Paralyzed by Fear: Rigid and Discrete Pricing Under Demand Uncertainty

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Motivation

- Aggregate inflation responds sluggishly to monetary shocks
  - but at the micro-level, prices do not appear to be particularly sticky
- Micro-level facts can be used to distinguish among models
  - Need degrees of freedom and/or parsimonious models
  - What micro-moments matter? In many models, just frequency and kurtosis (Alvarez et. al. (2016))
- This paper: a new model of rigid prices
  1. Firms face Knightian uncertainty about competitive environment
     - ★ March and Shapira (1987): managers exhibit uncertainty aversion
  2. Parsimonious: consistent with a large set of challenging empirical facts
  3. Those moments matter: strong monetary non-neutrality

Key Mechanism: competition uncertainty

1. Uncertainty about the demand function: $x(.)$
   - Not confident it belongs to a particular parametric family of functions
   - Uncertainty reduction local to location of observations, i.e. past prices

     $+$ ambiguity aversion $\Rightarrow$ kinks in *as if* expected demand at past prices
     - ★ if consider a price increase $\Rightarrow$ worry demand is relatively elastic
     - ★ if consider a price decrease $\Rightarrow$ worry demand is very inelastic

2. Uncertainty about relevant relative price: $x(p_{ijt} - p_{jt})$
   - Relevant price index of competition is unknown; review it infrequently
   - The law of motion of industry price level is uncertain (ambiguous)
     - ★ if act under belief that unobserved industry price rose (fell)

       $\rightarrow$ want to increase (decrease) its nominal price
     - ★ precisely the wrong action in case industry price actually fell (rose)

   $\Rightarrow$ act *as if* industry inflation is not forecastable in short-run

Key Implications

- Kinks from lower uncertainty at previously posted prices $\Rightarrow$ endogenous, time-varying and history-dependent cost of price change
- Leads to prices that are
  1. sticky : do not want to move and face higher uncertainty
  2. display memory : price changes likely to move back to 'safer' prices
  3. increasingly attractive: larger kinks if posted more often
  4. both flexible and sticky: endogenous cost of adjustment
  5. becoming stickier as a firm ages (and loses experimentation incentives)
- Novel empirical implications: prices with unusually high demand realizations are stickier, which we show is true in the data
- Significant and persistent real effects of monetary policy

Literature

1. Sticky prices
   1. Empirical
      - ★ Micro data: Bils & Klenow (2004), Klenow & Kryvtsov (2008), Nakamura & Steinsson (2008), Eichenbaum et al. (2011), Campbell and Eden(2014), Vavra (2014)
   2. Theory: pricing rigidities
      - ★ Real: Ball & Romer (1990), Kimball (1995), kinked demand curves (Stigler 1947, Stiglitz 1979)
      - ★ Nominal: Calvo, Taylor, menu costs (eg. Alvarez et al.(2011), Midrigan(2011), Kehoe & Midrigan, 2015), rational inattention (eg. Matejka (2015), Stevens (2014)), many others
2. Pricing under demand uncertainty
   - Parameter learning: Rothschild (1974), Willems (2011), Bachmann & Moscarini (2011), Baley and Blanco(2018), Argente and Yeh (2018)
3. Knightian uncertainty
   - Gilboa & Schmeidler (1989), Epstein & Schneider (2007)

Outline

1. **Analytical Model**
   - Learning under ambiguity about demand function
   - Optimal pricing
     - static and dynamic tradeoffs
2. Quantitative Model
   - Nominal Rigidity – ambiguity about relevant relative price
   - Quantitative Results
   - Monetary Policy implication

Analytical Model

- The firm faces log marginal cost c_t, sells single good for price p_t
- Time t profit:

$$\upsilon(p_t, q_t, c_t) = (e^{p_t} - e^{c_t})e^{q(p_t)}$$

  - demand:

$$q_t = x(p_t) + z_t$$

- Information:
  - not observe $x(p_t)$ and z_t separately
  - z_t is risky - i.e. know that

$$z_t \sim iidN(0, \sigma_z^2)$$

  - $x(.)$ is ambiguous – not know its probability distribution
  - the firm learns about $x(.)$ through past sales data $\{q^{t-1}, p^{t-1}\}$

Learning Framework

- Priors on $x(.)$ are Gaussian Process distr.: for any $\mathbf{p} = [p_1, ..., p_N]'$

$$x(\mathbf{p}) \sim N\left(\begin{bmatrix} m(p_1) \\ \vdots \\ m(p_N) \end{bmatrix}, \begin{bmatrix} K(p_1, p_1) & \dots & K(p_1, p_N) \\ \vdots & \ddots & \vdots \\ K(p_N, p_1) & \dots & K(p_N, p_N) \end{bmatrix} \right)$$

- The firm entertains a **set** of priors Υ, differing in mean function $m(p)$
- The firm has ex-ante information that $m(p)$:
  1. lies within an interval centered at true DGP $x^{DGP} = -bp_t$:

$$m(p) \in [-\gamma - bp, \gamma - bp]$$

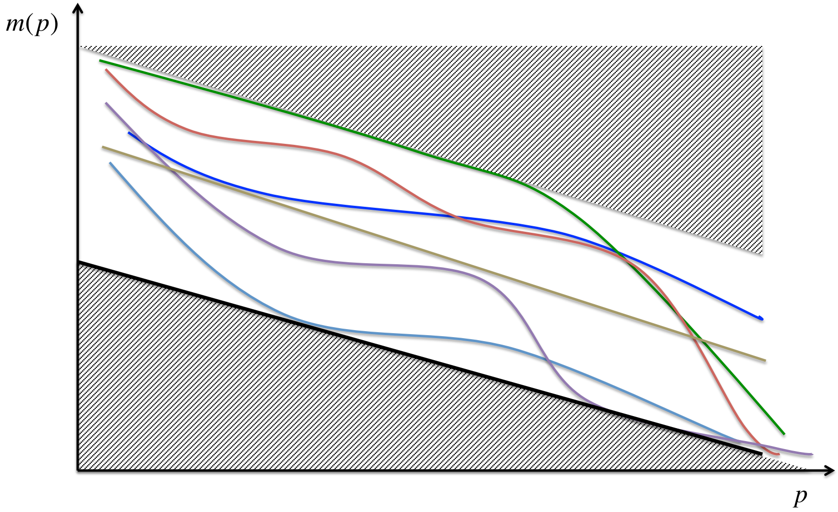
  2. is non-increasing, i.e. is a demand curve:

$$m(p') \leq m(p), \text{ for } \forall p' > p$$

  3. is differentiable, with derivative within an interval around true DGP

$$m'(p) \in [-b - \delta, b + \delta]$$

Admissible Prior Mean Functions



Conditional Beliefs and Profit Maximization

- The firm uses data $\varepsilon^{t-1} = (p^{t-1}, q^{t-1})$ to update each prior
- Recursive multiple priors utility (Epstein-Schneider (2007))

$$V\left(\varepsilon^{t-1}, c_t\right) = \max_{p_t} \min_{m(p)\in\Upsilon} E\left[\upsilon(\varepsilon_t, c_t) + \beta V\left(\varepsilon^{t-1}, \varepsilon_t, c_{t+1}\right) \middle| \varepsilon^{t-1}, c_t\right]$$

  - Min operator is conditional on price choice p_t
    - The firm looks for the p_t choice robust to the set of possible $m(p)$

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- Worst-case $m(p)$ – lowest expected demand $\widehat{x}_{t-1}(p_t; m(p))$:

$$m^*(p; p_t) = \operatorname*{argmin}_{m(p)\in\Upsilon} \widehat{x}_{t-1}(p_t; m(p))$$

Kinks in expected demand: a simple example

- Imagine $\varepsilon^{t-1} = \{p_0, \bar{q}_0, N_0\}$
  - $\alpha_{t-1}(p)$ is the associated signal-to-noise ratio for any p
- **Set** of conditional expectations, indexed by the different $m(p) \in \Upsilon$

$$\widehat{x}_{t-1}(p_t; m(p)) = \underbrace{(1 - \alpha_{t-1}(p_t))\, m(p_t)}_{\text{Prior of demand at } p_t} + \underbrace{\alpha_{t-1}(p_t)\,(q_0 + m(p_t) - m(p_0))}_{\text{Signal } + \Delta \text{ in Demand between } p_t \text{ and } p_0}$$

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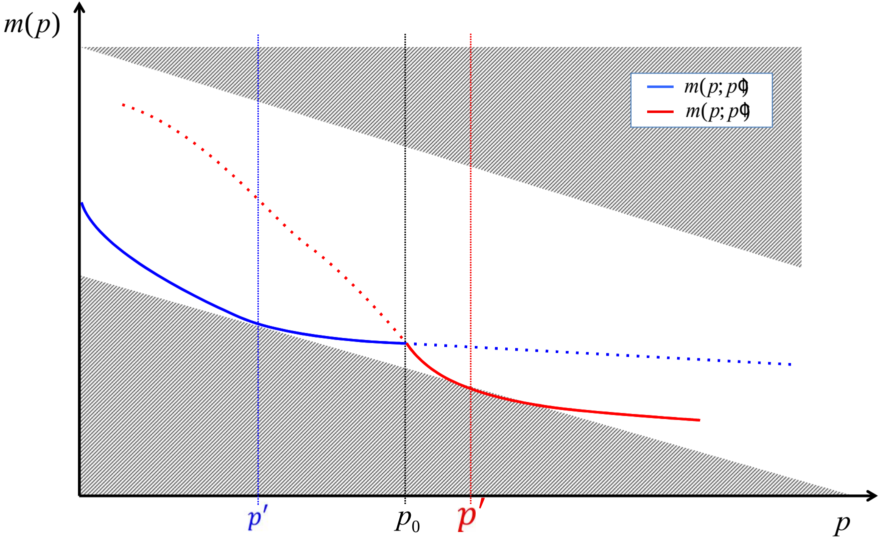
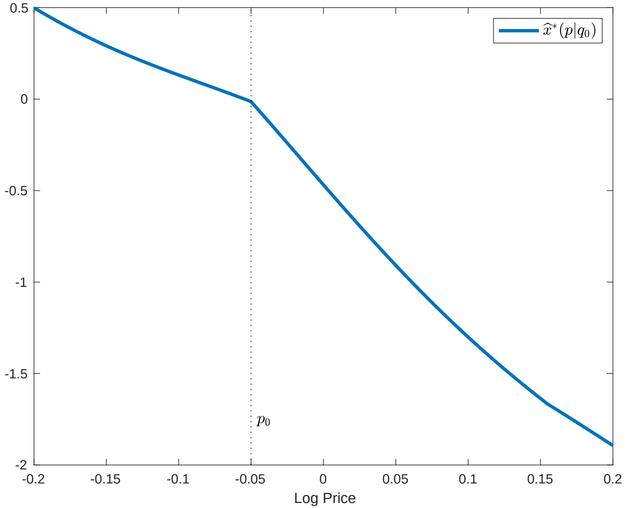
- Worst-case priors: minimize
  1. Prior demand at p_t:
  $$m^*(p_t) = -\gamma - bp_t$$
  2. Δ in demand from p_t to p_0: worst-case conditional on price choice p_t
     - For $p_t > p_0$: worry demand is elastic between p_t and p_0
     $$m^*(p_t) - m^*(p_0) = -(b + \delta)(p_t - p_0)$$
     - For $p_t < p_0$: worry demand is inelastic between p_t and p_0
     $$m^*(p_t) - m^*(p_0) = -(b - \delta)(p_t - p_0)$$

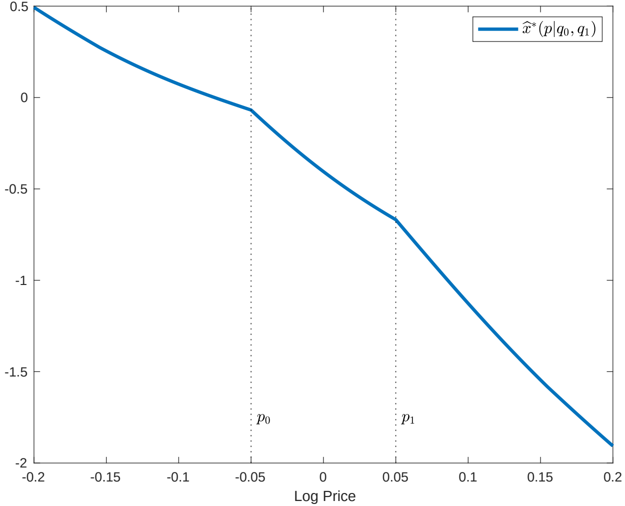
Worst-case prior is conditional on price



As if KINKED EXPECTED DEMAND



Two observed past price levels



Optimal pricing: Myopic (static) maximization

- Key implication: first-order cost of changing price from $p_i \in \varepsilon^{t-1}$

Result 1: Prices are sticky

$$\Pr\left(p_t^* = p_{t-1} | \varepsilon^{t-1}\right) = \int_{\underline{c}_{t-1,t-1}}^{\overline{c}_{t-1,t-1}} g(c_t | c_{t-1}) dc_t > 0$$

Result 2: Conditional on change prices display memory

$$\Pr\left(p_t^* = p_i \in \varepsilon^{t-1} | p_{t-1} \neq p_i, \varepsilon^{t-1}\right) = \int_{\underline{c}_{t-1,i}}^{\overline{c}_{t-1,i}} g(c_t | c_{t-1}) dc_t > 0$$

Result 3: Inaction widens if price is observed more often

$$\frac{\partial \underline{c}_{t-1,i}}{\partial N_i} < 0; \frac{\partial \overline{c}_{t-1,i}}{\partial N_i} > 0$$

Result 4: Good demand realizations ($\bar{q}_i$) increase stickiness

Optimal pricing: Forward-looking incentives

- Price choice affects profits today and information set tomorrow: ε^t
- History of observations ε^{t-1} is an infinitely long state variable $\Rightarrow$ general dynamic problem is intractable
- To get around this issue, we use following approximation
  - The firm understands $\varepsilon^t = \{p_t, q_t, \varepsilon^{t-1}\}$
  - But thinks no new information in future: $\varepsilon^{t+k} = \varepsilon^t$ for all $k > 0$
  - no ad-hoc restrictions on size or structure of ε^{t-1} needed
- Key forward-looking pricing incentives
  1. Experimentation: choose p_t in unexplored part so uncertainty is high
  2. Value relevant info: obtain signals that affect beliefs about demand near prices likely to be posted in the future $\rightarrow$ so p_t close to p_{t+k}

Option value of experimentation

- New information has an important option value component
  - If new signal q_t is bad, firm can switch future prices (learning is local)
- As a result, forward-looking price setting incentives can both counteract and reinforce myopic stickiness
  - Depends on the structure of initial information ε^{t-1}
- Analytical results for a couple of illuminating cases
  1. If firm has seen just one price level p_0 in neighborhood of future expected cost, then the price maximizing information value is $p_t \neq p_0$
  2. If firm has seen two distinct such prices, then there exists interval of costs such that $p_t = p_0$ maximizes information value
- Highlights importance of the structure of ε^{t-1}
  - for example: interesting life-cycle effects
  - history is endogenous in our quantitative model

Outline

1. Analytical Model

2. **Quantitative Model**

   - Nominal Rigidity – ambiguity about relevant relative price
   - Quantitative Results
   - Monetary Policy implication

Nominal prices

- Household: CES aggregator over goods produced by industries j
- Industry j: aggregates over interm. goods $\Rightarrow$ demand for good i

$$y_{i,j,t} = h(\underbrace{p_{i,t} - p_{j,t}}_{\equiv r_{it}}) \underbrace{- b(p_{j,t} - p_t) + y_t}_{=\text{demand for industry } j} + z_{i,t}$$

1. Firm i observes aggregate and own realizations: $\{p_t, y_t, p_{i,t}, y_{i,t}\}$
2. Firm i observes relevant prices $p_{j,t}$ infrequently, with prob. λ_T

- Ambiguity about competition: two layers

1. **demand function** set of GP over industry demand $h(.)$
2. **argument of demand function**: ambiguity about idustry price $p_{j,t}$
   - ★ Firm understands p_{jt} and aggregate p_t are co-integrated, but uncertain about short-run relationship $\phi(.)$ – set of GP distributions

$$p_{jt} - \tilde{p}_{jt} = \phi(p_t - \tilde{p}_{jt}) \in [-\gamma_p, \gamma_p], \text{ for } |p_t - \widetilde{p}_{j,t}| \leq \Gamma.$$

where $\tilde{p}_{jt}$ is value of last perfectly revealing signal on p_{jt}

Joint uncertainty

- Relative price of firm i = unambiguous estimate + ambiguous part

$$r_{it} = p_{it} - p_{jt} = \underbrace{p_{it} - \tilde{p}_{jt}}_{\text{unambiguous estimate} \equiv \tilde{r}_{it}} - \phi(p_t - \tilde{p}_{jt})$$

- Illustrate joint uncertainty: $t = 1$, firm born at $t = 0$
- The uncertain part of demand is

$$(1-\alpha)\left(\underbrace{m(r_{i,1}) - b\phi(p_1 - \widetilde{p}_{j,1})}_{\text{Prior of demand at } r_{i,1}}\right)$$

$$+\alpha\left\{y_{i,0} - \left[\underbrace{m\left(r_{i,0}\right) - m(r_{i,1})}_{\text{Prior on change in demand}} - b\underbrace{\left(\phi(p_0 - \widetilde{p}_{j,0}) - \phi(p_1 - \widetilde{p}_{j,1})\right)}_{\text{Perceived change in industry price}}\right]\right\}$$

- Firm picks price p_{i1} robust to joint uncertainty over $h(.)$ and p_{j1}

Joint worst-case beliefs

- For a price increase $\tilde{r}_{i1} > \tilde{r}_{i0}$, firm worries of a 'double whammy'
  1. Demand is elastic $\delta^* = \delta$
  2. Industry price index fell, increasing firm's effective relative price
- The joint worst-case beliefs induce the conditional demand schedule:

$$\widehat{x}^*(\tilde{r}_i) = \text{smooth terms} - \delta|\widetilde{r}_{i,1} - \widetilde{r}_{i,0}|$$

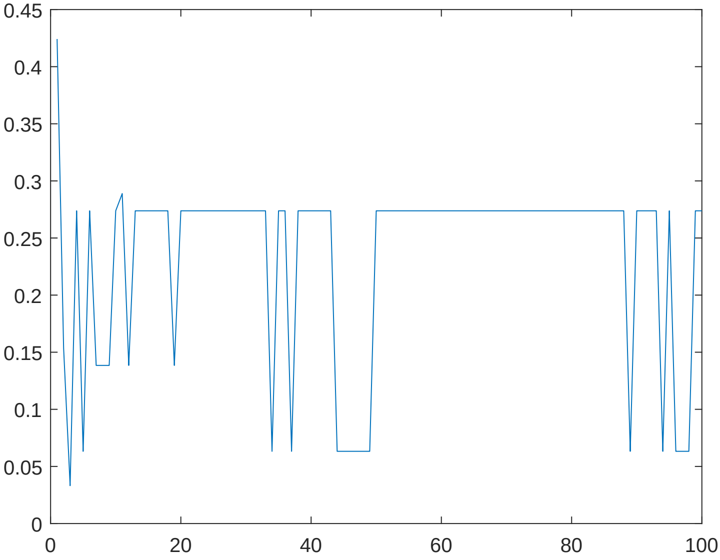
- Two key results:
  1. Relevant argument of worst-case demand is *unambiguous estimate* $\tilde{r}_{it}$
  2. Uncertainty in demand shape leads to kinks around previous $\tilde{r}_{i0}$
- When unambiguous signals on p_{jt} are not continuously available we obtain nominal rigidity and memory in nominal prices
  - indexation not optimal even though p_t is observed

Quantitative evaluation

- Macro model with measure zero of ambiguity-averse firms
  - Aggregate shocks: nominal spending and TFP
  - Endogenous aggregates evolve as with flex prices
  - Micro shocks: iid demand shocks z_{it} and idios. TFP
  - Firms exit with exogenous probability λ_ϕ
- Stochastic steady state
  - Data is endogenous – past prices become attractive reference prices, leading the firm to select from coarse set of prices
  - Never learns demand at all possible prices, friction remains in long-term
  - $\lambda_\phi > 0$ kills dependence on initial conditions
- Parameters:
  - macro: calibrate to standard moments on inflation and aggregate TFP
  - micro: calibrate and estimate using micro-data pricing moments

Calibrated Estimated

A typical path of nominal prices



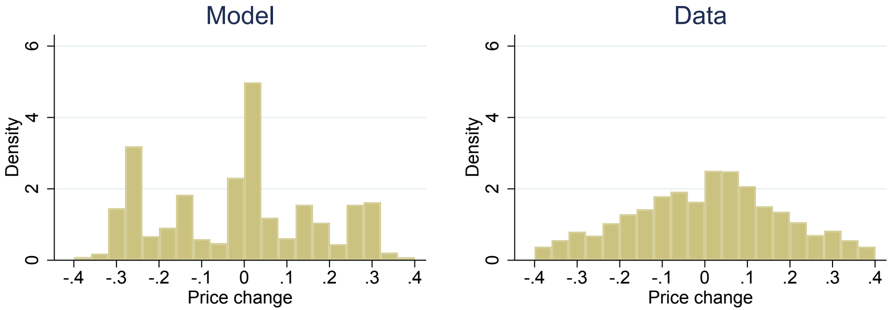
Other testable implications

- Without being targeted, the model fits other moments
  - "reference price" behavior
  - memory
  - declining hazard

		Data	Model
Panel A	Prob. modal P is max P	0.819	0.740
	Frac. of weeks at modal P (13-w window)	0.828	0.880
	Prob. price moves to modal P	0.592	0.669
Panel B	Prob. visiting old price (26-w window)	0.48	0.414
Panel C	Avg hazard slope (LPM)	-0.011	-0.015
	Old vs. young prices - Average slope	-0.104	-0.173

- Significant variation in flexibility and size of price change over life-cycle. In first 26 weeks,
  - Price change probability +23%
  - Avg price changes + 9%
  - consistent with the evidence of Argente and Yeh(2018)

Price change size distribution



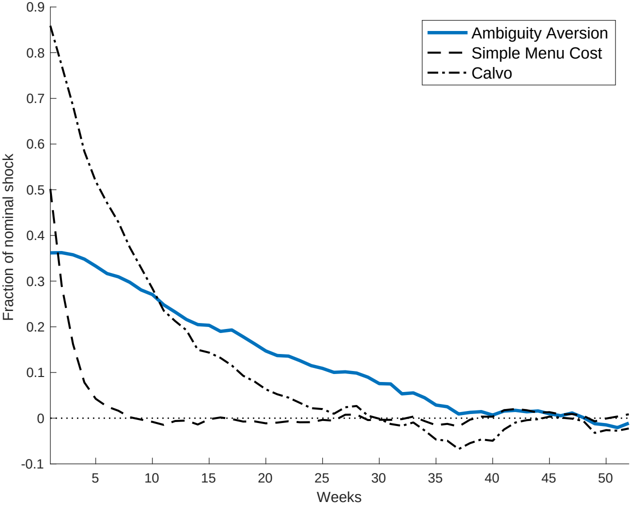
Quantity effects on price change probability

- Novel: higher past innovations $\widehat{z}_t$ lower prob. of price change
- Test in model and in data
  - Estimate $z_{i,t}$ in data from kitchen-sink demand regression
  - Run regression

$$\mathbb{1}(p_{i,j,t} \neq p_{i,j,t-1}) = \alpha_{ij} + \beta_Z \Delta \bar{z}_{ij,t-1} + \beta_N \bar{N}_{ij,t-1} + \varepsilon_{ijt}$$

	Data				Model	
$\bar{N}_{ij,t-1} \leq x$	$x = 12$		$x = 25$		$x = 12$	$x = 25$
$\Delta \bar{z}_{ij,t-1}$	-0.0087	-0.0086	-0.0058	-0.0057	-0.0083	-0.0065
$\bar{N}_{ij,t-1}$	-0.0373	-0.0290	-0.0466	-0.0264	-0.0253	-0.0195
Category/market FE	X		X			
Product/store FE		X		X		

Monetary Policy IRF



Micro-moments shape non-neutrality

- Standard sufficient statistic for non-neutrality (Alvarez et al, 2018)

$$Kurtosis/Frequency$$

  - more general when no history dependence (Baley & Blanco, 2019)
- Our model generates memory (consistent with data)
- Significant departure from the standard result

	Menu cost	Calvo	Our model
Kurtosis	1.25	6	2
MP effects	1.1%	6.5%	6.7%

- Because price movements tend to happen between kinks get long-lived neutrality even
  - as firms exhibit apparent flexibility
  - and there are large price changes

Conclusions

- Novel theory of nominal price rigidity
- Firms face uncertainty about competitive environment
  - Learn about demand function non-parametrically
  - Act *as if* kinked expected demand at previously observed prices
  - Interacted with uncertainty about relative price yields nominal rigidity
  - Consistent with a number of important additional micro-level facts
- Significant real monetary policy effects
  - especially due to their persistence
  - kurtosis not a sufficient statistic
- *Endogenous* cost of price change: history and state dependent rigidity
  - rich laboratory for counter-factual exercises
  - implications for policy

Parameters

- We calibrate macro params and/or for which we have direct evidence

Calibrated Parameters

Parameter	Value	Source/Target
		Macro Parameters
β	0.9994	period is a week, 3% annual int. rate
μ_s	0.00046	2.4% annual inflation
σ_s	0.0015	1.1% std. dev. nominal GDP growth
ρ_a	0.993	Vavra (2014)
σ_a	0.0017	Vavra (2014)
λ_ϕ	0.0075	mean lifespan of a product 2.5 yrs (Argente-Yeh2017)
σ_z	0.613	median demand forecast error IRI dataset
δ	b = 6	set to minimize degree of freedom

Back

Estimation for the rest

- Rest of parameters estimate via SMM

Estimated Parameters

Parameter	Value	Description
ρ_w	0.998	Persistence of idiosyncratic productivity
σ_w	0.008	St. dev. of idiosyncratic productivity shock
σ_x	0.691	Prior variance of $x(.)$
ψ	4.609	Prior covariance function smoothing parameter
λ_T	0.018	Frequency of price reviews
γ	0.614	ambiguity (width of tunnel on $m(r)$)

	Data	Model
Frequency of regular price changes	0.108	0.105
Median size of absolute regular price changes	0.149	0.154
75th pctile of $\lvert\Delta p_{it}\rvert$	0.274	0.277
Fraction of non-zero price changes that are increases	0.537	0.533
Frequency of modal price changes (13-week window)	0.027	0.026
Mean duration of pricing regimes	29.90	30.54

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