m_{t|t}$$

So, how do people "learn" what's going on with m_t? It works via $m_{t|t}$.

Learning to believe in sunspots is an endogenous process.

$$p_t = a\lambda \left(p_{t+1} - \eta_{t+1}\right) + \frac{a\rho(1-\lambda)}{1-a\rho} m_{t|t} + m_t$$

$$m_{t|t} = m_{t|t-1} + \kappa_\tau(p_t, p_{t|t-1}, ..., \eta_{t+1})$$

People can make inferences about m_t from which they may form expectations. Solution requires knowledge of the covariance structure and, in equilibrium, belief shocks η_{t+1} can play a role for what's observed today!

Key.

Variables depend both on current disturbances and current inferences about current disturbances.

But the inferences depend in turn on observations of some of the current variables.

Gives rise to an obvious circularity, with inferences depending on observations and observations depending on inferences. This circularity can lead to **self-confirming inferences**.

Pearlman, Currie & Levine (1986) have seen this. But decided not deal with it.

Action comes from how information is processed.

$$p_t = a\lambda\left(p_{t+1} - \eta_{t+1}\right) + \frac{a\rho(1-\lambda)}{1-a\rho} m_{t|t} + m_t$$

$$m_{t|t} = m_{t|t-1} + \kappa_\tau(p_t, p_{t|t-1}, ..., \eta_{t+1})$$

Question: can we formulate model so as to talk about importance of the *degree of imperfection* λ?

So what's the mechanism in, say, a stagnant price model?

Let's hit it by a sunspot shock and (expected) inflation-trajectory takes off.

$E_t \pi_{t+1} \uparrow$ real rate $\downarrow$ consumption path $\searrow$

Normally, Taylor Principle steps in by $i_t \uparrow$ to curtail beliefs

But if movements of real rate not fully understood (because the central bank is lacking the full picture)...

... increase may not sufficiently to throttle sunspots expectations. And sunspots self-fulfilled.

Question: How is Kalman gain affected by beliefs (in endogenous signal case)?

$$\kappa_\tau = \kappa_\tau(..., \gamma_b, \sigma_b^2)$$

Are bounds on belief loadings related to this?

Could it mean that (relatively) too much sunspots makes them less important?

Expectations, Fast and Slow

People are fast. Central bank is slow and its response to sudden sunspots not strong enough.

Reminds of the 2000s (apparently) loose policy, "the biggest deviation, comparable to the turbulent 1970s." [Taylor, 2007]...

... debate between Taylor and Bernanke all about using CPI or core PCE data (Doko-Tchatoka, Groshenny, Haque & Weder 2017) in policy rule

$$i_t = \phi \pi_t$$

If inflation data "slow-moving", then estimates make ϕ appear big.

Again, great paper.

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I hope I made some sense.