Raising an Inflation Target: the Japanese Experience with Abenomics

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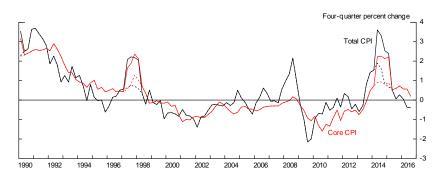
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Goal of the paper

- This paper studies the effects of increasing the inflation target in a liquidity trap.
- The motivation is to shed light on Japan's recent efforts to overcome deflation.



Abenomics

- Shinzo Abe became Japan's Prime Minister in 2012, running on a platform known as "Abenomics".
- Key element of Abenomics: aggressive monetary easing to overcome deflation.
 - November 2012: candidate Abe promises radical reorientation of monetary policy.
 - February 2013: BOJ adopts new inflation target of 2 percent.
 - April 2013: BOJ unveils "Quantitative and Qualitative Monetary Easing" (QQE).
 - October 2014 and December 2015: BOJ expands QQE.
 - February 2016: BOJ introduces "QQE with a Negative Interest Rate".
 - **September 2016**: BOJ introduces "QQE with a Yield Curve Control" and an "inflation-overshooting commitment".
- Sample period of the published paper ends in 2015:Q2.



Focus on Inflation Expectations Channel

"QQE aims to raise inflation expectations through the Bank's strong and clear commitment to achieving the price stability target of 2 percent and through large-scale monetary easing that underpins the commitment." (Gov. Kuroda, Sep. 2016)

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 - Asset purchases as a commitment device.

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- This paper studies the effects of raising inflation expectations through a change in the inflation target.
 - Asset purchases as a commitment device.
- This paper does not consider the direct effects of asset purchases on long-term nominal interest rates.

Preview of Main Results

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- Increasing the inflation target can have powerful effects on activity, especially at in a liquidity trap.
- However, such policy might have more limited effects, if the rise in the target is not fully credible.
- Japan's recent experience raises this concern as inflation expectations remain well below 2 percent.

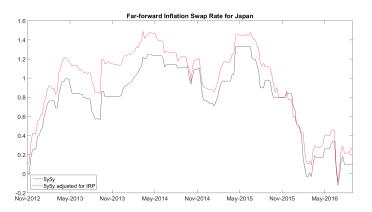
Inflation Expectations (Percent)

	5x5 inflation swap rate	10-year inflation swap rate	6-10 year ahead inflation by Consensus	
2012 Q3	0.0	0.3	0.8	
2015 Q2	1.2	1.0	1.6	
2015 Q4	0.8	0.8	1.4	
2015 Q2	0.2	0.3	1.3	

Sources: Bloomberg and Consensus Economics.



Inflation Swap Rate Adjusted for Inflation Risk Premia



Source: Rodriguez and Yoldas (2016, forthcoming IFDP Note)



- Data with limited theory: The effects of inflation target shocks using a VAR model
- Theory with limited data: Inflation target shocks in closed- and open-economy New-Keynesian DSGE models

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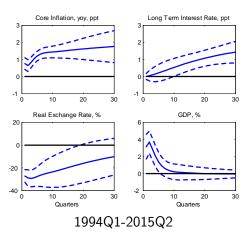
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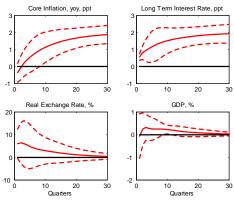
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 - 3. affects inflation and the interest rate one-for-one in the long run.
- Only short-run restriction is that inflation does not affect oil prices contemporaneously.

VAR: Responses to a 2ppt Inflation Target Shock

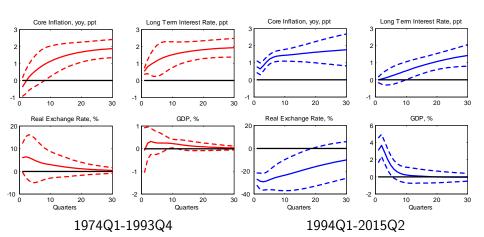


VAR Impulse Responses: no ZLB vs. ZLB



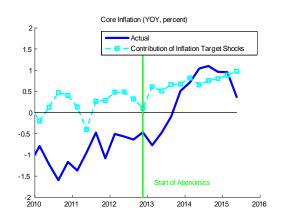
1974Q1-1993Q4

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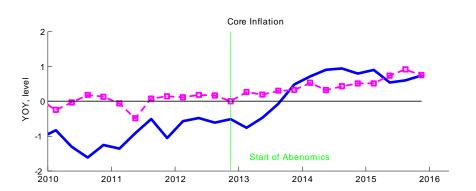
Size of Inflation Target Shock (2013:Q4-2015:Q2)

Historical decomposition of core inflation into the shocks identified by the VAR



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Summary of VAR Results

- Reflating the economy leads a short-run output boost.
- Response in a liquidity trap is much larger.
 The muted response of interest rates leads a substantial currency depreciation and a larger output boost.
- Are these shocks plausible/frequent?
 No. An inflation target shock of 2 percentage points is a 6 standard deviation shock in our sample.
- Has the identified inflation target moved to 2 percent?
 Only partially, and even smaller effects with more recent data.

A Closed Economy New-Keynesian Model

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- Model features nominal price and wage rigidities, habits in consumption, investment adjustment costs, and fiscal and monetary authorities.

Households maximize a lifetime utility function given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\mathsf{a}_{ct} \log \left(c_t - \varepsilon_c c_{t-1} \right) - \frac{1}{1+\eta} n_t^{1+\eta} \right)$$

where c_t is consumption in period t, \mathbf{a}_{ct} a consumption preference shock, and n_t hours worked.

NK Model Environment 1/3

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Their budget constraint is given by:

$$c_t + k_t + \phi_t = w_t n_t + (R_{kt} z_t + 1 - \delta) k_{t-1} + \text{div}_t - \tau_t - b_t + \frac{R_{t-1}}{\Pi_t} b_{t-1}$$

where k_t denotes capital, ϕ_t adjustment costs, w_t the wage rate, $(R_{kt}z_t+1-\delta)\,k_{t-1}$ capital income, ${\rm div}_t$ dividends, τ_t taxes, b_{t-1} one-period government debt, and Π_t gross inflation rate.

NK Model Environment 2/3

 Monopolistic competition in the goods and labor markets, coupled with staggered nominal adjustment à la Calvo.

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- Monopolistic competition in the goods and labor markets, coupled with staggered nominal adjustment à la Calvo.
- Firms that do not adjust their prices and wages index them to the previous period inflation rate with a elasticities given by ι_{π} and ι_{w} , respectively. The price and wage Phillips curves are:

$$\ln \pi_t - \iota_{\pi} \ln \pi_{t-1} = \beta \left(E_t \ln \pi_{t+1} - \iota_{\pi} \ln \pi_t \right) - \varepsilon_{\pi} \ln \left(X_{pt} / X_{pc} \right),$$

$$\omega_t - \iota_w \ln \pi_{t-1} = \beta \left(E_t \omega_{c,t+1} - \iota_w \ln \pi_t \right) - \varepsilon_w \ln \left(X_{wt} / X_{wc} \right)$$

where $\omega_t \equiv \frac{w_t \pi_t}{w_{t-1}}$ denotes wage inflation, and $\varepsilon_\pi = \frac{(1-\theta_\pi)(1-\beta\theta_\pi)}{\theta_\pi}$ and $\varepsilon_w = \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w}$ denote the elasticities of price and wage inflation to price and wage markups, respectively.

NK Model Environment 3/3

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- The government levies lump-sum taxes which respond to beginning of period debt, and buys g_t as a constant fraction of the final output each period.
- The economy-wide market clearing condition is

$$Y_t = c_t + i_t + g_t.$$

Inflation Target Shock

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- We also assume that the central bank follows an intertial Taylor rule subject to the ZLB:

$$r_t = \max\left(0, \phi_r r_{t-1} + (1-\phi_r)\left(1\% + \pi_t + \phi_\pi\left(\pi_t - \frac{\pi_t^*}{4}\right) + \frac{\phi_y}{4}\widetilde{y_t}\right)\right)$$

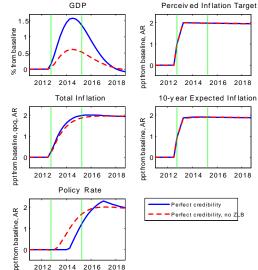
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• What happens when a new $\pi_t^* = 2\%$ target is announced (starting from 0%)?

Inflation Target Shock: ZLB (our benchmark) vs no ZLB



Results of Baseline NK Model

- Inflation target shock moves inflation and inflation expectations close to target quickly despite large price rigidity.
- Inflation target shock has powerful effects on GDP especially in a liquidity trap.
- However, the inflation target shock identified by the VAR is small and inflation expectations are well below 2 percent.

Introducing Imperfect Credibility

- No realistic amount of price rigidity can explain why long-run inflation expectations are not at 2 percent yet.
- We thus modify the model to allow for imperfect credibility about the inflation target.
- Want to capture two ideas:
 - agents are unsure about the BOJ's degree of commitment
 - agents are unsure as to what the BOJ will do in the future.

Modelling Imperfect Credibility

Following Erceg and Levin (2003, JME), we re-write the Taylor rule as:

$$r_{t} = \max\left(0, \phi_{r}r_{t-1} + (1-\phi_{r})\left(rr + \pi_{t} + \phi_{\pi}\left(\pi_{t} - \pi_{t}^{*}\right) + \frac{\phi_{y}}{4}\widetilde{y_{t}}\right) + \mathbf{e}_{t}\right)$$

 π_t^* : persistent monetary policy shock

: transitory monetary policy shock

Formally:

$$\left[\begin{array}{c} \pi_t^* \\ e_t \end{array}\right] = \left[\begin{array}{cc} 0.999 & 0 \\ 0 & 0.001 \end{array}\right] \left[\begin{array}{c} \pi_{t-1}^* \\ e_{t-1} \end{array}\right] + \left[\begin{array}{c} \varepsilon_{pt} \\ \varepsilon_{qt} \end{array}\right]$$

$$arepsilon_{pt} \sim N\left(0, \sigma_{p}^{2}
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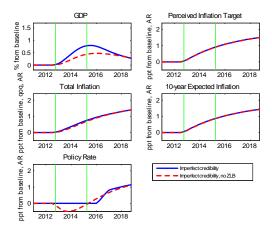
$$Z_t = \pi_t^* - (1 - \phi_r)^{-1} \phi_\pi^{-1} e_t$$
 inflation target persistent component transitory component transitory component

Imperfect Credibility: Some Intuition

- The BOJ challenge: it would like to change long-run inflation $(E_t\pi_{t+\infty})$ and r_t in a "stable manner", affecting $[\pi_t^*, E_t\pi_{t+1}^*, E_t\pi_{t+2}^*, \ldots]$
-but agents might not be able to tell whether the target and the interest rate are changing on a permanent or transitory basis.
- In other words, agents cannot tell whether the current deviations from the historical policy rule are going to last "forever" (π_t^*) or not (e_t) .
- We calibrate the imperfect credibility by the signal-to-noise ratio, σ_p^2/σ_q^2 :
 - σ_p^2/σ_q^2 high: inflation target shock fully credible (as before)
 - σ_p^2/σ_q^2 low: inflation target shock less than fully credible.



Impulse Responses: Perfect vs Imperfect Credibility



Calibrate signal-to-noise to get rise in expected inflation as in data and through the VAR: effect on GDP is smaller.

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- The lack of credibility increases inflation persistence and, at the ZLB, also dampens the output response (the opposite is true away from ZLB, e.g. Goodfriend and King, 2005).
- At the ZLB, inflation target shocks are more powerful the more agents expect them to be permanent (the larger the signal-to-noise ratio σ_p^2/σ_q^2 .)

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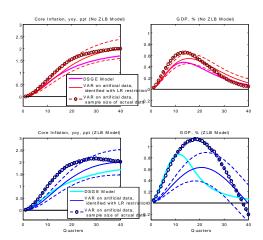
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 - 1. When the ZLB is caused by a sequence of fundamental shocks, the underlying dynamics of the economy may become highly nonlinear (Christiano, Eichenbaum and Rebelo, 2011 JPE).
 - 2. Long-run restrictions, although theoretically appealing, may be unreliable in small samples (Faust and Leeper, 1997 JBES).

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- Compare the impulse responses with the "true" responses generated by our DSGE model.
- Note: We take the "true" responses to be the responses of the macro variables in deviation from a baseline in which policy rates are at 1 percent, the output gap is closed, and inflation is zero, when the economy is outside the ZLB; or a baseline in which the policy rate is zero and expected to be at zero for 6 quarters, when the economy is at the ZLB.

The VAR identification scheme yields impulse responses that "look like" the true effects of the inflation target shock in the DSGE model.





International Effects of Abenomics

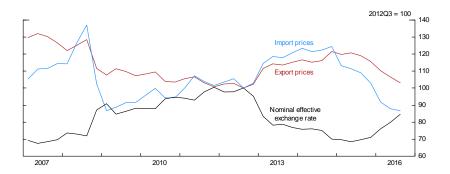
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International Effects of Abenomics

- How much progress has Japan made so far?
 Closed-economy NK model suggests limited progress.
- However, international variables may suggest otherwise.
 Exchange rate and trade price movements have been large since Abenomics.
 Want to understand their role.

External Prices since the Start of Abenomics



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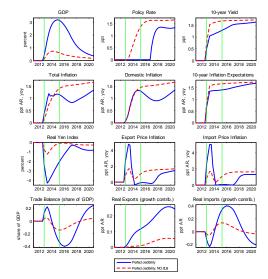


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 - Japanese exporters change their prices (in dollars) very infrequently
 Exports respond little to exchange rate.
 - 2. U.S. and ROW exporters adjust their prices (in yen) more frequently -> Imports respond strongly to exchange rate.

Inflation Target Shock in SIGMA





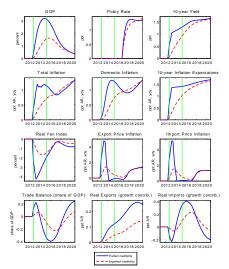
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- The depreciation gives small but transient boost to GDP.
 The shock mostly affects GDP through a domestic demand channel.
- Inflation rises towards its target very slowly.
- However, model unable to capture large yen depreciation seen in the data and through the VAR.
 Layer depreciation shock on top of inflation target shock to match the 6 percent depreciation implied by the inflation target shock identified in the VAR.

Inflation Target and Depreciation Shocks in SIGMA





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- The surge in total inflation is reversed quickly as the inflationary impulse of depreciation dies out.
- Inflation eventually rises towards its target very slowly.

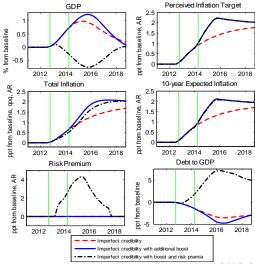
Overshooting the Inflation Target

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- In Sep. 2016, the BOJ introduced an "inflation-overshooting commitment", that is to keep expanding the monetary base inflation exceeds the 2 percent target and stays above the target in a stable manner.
- We have analyzed the effects of temporarily increasing the target from 2 to 3 percent for 6 quarters using the closed economy DSGE under imperfect credibility (Appendix A.2).

Overshooting the Target in the Closed Economy DSGE





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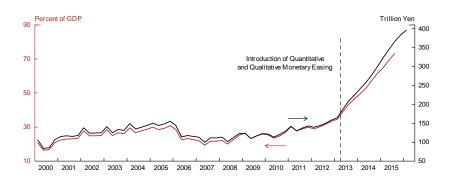
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 - 3. What are the long-run consequences of a higher inflation target?

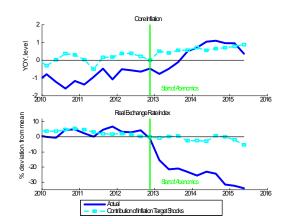
Abenomics: the BOJ's QQE

QQE calls for a rapid and **open-ended** expansion of the BOJ balance sheet until the 2 percent target is reached.



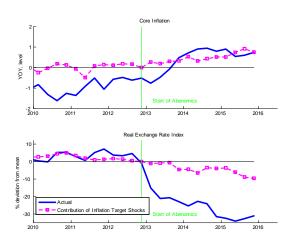
Size of Inflation Target and Exchange Rate Shocks

Historical decomposition of core inflation and the real exchange rate into the shocks identified by the ${\sf VAR}$



Size of Inflation Target and Exchange Rate Shocks

Historical decomposition of core inflation and the real exchange rate into the shocks identified by the ${\sf VAR}$





When the ZLB does not bind we can rewrite the Taylor rules as:

$$\begin{split} r_t &= \phi_r r_{t-1} + (1 - \phi_r) \left(rr + \pi_t + \phi_\pi \left(\pi_t - \pi_t^* \right) + \frac{\phi_y}{4} \widetilde{y_t} \right) + e_t \\ &= \phi_r r_{t-1} + (1 - \phi_r) \left(rr + \pi_t + \phi_\pi \pi_t - \phi_\pi \pi_t^* + \frac{\phi_y}{4} \widetilde{y_t} + \frac{e_t}{1 - \phi_r} \right) \\ &= \phi_r r_{t-1} + (1 - \phi_r) \left(rr + \pi_t + \phi_\pi \pi_t - \phi_\pi \pi_t^* - \frac{-\phi_\pi e_t}{(1 - \phi_r) \phi_\pi} + \frac{\phi_y}{4} \widetilde{y_t} \right) \\ &= 0, \phi_r r_{t-1} + (1 - \phi_r) \left(rr + \pi_t + \phi_\pi \pi_t - \phi_\pi \left(\pi_t^* - \frac{e_t}{(1 - \phi_r) \phi_\pi} \right) + \frac{\phi_y}{4} \widetilde{y_t} \right) \\ &= \phi_r r_{t-1} + (1 - \phi_r) \left(rr + \pi_t + \phi_\pi \pi_t - \phi_\pi \left(Z_t \right) + \frac{\phi_y}{4} \widetilde{y_t} \right) \\ &= \phi_r r_{t-1} + (1 - \phi_r) \left(rr + \pi_t + \phi_\pi \pi_t - \phi_\pi \left(Z_t \right) + \frac{\phi_y}{4} \widetilde{y_t} \right) \end{split}$$