INDETERMINACY AND IMPERFECT INFORMATION

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MOTIVATION

Real-time uncertainty about economic conditions: What are the policy implications?

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Potential application

Consequences of tracking unobserved objects

$$i_t = \bar{r}_{t|t} + \bar{\pi} + \phi_\pi(\pi_{t|t} - \bar{\pi}) + \phi_x(y_{t|t} - \bar{y}_{t|t}) + \dots$$

For example: Orphanides (2003, JME)
w/backward-looking Keynesian model

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We set out to consider the implications in forward-looking models . . .

THIS PAPER

Multiple equilibria in linear RE models

- Well understood when all agents have perfect information: Lubik and Schorfheide (2003), Farmer et al. (2015)
- Indeterminacy gives rise to fluctuations driven by non-fundamental "belief shocks"
- Taylor principle rules out belief shocks and indeterminacy

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We show that under asymmetric information . . .

- . . . belief shocks come into play, even when the full-information model has a unique equilibrium
- . . . scope of belief shocks is limited (novel compared to full-info indeterminacy)

SETUP

Two kinds of agents:

1. **Fully informed** public
2. **Imperfectly informed** central bank

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Equilibrium conditions

- **Linear RE difference system** describes optimal behavior
- Time-invariant, linear, Gaussian equilibria:
  Optimal expectations described by **Kalman filter**
- Standard criterion: stable outcomes

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RELATED LITERATURE

- **Monetary policy under uncertainty:**

  Svensson and Woodford (2004), Aoki (2006), Nimark (2008), Carboni & Ellison (2010)

- **Indeterminacy in rational expectations models:**

  Benhabib & Farmer (1994), Lubik & Schorfheide (2003), Farmer, Khramov and Nicolo (2015), Tchatoka, Groshenny, Qazi & Weder (2017), Ascari, Bonomolo & Lopes (2019)

- **Imperfect information:**

  Pearlman, Currie & Levine (1986), Nimark (2011), Benhabib, Wang & Weng (2015), Mertens (2016), Rondina & Walker (2017)

- **Indeterminacy and learning:**

  Lubik & Matthes (2016)

AGENDA

1. **A Simple Example Economy**
2. **General Framework**
3. **NK Model**
4. **Determinacy Without Optimal Projections**
5. **Conclusions**

A SIMPLE EXAMPLE ECONOMY

Textbook case of Woodford (2003), Gali (2008) ...

An exogenous real rate

$$r_t = \rho\, r_{t-1} + \varepsilon_t \quad E_{t-1}\varepsilon_t = 0$$

Fisher equation and natural-rate policy rule

$$\begin{aligned} i_t &= r_t + E_t \pi_{t+1} \\ i_t &= r_t + \phi \pi_t \qquad\qquad \phi > 1 \end{aligned}$$

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Full-information equilibrium

$$\pi_{t+1} = \phi \pi_t + \eta_{t+1} \qquad \eta_{t+1} \equiv \pi_{t+1} - E_t \pi_{t+1}$$

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Only one stable solution: $\pi_t = 0$ and $\eta_t = 0 \; \forall \, t$

Interest rate rule w/o natural rate tracking

$$i_t = \phi \, \pi_t \quad \text{with} \quad |\phi| > 1$$

Full information outcomes

$$\pi_t = \bar{g} \, r_t \quad \text{and} \quad \eta_t = \bar{g} \, \varepsilon_t \quad \text{with} \quad \bar{g} = \frac{1}{\phi - \rho}$$

LESSONS FROM FULL-INFORMATION BENCHMARK
To avoid multiple equilibria . . .

Policy rule: $i_t = \phi \pi_t + \ldots$

- Must respond to endogenous variables, like $\boldsymbol{\pi_t}$, not just to exogenous states
- Taylor principle, $|\phi| > 1$: "Threat" of explosive behavior invalidates many candidate equilibria
- See, for example, Bullard and Mitra (2002), Gali (2011)

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$|\phi| < 1$ **did contribute to Great Inflation**

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- $|\phi| > 1$ with real-time projections $i_t = \phi\pi_{t|t} + \ldots$

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- $|\phi| > 1$ with real-time projections $i_t = \phi\pi_{t|t} + \dots$
- **and a backward-looking model**

IMPERFECT INFORMATION SETUP

Private sector: perfectly informed

Central Bank: imperfectly informed

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- **Homogenous private sector**
- **Fully informed about state of the economy: S^t**
- $E_t(\cdot) \equiv E(\cdot|S^t)$

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Central Bank: imperfectly informed

- Observes measurement vector $Z_t = HS_t$
- Central bank's projection of any variable x_t is

$$x_{t|t} = E\left(x_t|Z^t\right) \quad \text{and} \quad x_{t+h|t} = E\left(x_{t+h}|Z^t\right)$$

- Nested info sets, Z^t is spanned by S^t:

$$E\left(E_t x_{t+h}|Z^t\right) = x_{t+h|t}$$

POLICY UNDER LIMITED INFORMATION

Limited Information Policy Rule

- Nominal interest rate must be function of Z^t
- Svensson & Woodford (2004): certainty-equivalent rules

$$i_t = r_{t|t} + \phi \pi_{t|t}$$

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Certainty equivalence of full-info solution implies:

$$\pi_{t|t} = 0 \quad \text{and} \quad \pi_{t|t-1} = 0$$

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With Taylor rule, when $\pi_t = \bar{g}\, r_t$ under full-info

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EQUILIBRIUM CONDITIONS W/LIMITED INFORMATION

Fisher equation cum policy rule:

$$r_{t|t} + \phi\pi_{t|t} = r_t + E_t\pi_{t+1}$$

Combines two kinds of expectations

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$$\pi_{t+1} = -(r_t - r_{t|t}) + \eta_{t+1}$$

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η_t **must support the projection condition**

ENDOGENOUS FORECAST ERRORS

As in Farmer et al: project η_t on fundamentals

$$\eta_t = \gamma_\varepsilon \varepsilon_t + \ldots + \gamma_b b_t, \quad b_t \sim N(0,1)$$

- The "..." stands in for other fundamental shocks
- b_t: belief shock, orthogonal to fundamentals

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Full-information case with $i_t = r_t + \phi \pi_t$ $\quad (|\phi| > 1)$

$$\pi_t = 0 \quad \Longrightarrow \quad \eta_t = 0 \quad \Longrightarrow \quad \gamma_b = 0$$

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Imperfect information case: "projection condition"

$\gamma_\varepsilon, \gamma_b$, etc. must support $\pi_{t|t} = 0$

INFORMATION SETS

We consider different classes of information sets

Exogenous signal

Linear combination of exogenous variables

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Closed-form solutions in special case

$$\boldsymbol{Z_t = \begin{bmatrix} \pi_t + \nu_t \\ r_t \end{bmatrix}}$$

Endogenous signal

Signal includes endogenous variables

$$\boldsymbol{Z_t = \pi_t + \nu_t}$$

AGENDA

CASE 1: EXOGENOUS SIGNAL

Central bank's information set

$$Z_t = r_t + \nu_t$$

$$r_{t|t} = \rho \, r_{t-1|t-1} + \kappa_r \left(r_t - r_{t|t-1} + \nu_t \right)$$

$$0 \leq \kappa_r \leq 1$$

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γ_b **unrestricted since** $\mathrm{Cov}\left(b_t, Z_t\right) = 0$

WHAT'S GOING ON?

Remember the Policy Rule

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No response to inflation to begin with...

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Similar reasoning with Taylor-type policy rule

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1. **A Simple Example Economy**
   - Case 1: Exogenous Signal
   - Case 2: Closed-Form Solutions in Special Case
   - Case 3: Endogenous Signal
2. **General Framework**
3. **NK Model**
4. **Determinacy Without Optimal Projections**
5. **Conclusions**

CLOSED-FORM SOLUTIONS IN SPECIAL CASE

Natural-rate rule: $\bar{g} = 0$, Taylor rule $\bar{g} = 1/(\phi - \rho)$

Info set

$$\boldsymbol{Z_t} = \begin{bmatrix} \boldsymbol{\pi_t + \nu_t} \\ \boldsymbol{r_t} \end{bmatrix} \qquad \Rightarrow \quad \boldsymbol{r_{t|t} = r_t}$$

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with $$\gamma_\nu = -\frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{\gamma_b^2}{\sigma_\nu^2}}, \qquad |\gamma_b| \leq \frac{1}{2}\sigma_\nu$$

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Key results

- Non-zero, but bounded belief shock loadings
- Multiple pairs of valid shock loadings (γ_ν, γ_b)

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Filtering depends on π_t

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Recall: $\pi_{t|t-1} = 0$

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Projection condition restricts η_t and bounds γ_b

$$\mathrm{Cov}\left(\pi_t, Z_t\right) = \mathrm{Var}\left(\pi_t\right) + \mathrm{Cov}\left(\pi_t, \nu_t\right) = 0$$

VARIANCE BOUND

Simple model with $Z_t = \pi_t + \nu_t$, natural-rate rule $i_t = r_{t|t} + \phi \pi_{t|t}$

Ingredients

Projection condition: $\pi_{t|t} = 0$

Info set: $Z_t = Z_{t|t}$

$$\Leftrightarrow \ \pi_t - \pi_{t|t} = -(\nu_t - \nu_{t|t})$$

Variance bound

$$\begin{aligned} \mathrm{Var}(\pi_t) &= \mathrm{Var}(\pi_{t|t}) + \mathrm{Var}(\pi_t - \pi_{t|t}) \\ &= 0 + \mathrm{Var}(\nu_t - \nu_{t|t}) \\ &\leq \mathrm{Var}(\nu_t) \end{aligned}$$

VARIANCE BOUND

Simple model with $Z_t = \pi_t + \nu_t$

The variance bound generalizes also to the case of a Taylor-type rule with $\pi_t = \bar{g}\, r_t$ under full-info

Ingredients

Projection condition: $\pi_{t|t} = \bar{g}\, r_{t|t}$

Info set: $Z_t = Z_{t|t}$

$$\Leftrightarrow \pi_t - \pi_{t|t} = -(\nu_t - \nu_{t|t})$$

Variance bound

$$\begin{aligned}
\text{Var}(\pi_t) &= \text{Var}(\pi_{t|t}) + \text{Var}(\pi_t - \pi_{t|t}) \\
&= \bar{g}^2\, \text{Var}(r_{t|t}) + \text{Var}(\nu_t - \nu_{t|t}) \\
&\leq \bar{g}^2\, \text{Var}(r_t) + \text{Var}(\nu_t)
\end{aligned}$$

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SETUP OF OUR GENERAL FRAMEWORK

Linear RE system for $S_t = \begin{bmatrix} X_t' & Y_t' \end{bmatrix}'$

$$\begin{aligned} E_t S_{t+1} + \hat{J} S_{t+1|t} &= A S_t + \hat{A} S_{t|t} + A_i \, i_t \\ i_t &= \Phi_i i_{t-1} + \Phi_J S_{t+1|t} + \Phi_A S_{t|t} \\ Z_t &= H \, S_t \end{aligned}$$

- X_t backward-looking, with exogenous errors ε_t
- Y_t forward-looking, with endogenous errors η_t
- i_t policy controls and Z_t measurement

SETUP OF OUR GENERAL FRAMEWORK

Linear RE system for $S_t = \begin{bmatrix} X_t' & Y_t' \end{bmatrix}'$

$$\begin{aligned} \boldsymbol{E}_t \boldsymbol{S}_{t+1} + \hat{\boldsymbol{J}} \boldsymbol{S}_{t+1|t} &= \boldsymbol{A} \boldsymbol{S}_t + \hat{\boldsymbol{A}} \boldsymbol{S}_{t|t} + \boldsymbol{A}_i \, \boldsymbol{i}_t \\ \boldsymbol{i}_t &= \boldsymbol{\Phi}_i \boldsymbol{i}_{t-1} + \boldsymbol{\Phi}_J \boldsymbol{S}_{t+1|t} + \boldsymbol{\Phi}_A \boldsymbol{S}_{t|t} \\ \boldsymbol{Z}_t &= \boldsymbol{H} \, \boldsymbol{S}_t \end{aligned}$$

- $\boldsymbol{X}_t$ backward-looking, with exogenous errors $\boldsymbol{\varepsilon}_t$
- $\boldsymbol{Y}_t$ forward-looking, with endogenous errors $\boldsymbol{\eta}_t$
- $\boldsymbol{i}_t$ policy controls and $\boldsymbol{Z}_t$ measurement

Assumptions

- Unique full-info equilibrium, and projection condition
- Gaussian errors
- Linear, time-invariant and stationary equilibrium

Kalman filter represents CB expectations

GENERIC INDETERMINACY

Equilibrium state vector dynamics

$\overline{\mathcal{S}}_t$ tracks projections and actual values of $\boldsymbol{Y_t, X_t, i_t, i_{t-1}}$

$$\overline{\mathcal{S}}_{t+1} = \overline{\mathcal{A}}\,\overline{\mathcal{S}}_t + \overline{\mathcal{B}}\begin{bmatrix}\boldsymbol{\varepsilon}_{t+1}\\ \boldsymbol{\eta}_{t+1}\end{bmatrix}$$

with $$\overline{\mathcal{A}} = \begin{bmatrix}(\boldsymbol{A} - \boldsymbol{KC}) & \boldsymbol{0}\\ \mathcal{K}_x \boldsymbol{C} & \mathcal{P}\end{bmatrix}$$

$\overline{\mathcal{A}}$ is generally stable since

- $\mathcal{P}$ stable, known from full-info and stable
- $(\boldsymbol{A} - \boldsymbol{KC})$ stable, provided Kalman filter exists

No restrictions on η_t from $\overline{\mathcal{A}}$

(but some further restrictions from projection condition)

AGENDA

DYNAMIC INFORMATION FRICTION IN A NK MODEL

Private sector PC and IS curves

$$(1-\gamma\beta)\boldsymbol{\pi}_t = \gamma\boldsymbol{\pi}_{t-1} + \beta E_t\boldsymbol{\pi}_{t+1} + \boldsymbol{\kappa}\boldsymbol{x}_t$$

$$\boldsymbol{i}_t = \bar{\boldsymbol{r}}_t + E_t\boldsymbol{\pi}_{t+1} + \boldsymbol{\sigma}(E_t\boldsymbol{x}_{t+1} - \boldsymbol{x}_t)$$

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Exogenous natural rate process

$$\bar{r}_t = \sigma E_t \Delta \bar{y}_{t+1} \qquad \Delta \bar{y}_t = \rho_y \Delta \bar{y}_{t-1} + \varepsilon_t^y$$
$$y_t = \bar{y}_t + x_t$$

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Monetary policy with noisy measurements

$$i_t = \phi_\pi \pi_{t|t} + \phi_x x_{t|t}$$

$$Z_t = \begin{bmatrix} \pi_t + \nu_t^\pi \\ y_t + \nu_t^y \end{bmatrix}$$

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CALIBRATION

Textbook values, noise variances from Ahmadi, Mathes & Wang (2017)

NK model parameters

β	Discount Factor	0.99
σ	Substitution Elasticity	1.00
ϕ	Labor Elasticity	1.00
γ	Inflation Indexation	0.25
κ	PC Slope	0.17
ϕ_π	Taylor Rule Coefficient	2.50
ϕ_x	Taylor Rule Coefficient	0.50

Natural rate process

ρ_y	AR(1) - Coefficient	0.75
σ_y	StD. Output Growth	0.30

Noise variances

σ_π	StD. Measurement Error	0.80
σ_x	StD. Measurement Error	1.39

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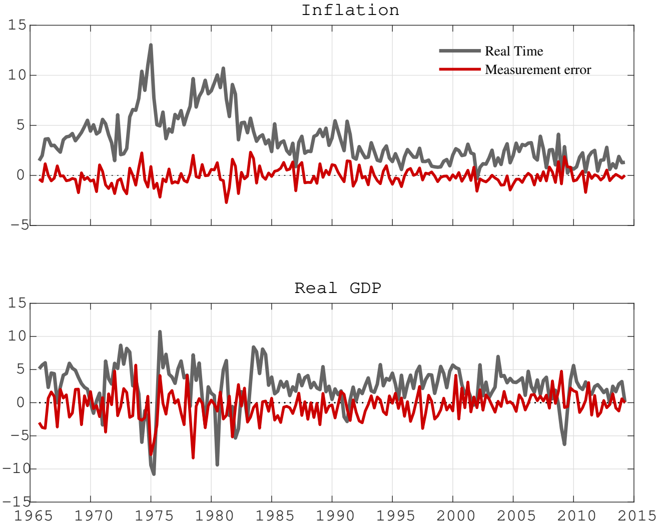
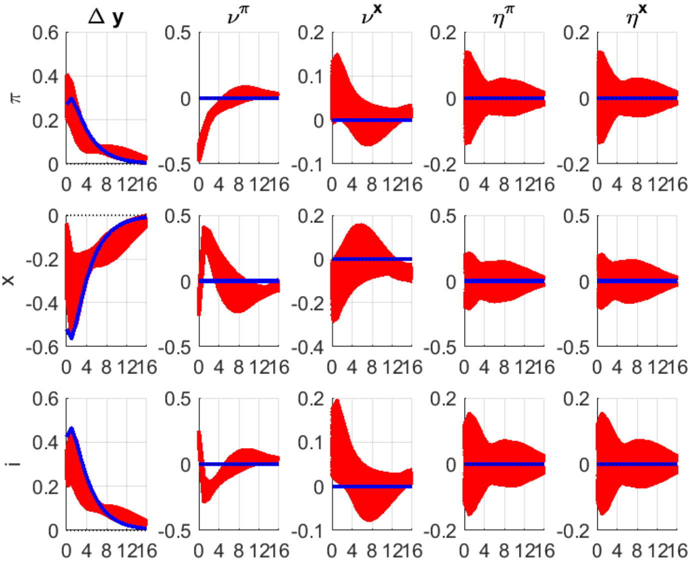
MEASUREMENT ERRORS IN MACRO DATA
Ahmadi, Matthes and Wang (2017, JEDC)



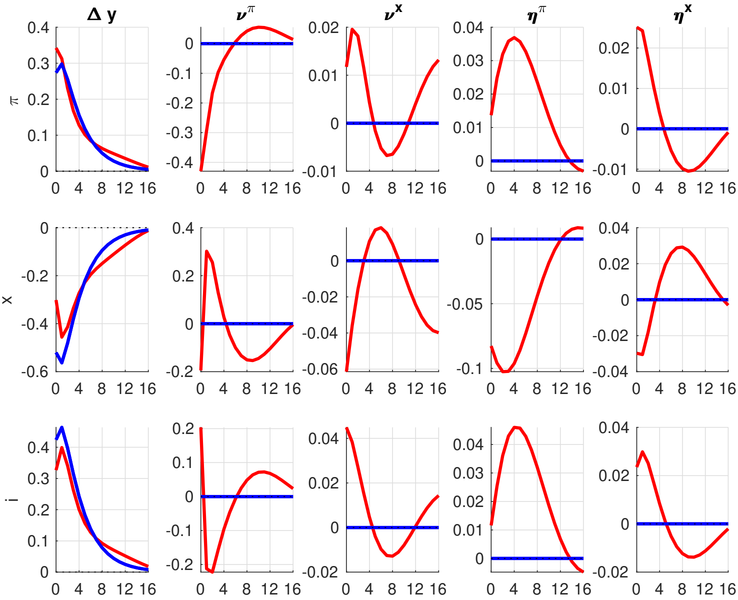
MULTIPLE EQUILIBRIA IN NK MODEL

IRF: Limited-information (red), full information (blue)



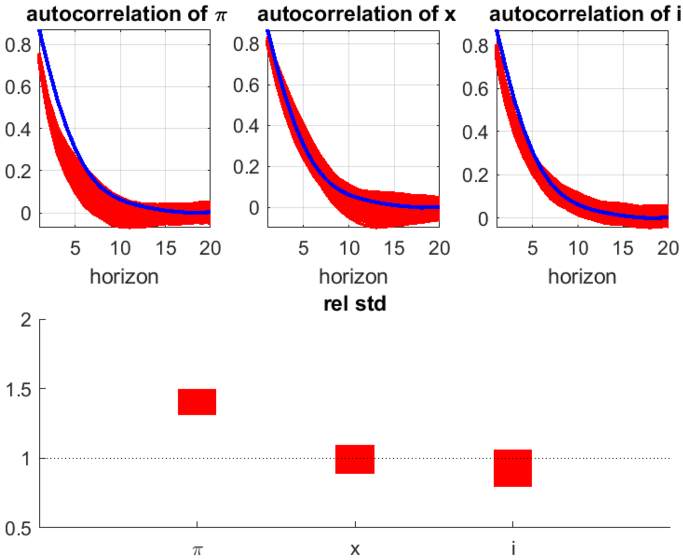
EXAMPLE EQUILIBRIUM IN NK MODEL

IRF: Limited-information (red), full information (blue)



SECOND MOMENTS

Limited-information (red), full information (blue)



AGENDA

1. A Simple Example Economy
2. General Framework
3. NK Model
4. **Determinacy Without Optimal Projections**
5. Conclusions

SIGNAL VS PROJECTIONS IN POLICY RULE

Simple example with $Z_t = \pi_t + \nu_t$

Alternative policy rule:

$$i_t = \phi Z_t = \phi(\pi_t + \nu_t)$$

Unique equilibrium for $|\phi| > 1$

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In our case, multiplicity arises due to endogenous attenuation of optimal projections

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SUMMARY

We simply rule out unbounded outcomes

- *no* unbounded nominal variables
- *no* switching between stable and unstable trajectories

Signal extraction adds only stable variables

- When steady-state Kalman filter exists, deviations between true values and projections are stationary
- Existence of steady state Kalman filter ensured by "detectability" and "stabilizability"

$\Rightarrow$ **Generic indeterminacy**

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$\Rightarrow$ **Generic indeterminacy**

Key element of our setup

Central bank cannot observe information set driving public's forward-looking decisions

KEY LESSONS FROM OUR PAPER

Informational frictions can give room for non-fundamental shocks to drive outcomes

Determinacy conditions depend on information structure

Consistency of beliefs between agents places bound on non-fundamental fluctuations

- **Empirical work:** Orphanides (2001) meets Lubik & Schorfheide (2004)
- Revisit **optimal policy problem**
- ... and **implementation** of desired equilibrium
- **Equilibrium selection**