The Central Bank's Dilemma:

Look through supply shocks or control inflation expectations?

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Introduction

- Recent inflation spurt elicited similar central bank responses
- Initially most central banks didn't respond
  - rationale: *inflation due to temporary supply shocks*
- Sudden pivot after months of continuing bad inflation data
  - rationale: *need to keep inflation expectations anchored*
  - fear of wage-price spiral

This Paper

- Can observed central bank response arise even without policy errors?
  - Look-through supply-drive inflation shocks initially
  - Sudden monetary tightening
- Can this behavior also be consistent with the offered policy rationales?
  - inflation is supply driven
  - sustained inflation shock induces wage-price spiral

Two Modifications to Standard Models

- Relax rational expectations
- Introduce bounded rationality: *level-k thinking*
  - makes inflation expectations a slow-moving function of current inflation
- Assume wages stickier than prices
  - generates potential wage-price spiral

Results 1

- Structure includes rational expectations and adaptive expectations as special cases
- Under rational expectations, optimal policy looks-through supply-driven inflation shocks
- Under adaptive expectations, optimal policy always responds proportionally to supply-driven inflation
- Neither case generates sudden policy pivot

Results 2

- Optimal response to supply shocks under *level-k thinking*
  - initially look-through but pivot sharply if inflation deviations cross threshold
- Arises when either
  - the central bank cares “enough” about employment
  - with a very flat Phillips curve

Model

- Builds on Blanchard-Kiyotaki (1987)
- Closed economy with households, firms and a central bank
- Households supply differentiated labor: wage-setting power
- Firms supply differentiated goods: price-setting power
- Wages set before observing shocks for the period
- Wages and prices are both set for one-period

Households

- n individuals who maximize their expected lifetime utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\ln C_{it} - \eta N_{it}\right]$$

- C_{it} is the final good produced by combining intermediates

$$C_{it} = \left(\sum_{j=1}^{m} C_{ijt}^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}$$

- Individual budget constraint

$$\sum P_{jt} C_{ijt} + B_{it+1} = W_{it} N_{it} + D_{it} + \tau_{it} + B_{it}(1 + \iota_t)$$

Households II

- Individuals make three decisions at every date t
  - choose optimal mix of intermediate consumption goods C_{ijt}
  - choose B_{it+1} and W_{it+1}
- Bellman equation

$$V\left(\frac{B_{it}}{P_t}, \frac{W_{it}}{P_t}\right) = max \left\{ \ln C_{it} - \eta N_{it} + \beta \mathbb{E}_t V\left(\frac{B_{it+1}}{P_{t+1}}, \frac{W_{it+1}}{P_{t+1}}\right) \right\}$$

- Wage is a state variable here

Firms

- m firms produce intermediate goods

$$Y_{jt} = \theta_{jt} \left(\sum_{i=1}^{n} N_{ijt}^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}}$$

- N_{ij} is labor of type i employed by firm j
- θ_{jt} is the firm's stochastic productivity
- Firms maximize profits by choosing labor and price of product
- Firms decide after observing wages and productivity

Wage-setting rule

- Households choose W_{it+1} at date t to solve their dynamic problem
- Wages are set based on expected productivity, prices and labor demand

$$W_{it+1} = \left(\frac{\rho\eta}{(\rho - 1)m^{\frac{1}{\gamma-1}}} \right) \left(\frac{\mathbb{E}_t N_{t+1}}{\mathbb{E}_t (N_{t+1}/P_{t+1} C_{it+1})} \right)$$

Productivity process

Assumption 1 $\theta_{jt} = \theta_t$ for all j

Assumption 2 $\ln \theta_t = \ln \theta_{t-1} + \epsilon_t$ where $\epsilon_t \sim iid \ (0, \sigma_\theta^2)$

- All firms receive same productivity draw every period
- There is only aggregate uncertainty
- Interpret productivity shock as aggregate oil price shock

Aggregate Wage and Price

- Aggregate price level

$$\ln P_t = \ln W_t - \ln \theta_t + \Lambda_P$$

- Aggregate wage

$$\ln W_t = \mathbb{E}_{t-1} \ln P_t + \mathbb{E}_{t-1} \ln \theta_t + \mathbb{E}_{t-1} \ln \left(\frac{N_t}{\bar{N}} \right) + \Lambda_W$$

Phillips Curve

- Phillips curve in the model

$$\pi_t - \pi^* = \mathbb{E}_{t-1}(\pi_t - \pi^*) + \mathbb{E}_{t-1}\left(\ln N_t - \ln \bar{N}\right) - (\ln \theta_t - \mathbb{E}_{t-1} \ln \theta_t)$$

- Key features of this Phillips curve
  - inflation at date t is driven by expectations at date $t-1$
  - no divine coincidence: stabilizing inflation expectations does not stabilize output
  - productivity shocks have direct effect on inflation

Monetary Policy Rule

- Monetary policy ϕ is set to have employment obey

$$N_t = \bar{N} \left(\frac{1 + \pi_t}{1 + \pi^*} \right)^{-\phi_t}$$

- Formulation directly recognizes an employment tradeoff in reducing inflation
- Use Euler equation to derive path of ι that implements rule
- Formulation more convenient for highlighting link between expectation formation and policy

Euler

Equilibrium System

- Inflation and employment

$$\hat{\pi}_t = \mathbb{E}_{t-1}\hat{\pi}_t + \mathbb{E}_{t-1}\hat{N}_t - \hat{\theta}_t$$
$$\hat{N}_t = -\phi_t[\mathbb{E}_{t-1}\hat{\pi}_t + \mathbb{E}_{t-1}\hat{N}_t - \hat{\theta}_t]$$

- Notation

$$\begin{aligned} \hat{\pi}_t &= \pi_t - \pi^* \\ \hat{N}_t &= \ln N_t - \ln \bar{N} \\ \hat{\theta}_t &= \ln \theta_t - \mathbb{E}_{t-1} \ln \theta_t \end{aligned}$$

Level-k thinking

- Start with initial seed (level-0) about aggregate expectation
- Compute aggregate outcome under initial seed
- Update aggregate expectation and recompute aggregate outcome
- Repeat *k*-times for level-k thinking
- Finite k iterations reflects bounded rationality

Level-k thinking II

- Let initial seed (level-0) expectation be

$$\mathbb{E}_{t-1}\hat{\pi}_t^0 = \hat{\pi}_{t-1}$$
$$\mathbb{E}_{t-1}\hat{N}_t^0 = \hat{N}_{t-1}$$

- Equilibrium system

$$\hat{\pi}_t^{KLT} = (1-\phi_t)^k \left[\hat{\pi}_{t-1} + \hat{N}_{t-1}\right] - \hat{\theta}_t$$
$$\hat{N}_t^{KLT} = -\phi_t \left[(1-\phi_t)^k \left\{\hat{\pi}_{t-1} + \hat{N}_{t-1}\right\} - \hat{\theta}_t\right]$$

Policy Problem

- Policymaker's problem

$$\min_{\phi_t} \sum_{t=0}^{\infty} \beta_G^t \mathbb{E}_{t-1} \left(\hat{\pi_t}^2 + \mu \hat{N}_t^2 \right)$$

- Define $x_t \equiv \hat{\pi}_t + \hat{N}_t$

- Restated problem:

$$\min_{\phi_t} \sum_{t=0}^{\infty} \beta_G^t \mathbb{E} \left[(1 + \mu \phi_t^2) \left((1 - \phi_t)^{2k} x_{t-1}^2 + \sigma_\theta^2 \right) \right]$$

subject to

$$x_t = (1 - \phi_t)^{k+1} x_{t-1} - (1 - \phi_t) \hat{\theta}_t$$

Rational Expectations

- Optimal policy when $k \to \infty$

$$\phi_t^{RE} = 0$$

- Look through any deviations of inflation from target
- Inflation expectations anchored at π^* under policy commitment
- No reason to react to inflation

Adaptive Expectations

- Optimal policy when $k = 0$

$$\phi_t^{AE} = \frac{\beta_G a_1}{\mu + \beta_G a_1} \in (0, 1), \quad a_1 > 0$$

- No policy pivot: ϕ^{AE} is constant
- Policy more hawkish than under rational expectations

Level-k thinking: analytical results

- Analyze special case $\beta_G = 0, k = 1$
- Myopic policymaker
- Full dynamic problem solved quantitatively below

Level-k thinking: illustrating possibilities

- Define $\tilde{x}_{t-1} \equiv \frac{x_{t-1}^2}{\sigma_\theta^2}$
- First order condition (FOC)

$$\mu\phi_t \left[(1-\phi_t)^2 \tilde{x}_{t-1} + 1\right] = (1+\mu\phi_t^2)(1-\phi_t)\tilde{x}_{t-1}$$

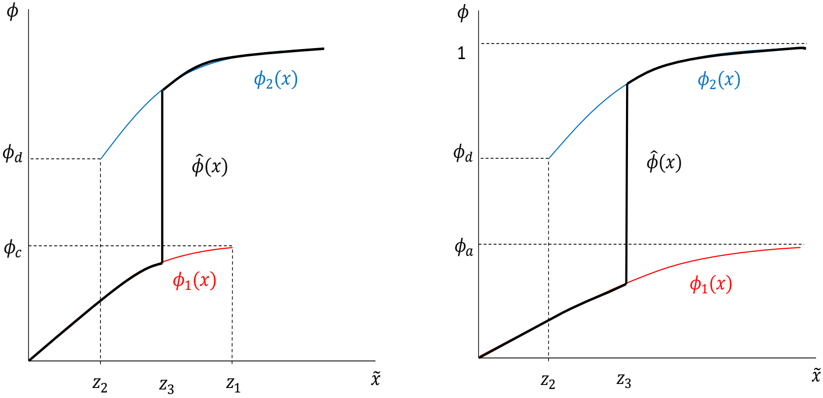
- FOC is a cubic: multiple solutions

Optimal Policy

- Optimal ϕ_t depends on $\tilde{x}_{t-1}$
- Multiple solutions implies ϕ is likely a correspondence
- For $\mu > 4$ there exist two functions: $\phi_1(\tilde{x}),\ \phi_2(\tilde{x})$
  - functions represent local optima
  - functions have overlapping domains
- Need to determine global optima in the overlapping zone

Proposition: Policy Pivot

If μ is sufficiently big, there exists a unique cutoff for $\tilde{x}_{t-1}$, such that at this cutoff, the global optimum $\hat{\phi}(\tilde{x}_{t-1})$, jumps up discontinuously.



$4 < \mu < 8$ $\mu > 8$

Intuition for Pivot: Non-convexity

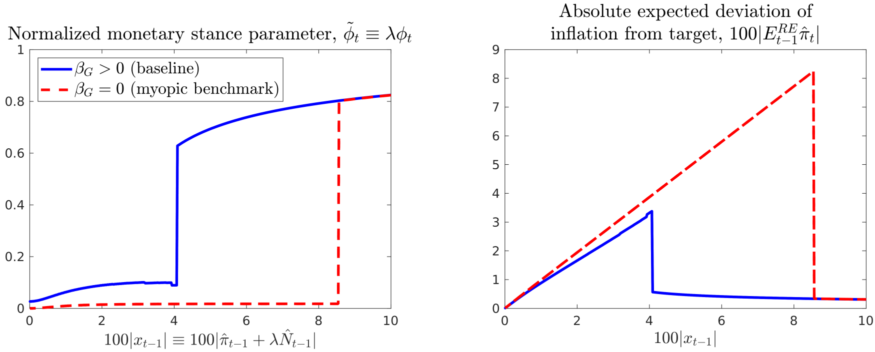
- Policy tightening reduces employment *directly*
- Tightening raises employment *indirectly* by reducing inflation expectations
- Policy elasticity of inflation expectations rising in ϕ
- Indirect effect rises with ϕ and is greater the larger is $\hat{\pi}$
- Direct effect overwhelmed by indirect effect at high enough $\hat{\pi}$

Proposition: Soft Landing

If ϕ *and* ϕ' *represent the two stances of monetary policy at the pivot point, then* $\phi(1-\phi) = \phi'(1-\phi')$*. This implies that at the optimal point of pivot, inflation is expected to fall but employment is expected to stay constant. However, the variance of employment increases discontinuously at the pivot point.*

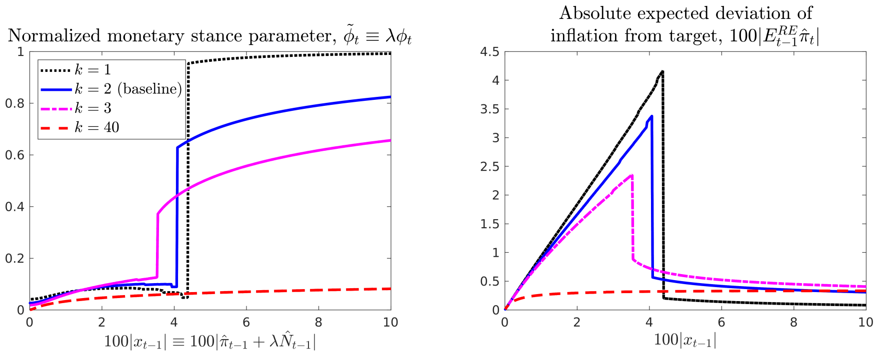
- No jump in employment at pivot (soft landing)

Dynamic Model: Varying β_G



- Higher β_G induces more concern about inflation expectations
- Lower threshold for pivot

Different levels of thinking: Varying k



- Higher k shifts pivot point lower and reduces size of pivot
- Pivot disappears for sufficiently high k

Conclusions

- Paper provides framework for studying monetary policy response to supply shocks
- Key ingredients
  - bounded rationality: level-k thinking
  - prices more flexible than wages
- Implications of structure
  - Phillips curve without divine coincidence
  - expectations are slow-moving object
- Looking through supply shocks can be optimal, till some point

Slope of Phillips Curve

- Phillips curve in the model is

$$\hat{\pi}_t = \mathbb{E}_{t-1}\hat{\pi}_t + \mathbb{E}_{t-1}\hat{N}_t - \hat{\theta}_t$$

- Slope is unity: restrictive and empirically debatable

- Generalization with GHH preferences

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t \ln\left(C_{it} - \eta\theta_t N_{it}^{1+\lambda}\right),$$

- Revised Phillips curve

$$\hat{\pi}_t = \mathbb{E}_{t-1}\hat{\pi}_t + \lambda\mathbb{E}_{t-1}\hat{N}_t - \hat{\theta}_t,$$

Revised interpretation of μ

- Define $\tilde{\mu} \equiv \frac{\mu}{\lambda^2}$ and $\tilde{\phi}_t \equiv \lambda \phi_t$
- Policy problem can be written as

$$\min_{\tilde{\phi}_t} \sum_{t=0}^{\infty} \beta_G^t \mathbb{E}\left[(1 + \tilde{\mu}\tilde{\phi}_t{}^2)\left((1-\tilde{\phi}_t)^{2k} x_{t-1}^2 + \sigma_\theta^2\right)\right]$$

subject to

$$x_t = (1-\tilde{\phi}_t)^{k+1} x_{t-1} - (1-\tilde{\phi}_t)\hat{\theta}_t$$

- Same problem but with $\tilde{\phi}$ and $\tilde{\mu}$ replacing ϕ and μ
- Propositions with μ go through with $\tilde{\mu}$

Euler equation

- The Euler equation is

$$\iota_{t+1} - \bar{\iota} = \mathbb{E}_t(\ln N_{t+1} - \ln N_t) + \mathbb{E}_t(\pi_{t+1} - \pi^*)$$

- Solving forward, this gives

$$\ln N_t - \ln \bar{N} = -\sum_{h=1}^{\infty} \mathbb{E}_t \cdot \cdot \mathbb{E}_{t+h-1}[\iota_{t+h} - \bar{\iota} - (\pi_{t+h} - \pi^*)]$$

Rule